A HANDBOOK

OF

ELECTRICAL TESTING.

BY

H. R. KEMPE,

MEMBER OF THE SOCIETY OF TELEGRAPH-ENGINEERS AND ELECTRICIANS;
ASSOCIATE MEMBER OF THE INSTITUTION OF CIVIL ENGINEERS.

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NOTE.

In the present Edition I have not only taken advantage, as far as possible, of the many friendly suggestions which have been made to me for the improvement of the original work, but have added a considerable amount of new matter, besides throughly revising the old. I have especially to thank Messrs. Elliott Brothers for the illustrations of apparatus which have been added. I am also greatly indebted to Messrs. W. T. Glover and Co. for permission to insert their valuable table (Table III.) of the Weights, Resistances, &c., of Pure Copper Wire.

H. R. K.

Engineer-in-Chief's Office,
Electrician's Department,
General Post Office.

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A

HANDBOOK

OF

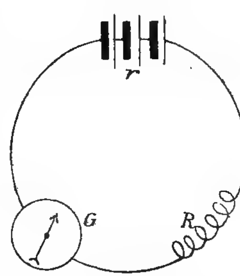
ELECTRICAL TESTING.

CHAPTER I.

SIMPLE TESTING.

1. In order to be able to make measurements of any kind, it is necessary to have certain standard units with which to make comparisons. For example, in the case of length, or weight, we have as standards the foot and the pound. Some of the units are dependent upon two of the other units; the unit of "work," for example, is the foot-pound, or the work done in raising a pound 1 foot high. Now in electrical measurements we require units of a like character. Those with which we have to deal chiefly are *electromotive force*, the unit of which is called the *volt*; *resistance*, the unit of which is the *ohm*; also we have the unit of *current*, which is dependent upon the two foregoing units, and which is called the *ampère*.

Fig. 1.



2. If the two poles of a battery be joined by a conductor a current will flow, and the strength of this current will vary directly as the electromotive force of the battery, and inversely as the total resistance in the circuit. This relation is known as "Ohm's law." If the electromotive force is expressed in volts and the resistance in ohms, then the resulting current will be in ampères.

3. Suppose now a battery of a resistance r and electromotive force E, a galvanometer of a resistance G, and a wire of a resistance R, be joined up in circuit, as shown by Fig. 1. By

the foregoing law, the strength of current C, which will flow out of the battery and through the galvanometer, will be

$$C = \frac{E}{R + r + G}.$$

The current, in flowing through the galvanometer, produces a deflection of its needle, which deflection will remain constant provided the electromotive force of the battery and also the resistances remain constant. If now R be a wire whose resistance we require to find, and which we can replace by another wire the value of whose resistance can be varied at pleasure, then by adjusting this latter so that the deflection of the galvanometer needle becomes the same as it was before the change of resistances was made, this resistance gives the value of our unknown resistance R.

This method of testing, known as the *substitution* method, although exceedingly simple, is a very good and accurate one if a little ordinary care be taken in making it. Its correctness is only limited by the sensibility of the galvanometer to small changes of strength in the current affecting it, and by the accuracy with which the variable resistance can be adjusted.

4. Next, suppose the galvanometer to have its scale so graduated that the number of divisions on it will, by the deflection of the needle, accurately represent the comparative strength (C) of currents which may pass through it. Let the battery, galvanometer, and resistance be joined up as at first, then, as before,

$$C = \frac{E}{R + r + G}; \text{ or, } E = C\,(R + r + G).$$

Now remove R, and insert any other known resistance ρ, in its place. Calling the new strength of current C_1, then

$$C_1 = \frac{E}{\rho + r + G}; \text{ or, } E = C_1\,(\rho + r + G).$$

But we have seen that $E = C\,(R + r + G)$, therefore

$$C\,(R + r + G) = C_1\,(\rho + r + G),$$

or

$$R + r + G = \frac{C_1}{C}\,(\rho + r + G),$$

that is

$$R = \frac{C_1}{C}\,(\rho + r + G) - (r + G). \qquad [1]$$

Now, as we have supposed the deflections of the galvanometer needle to be directly proportional to the strengths of current which produce them, we may, instead of C and C_1, write in our formulæ the deflections of the galvanometer needle which those strengths produce. Calling, then, a the deflection obtained with the strength C, and a_1 that with the strength C_1, our formula [1] becomes

$$R = \frac{a_1}{a}(\rho + r + G) - (r + G). \qquad [2]$$

In order to find R, it is necessary to know G, which is usually marked on the galvanometer by the manufacturer. r also must be known; but as it is difficult to determine its value accurately, it is best to use a battery whose resistance is very small in comparison with the other resistances in the circuit, and which may consequently be neglected, we may therefore write our formula

$$R = \frac{a_1}{a}(\rho + G) - G. \qquad [3]$$

Having then obtained a with R and a_1 with ρ, we can find the value of R.

For example.

With a galvanometer whose resistance was 100 ohms, and a battery whose resistance could be neglected, we obtained with a resistance R a deflection of 20 divisions (a), and with a resistance of 200 ohms (ρ) a deflection of 30 divisions (a_1). What was the unknown resistance R?

$$R = \frac{30}{20}(200 + 100) - 100 = 350 \text{ ohms.}$$

5. Next, suppose it is required to find the resistance of a galvanometer.

From equation [3], by multiplying up, we find that

$$R\,a = \rho\,a_1 + G\,a_1 - G\,a,$$

by arranging the quantities

$$G\,a_1 - G\,a = R\,a - \rho\,a_1,$$

or

$$G\,(a_1 - a) = R\,a - \rho\,a_1,$$

therefore

$$G = \frac{R\,a - \rho\,a_1}{a_1 - a}. \qquad [4]$$

If, then, with a known resistance R, we obtain a deflection of a divisions, and with a known resistance ρ we obtain a deflection of a_1 divisions, we can determine G.

For example.

With a galvanometer (G) and a battery whose resistance could be neglected, we obtained with a resistance of 350 ohms (R) a deflection of 20 divisions (a), and with a resistance of 200 ohms (ρ) a deflection of 30 divisions (a_1). What was the resistance of the galvanometer?

$$G = \frac{350 \times 20 - 200 \times 30}{30 - 20} = 100 \text{ ohms}.$$

6. Lastly, when the resistance of our battery is considerable, and it is required to find its value, from equation [2] by multiplying up, we find

$$R\,a = \rho\,a_1 + r\,a_1 + G\,a_1 - r\,a - G\,a,$$

by arranging the quantities

$$r\,a_1 - r\,a = R\,a - \rho\,a_1 - G\,a_1 + G\,a$$

or

$$r\,(a_1 - a) = R\,a - \rho\,a_1 - G\,(a_1 - a),$$

that is

$$r = \frac{R\,a - \rho\,a_1}{a_1 - a} - G. \qquad [5]$$

For example.

With a galvanometer whose resistance was 100 ohms (G), and a battery (r), we obtained with a resistance in circuit of 300 ohms (R) a deflection of 30 divisions (a), and with a resistance in circuit of 150 ohms (ρ) a deflection of 40 divisions (a_1). What was the resistance of the battery?

$$r = \frac{300 \times 30 - 150 \times 40}{40 - 30} - 100 = 200 \text{ ohms}.$$

7. The formulæ may be considerably simplified if we so adjust our resistances that one deflection becomes half the other,

or, in other words, if we make $\alpha = \frac{a_1}{2}$. Formula [3] for determining any resistance then becomes

$$R = \frac{a_1}{\frac{a_1}{2}}(\rho + G) - G = 2\,\rho + 2\,G - G.$$

that is

$$R = 2\,\rho + G.$$

8. Similarly we should find that formula [4] for determining the resistance of a galvanometer becomes

$$G = R - 2\,\rho\,;$$

and formula [5] for determining the resistance of a battery,

$$r = R - (2\,\rho + G)\,;$$

R being in all cases the resistance which gives the small deflection, and ρ being the smaller resistance which doubles it.

9. When the resistance we have to measure is very high compared with the resistance of the galvanometer and battery used for measuring, then in our equation

$$R = \frac{a_1}{a}(\rho + r + G) - (r + G),$$

we may practically, especially when great accuracy of measurement is not required, put G as well as r equal to 0, in which case,

$$R = \frac{a_1}{a}\,\rho.$$

To measure a resistance according to this formula, we should first join up, as shown by Fig. 1, our battery, galvanometer, and standard resistance, as it is called, which in our formula is ρ; and having noted the deflection a_1, should multiply the latter by ρ; this gives us what is called the *constant*. R (the resistance to be determined) is then inserted in the place of ρ; a new deflection α is obtained, by which we divide the constant, and thus get the value of R.

This method of measuring resistances is the one generally employed in taking tests for *insulation* resistance of telegraph lines, the standard resistance ρ being usually 1000 ohms.

When the insulation resistances of several lines are to be

measured, the constant would first be taken and worked out, and the several lines to be measured being inserted one after the other in the place of the resistance ρ, the deflections are noted; then the constant being divided by the several deflections, the resistances are thus obtained.

For example.

With a battery, a galvanometer, and a resistance of 1000 ohms (ρ) in circuit, we obtained a deflection of 20 divisions (α_1), then

$$\text{Constant} = 1000 \times 20 = 20000.$$

Taking away our resistance and inserting

Wire No. 1,	we obtained	a deflection of	5	divisions.
,, 2,	,,	,,	6	,,
,, 3,	,,	,,	12	,,
,, 4,	,,	,,	3	,,

The resistances of our wires would then be

$$\begin{aligned} \text{No. } 1,\ 20000 \div 5 &= 4000 \text{ ohms.} \\ ,,\ 2,\ 20000 \div 6 &= 3333\ \ ,, \\ ,,\ 3,\ 20000 \div 12 &= 1666\ \ ,, \\ ,,\ 4,\ 20000 \div 3 &= 6666\ \ ,, \end{aligned}$$

These results are the *total* insulation resistances of the wires, which may be of various lengths. To get comparative results, it is necessary to obtain the insulation resistance of some unit length of each wire, such as a mile.

Now, it will be readily seen that the greater the length of the wire the *greater* will be the leakage, and consequently the *less* will be the insulation resistance, or, in other words, this resistance will vary *inversely* as the length of the wire. To obtain, then, the insulation resistance, or insulation, as it is simply called, all we have to do is to multiply the *total* insulation by the length of wire. Thus, for example, if No. 1 wire was 100 miles long, its insulation per mile would be $4000 \times 100 = 400{,}000$ ohms. It is usual to fix a standard insulation per mile, and if the result is below that standard, the line is considered faulty. 200,000 ohms per mile is the standard adopted by the Postal Telegraph Department.

10. The rule of multiplying the total insulation by the mileage of the wire to get the insulation per mile, is not strictly correct, more especially for long lines, as it assumes that the leakage is the same at every point along the line. This, how-

ever, is clearly not the case, as a little of the current leaking out at one point leaves a smaller quantity to leak out at the next. In fact, we really measure the last portions of the line with a weaker battery than we do the first. The true law is, however, somewhat complex, and will be considered hereafter.*

11. We have hitherto considered the galvanometer deflections to be directly proportional to the currents producing them, but in no galvanometer is this the case if the deflections are measured in *degrees;* in such a case they are proportional to some function of those degrees, such as the *tangent.* Thus, if we were reading off the scale of degrees on a *tangent* galvanometer, that is to say a galvanometer in which the strengths of current are directly proportional to the *tangents* of the angles of deflection which those currents produce, we should have to find the tangents of those degrees of deflection before multiplying and dividing.

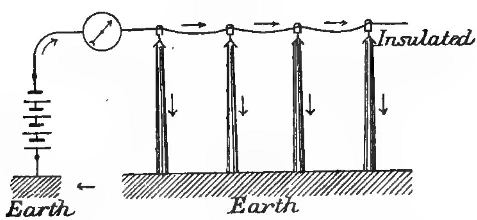
For example.

If with a tangent galvanometer we obtained with our standard resistance of 1000 ohms a deflection of 20°, and with the unknown resistance (R) a deflection of 15°, we should have

$$R = \frac{\tan 20^\circ \times 1000}{\tan 15^\circ} = \frac{\cdot 364 \times 1000}{\cdot 268} = 1358 \text{ ohms.}$$

When measuring the insulation resistance of a line of telegraph, having taken the constant, we should join up our instruments and line, as shown by Fig. 2. In making a measurement

FIG. 2.



of this kind, it is usual to have the positive pole of the battery to earth, so that a negative (zinc) current flows out to the line, as a zinc current will show best any defective insulation in the wire, a positive current having the effect, to a certain extent, of *sealing* a fault up, more especially if any underground work which may be in the circuit is defective.

* See Appendix.

The foregoing method of measurement is, as a rule, sufficiently accurate for all practical purposes. Greater accuracy may, however, be obtained with but little extra trouble by allowing for the resistance of our battery and galvanometer in the following manner:

Instead of multiplying the *constant* deflection by the 1000 ohms standard resistance, multiply it by 1000 plus the resistance of the galvanometer and battery, and having divided the result by the deflection obtained with the line wire in circuit, subtract from the result the resistance of the galvanometer and battery.

For example.

With a standard resistance of 1000 ohms, a tangent galvanometer of a resistance of 50 ohms, and a battery of a resistance of 100 ohms, we obtained a deflection of 30°, and with the line wire in circuit a deflection of 10°. What was the exact insulation resistance of the line?

$$\left.\begin{matrix}\text{Insulation}\\ \text{resistance}\end{matrix}\right\} = \frac{\tan 30^\circ\,(1000 + 50 + 100)}{\tan 10^\circ} - (50 + 100)$$

$$= \frac{\cdot 577 \times 1150}{\cdot 176} - 150 = 3760 \text{ ohms.}$$

When a large number of wires have to be measured for insulation daily, it is very convenient to have a table constructed on the following plan:

EARTH READINGS.

Constant Readings through 1000 ohms.

	1°	2°	3°	4°
20°	20852	10423	6945·0	5205·0
21°	21992	10993	7324·6	5489·5
22°	23146	11570	7709·3	5777·9
23°	24318	12155	8099·5	6070·2
24°	25507	12750	8495·5	6367·1

In this table the first vertical column represents the deflections in degrees obtained with a tangent galvanometer through a standard resistance of 1000 ohms, and the top row of degrees

are the deflections obtained with the line wire in circuit. The numbers at the points of intersection of a vertical with a horizontal column give the resistances corresponding to those deflections, these resistances being calculated from the formula

$$\left.\begin{array}{l}\text{Insulation}\\\text{resistance}\end{array}\right\} = \frac{\tan \textit{constant reading} \times 1000}{\tan \textit{earth reading}}.$$

Thus the *constant* deflection, or reading, with the 1000 ohms standard resistance being 22°, and the deflection with the line wire (the earth reading) being 2°, the resistance required is seen at a glance to be 11,570 ohms.

Before proceeding to the more intricate systems of measurement, we will consider some of the instruments which would be used in making measurements such as we have described.

CHAPTER II.

RESISTANCE COILS.

12. THE essential points of a good set of resistance coils are, that they should not vary their resistance appreciably through change of temperature, and that they should be accurately adjusted to the standard units, which adjustment ought to be such that not only should each individual coil test according to its marked value, but the total value of all the coils together should be equal to the numerical sum of their marked values. It will be frequently found in imperfectly adjusted coils that although each individual coil may test, as far as can be seen, correctly, yet when tested all together their total value will be one or two units more or less than the sum of their individual values; because, although an error of a fraction of a unit may not be perceptible in testing each coil individually, yet the accumulated error may be comparatively large.

The wire of the coils is, as a rule, of German silver, the specific resistance of which metal is but little affected by variations of temperature. The wire is usually insulated by two coverings of silk, and is wound double on ebonite bobbins, the object of the double winding being to eliminate the extra current which would be induced in the coils if the wire were wound on single. By double winding, the current flows in two opposite directions on the bobbin, the portion in one direction eliminating the inductive effect of the portion in the other direction. When wound, the bobbins are saturated in hot paraffin wax, which thoroughly preserves their insulation, and prevents the silk covering from becoming damp, which would have the effect of partially short circuiting the coils and thereby reducing their resistance.

The small resistances are made of thick wire, the higher ones of thin wire to economise space.

When bulk and weight are of no consequence, it is better to have all the coils made of thick wire, more especially if high battery power is used in testing, as there is less liability of the coils to become heated by the passage of the current through them.

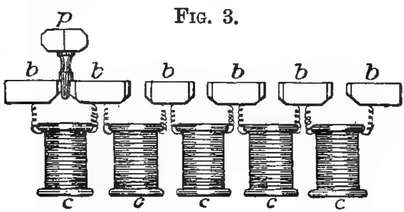
The individual resistances of a set of coils are generally of

such values that, by properly combining them, any resistance from 1 to 10,000 can be obtained. One arrangement in general use has coils of the following values: 1, 2, 2, 5, 10, 10, 20, 50, 100, 100, 200, 500, 1000, 1000, 2000, 5000 ohms.

These numbers enable any resistance from 1 to 10,000 to be obtained, using a minimum number of coils without fractional values. With these numbers, however, it is a matter of some little difficulty to see at once what coils it is necessary to put into circuit in order to obtain a particular resistance; and as it is often necessary to be quick in changing the resistances, the following numbers are frequently used: 1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400, 1000, 2000, 3000, 4000, which enables any particular resistance, that is required to be inserted, to be seen almost at a glance.

The way in which the different coils are put in circuit is shown by Fig. 3. The ends of the several resistances, *c*, *c*, *c*, · · · · are inserted between the brass blocks, *b*, *b*, *b*, · · · · . Any of the

FIG. 3.



coils can then be cut out of the circuit between the first and last blocks, by inserting plugs, *p*, as shown, which short-circuit the coils between them; thus, if all the plugs were inserted, there would be no resistance in circuit, and if all the plugs are out, all the coils would be in circuit.

13. There are various ways of arranging the coils in sets; one of the most common is that shown in outline by Fig. 4, and in general view by Fig. 5. This form is much used in submarine cable testing. The brass blocks, here shown in plan, are screwed down to a plate of ebonite which forms the top of the box in which the coils are enclosed. The ebonite bobbins are fixed to the lower surface of the ebonite top, the ends of the wires being fixed to the screws which secure the brass blocks. The holes shown in the middle of the brass blocks are convenient for holding the plugs that are not in use.

It will be seen that six terminals, A, B, C, D, E, F, are provided; when we only require to put a resistance in circuit, the two

terminals D and E would be used. The use of the other terminals and of the movable brass strap S, will be explained hereafter.

FIG. 4.

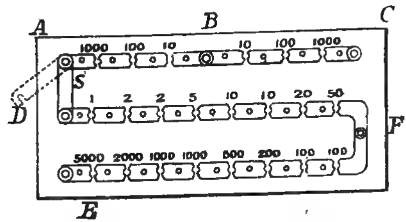
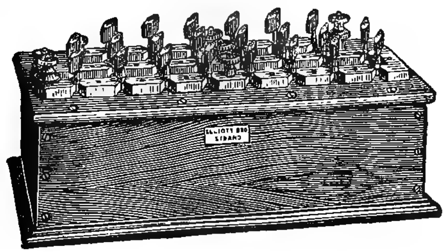


FIG. 5.



14. In using a set of resistance coils, one or two precautions are necessary.

First of all, it should be seen that the brass shanks of the plugs are clean and bright, as the insertion of a dirty plug will not entirely short-circuit the coil it is intended to cut out. It is a good plan, before commencing to test, to give the plugs a scrape with a piece of glass or emery paper, taking care to rub off any grains of grit which may remain sticking to them after this has been done.

When a plug is inserted, it should not be simply pushed into the hole, but a twisting motion should be given it in doing so, so that good contact may be ensured; too much force should not be used, as the ebonite tops may be thereby twisted off in extracting the plugs. Care also should be taken that the neighbouring plugs are not loosened by the fingers catching them during the operation of shifting a plug.

Before commencing work it is as well to give all the plugs a twist in the holes, so as to ensure that none of them are left loose. On no account must the plugs be greased to prevent

their sticking, and their brass shanks should be touched as little as possible with the fingers.

15. For taking the insulation resistance of a line in the manner described in the last chapter, such an elaborate set of coils is not of course wanted. A single coil of a resistance of 1000 ohms in a box with two terminals, to which the ends of the coils are attached, is all that is required.

16. One of the most useful sets of coils for general purposes is represented in outline by Fig. 6, and in general view by Fig. 7. The general arrangement of resistances, it will be seen, is the same as that shown by Fig. 4. Two keys, however, are provided (drawn in Fig. 6 in elevation, for distinctness), the contact point of the left-hand key being connected, as shown by the dotted line, with the middle brass block of the upper set of resistances, the terminal B′ at the end of the key corresponding, in fact, when the key is pressed down, with the terminal B shown in Fig. 4. In like manner the terminal A′ corresponds with the terminal A. In the place of the movable piece of

FIG. 6.

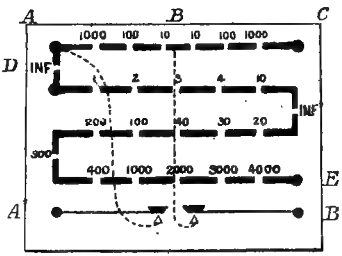
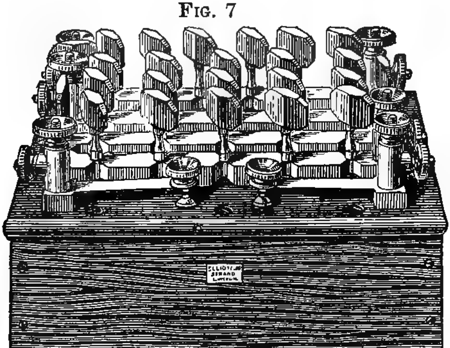


FIG. 7

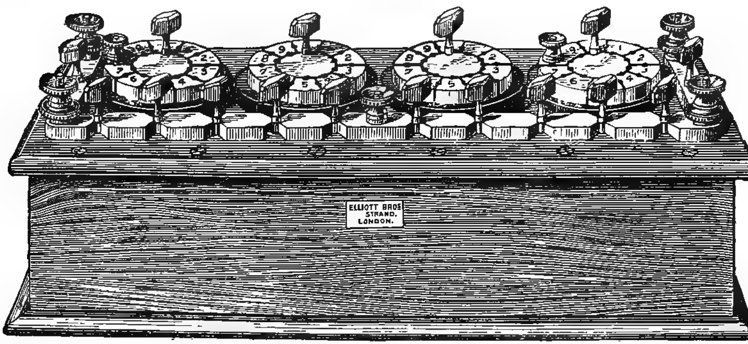


brass between A and D, a plug marked INF. (infinity) is provided, which answers the same purpose; an infinity plug is also placed at the first bend of the coils on the right hand of the figure.

When we require simply to insert a resistance in a circuit, we should use the terminals A′ and E, the left-hand key being pressed down when the deflection of the galvanometer needle is to be noted. The current can thus be conveniently cut off or put on when required, by releasing or depressing the key. Care should be taken that the two *infinity* plugs are firmly in their places, to ensure their making good contact. For the same purpose the key contacts should be occasionally touched with emery paper or a fine file.

Another set of coils is represented in general view by Fig. 8; these will be referred to hereafter (Chapter VIII.).

FIG. 8.

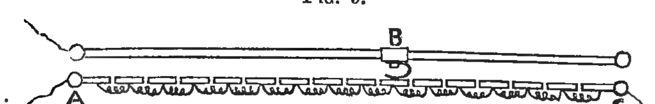


SLIDE RESISTANCE COILS.

17. Fig. 9 shows the principle of this method of arranging Resistance Coils.

The coils, which are generally all of equal value, are fixed between brass blocks, as in Fig. 3, but instead of plugs being inserted between them to cut the various coils out of circuit, a

FIG. 9.



sliding piece, B, is provided which can be moved along a rod with which it is in connection. The slider has a spring fixed to it which presses against the brass blocks; it is evident, then,

that any required resistance can be inserted between A and B, that is between A and a terminal fixed to the end of the rod, by simply sliding the piece B along the rod.

The object of arranging the coils in this manner is more particularly to enable the ratio of A B to B C to be varied, whilst the sum of the two, that is to say the whole length, A C, remains constant. This is sometimes required to be done.

These coils are sometimes arranged in a circle instead of a straight line, the contact-piece B being a spring forming a radius of the circle. This is a very compact and useful plan.

18. For some tests a long straight wire of German silver or other alloy is employed in the place of the resistance coils. It is important that this wire should be made of a perfectly uniform alloy, and should be of the same diameter throughout, so that its resistance may be directly proportional to its length; thus if the slider were at the middle point of the wire, the resistance on each side should be exactly the same.

If, as is sometimes the case, it is required to use a long wire of this kind, it would be inconvenient to have it straight; in such a case, therefore, the wire is wound spirally on a cylinder of ebonite or other insulating material, the two ends being connected to the metal axes, these latter being in connection with terminals. The sliding contact-piece is moved along parallel with the axes of the cylinder by a screw which gears with the cylinder, and which is therefore revolved by the handle which turns the latter; the contact of the slider with the wire is made when required by pressing the former with the finger. The arrangement, in fact, is a modified form of Jacobi's Rheostat.

19. It is evident that a much finer adjustment of resistance can be obtained by the slide wire than by the slide resistance coils, but inasmuch as the length of the wire and the smallness of its diameter must be limited, it does not admit of very large variations of resistance being obtained. By combining, however, a slide-wire resistance with plug resistance coils, this difficulty can be got over, though in tests which we shall describe it is preferable to use the slide coils.

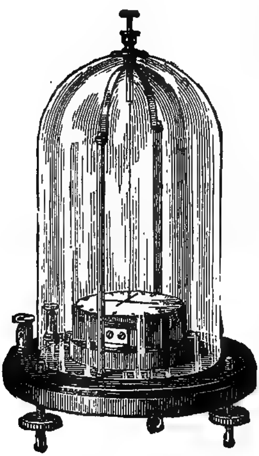
20. Slide resistance coils, though very convenient, are not absolutely necessary for varying the ratio of the resistances in the manner described; for it is evident that A B and B C could be two sets of resistance coils in which, to adopt the slide resistance principle, the resistances would have to be increased in one set and diminished in the other, or *vice versâ*, care being taken that the same resistance is added in one set as is taken out in the other.

CHAPTER III.

GALVANOMETERS.

21. For the class of tests in which it is required, by adjusting certain resistances, either to bring the needle to zero, or to the same deflection in making two measurements, as described on pages 1 and 2, a galvanometer having its scale graduated to degrees would be sufficient. It should be provided with an astatic pair of needles, suspended by a cocoon fibre, whose end is attached to a piece of metal connected to a screw, by the twisting of which the needles can be lowered down on to the coil, so that when it is not in use there would be no danger of the fibre being fractured by moving the instrument about. Such an instrument is shown by Fig. 10.

Fig. 10.



When the galvanometer is to be used it should be placed on a firm table, and the screw connected to the fibre turned until the needles swing clear of the coil. The instrument should then be placed in such a position that the top needle stands as nearly as possible over the zero points. It should next be carefully levelled by means of the levelling screws attached to its base, until the metal axis which connects the two needles together is exactly in the centre of the hole in the scale-card through which it passes.

The adjustment of the needles to zero is much facilitated in the instrument by making the coil movable about the centre of the scale-card by means of a simple handle attached direct to the coil. The final touch can thus be given without shaking the needles, which would render exact adjustment difficult.

In some galvanometers there is a scale graduated to degrees attached to the coil, so that the angle through which it is turned can be seen if required. This scale is employed when using the instrument as a *Sine* galvanometer.

The Sine Galvanometer.

22. We before stated that the strengths of currents producing certain deflections are not directly proportional to those deflections, but to some function of them, such as the *tangent*. In measuring strengths of currents by means of a *sine* galvanometer we proceed as follows:—

The needle is first adjusted to zero. The current whose strength is to be measured is then allowed to flow, and a deflection of the needle produced. The *coil* is now turned round; this causes the needle to diverge still more with respect to the stand of the instrument, but the angle which it makes with the coil becomes less the farther the latter is turned, and finally a point is reached at which the needle is again parallel to the coil—that is, its ends are again over the zero points on the scale-card. The reason of this is, that the deflective action of the coil on the needle is always the same, provided the current strength does not vary, but the farther the needle moves from the magnetic meridian, the greater becomes its tendency to return to that meridian, and finally, when the needle becomes parallel to the coil, the deflective force of the latter exactly balances the reactive force of the earth's magnetism.

The strength of the current which produces the deflection of the needle will then be directly proportional to the *sine* of the angle through which the coil is turned.

The sine galvanometer is a very accurate instrument if properly managed, its results being entirely independent of the shape of the coil, size of the needle, &c. The only precaution necessary is to see that when the needle is at zero at starting it is brought back exactly to zero. Indeed it is not absolutely necessary that the starting point be zero,—the law of the *sines* holds good if the needle be at, say, 5° when commencing, but in this case, by the turning of the coil, the needle must be brought back to 5°, and not to zero.

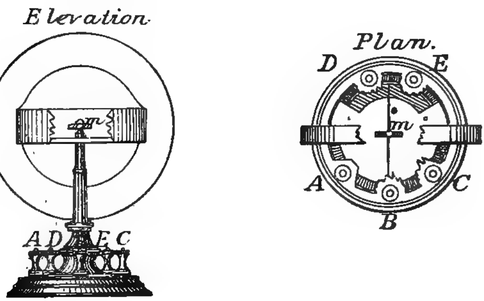
The Tangent Galvanometer.

23. The tangent galvanometer, which is perhaps the most useful and convenient instrument for general purposes, consists essentially of coils of wire wound in a deep groove in the circumference of a circular ring, usually of brass, about 6 or 7 inches in diameter, with a small magnetic needle at its centre moving over a graduated circle. To render the instrument correct, it is necessary that the length of the needle be small compared

with the diameter of the coil; about three-quarters of an inch for a 6 or 7-inch ring is a convenient size, and gives sufficiently accurate results for all practical purposes.

Fig. 11 shows a very convenient form of this galvanometer, a modification of which is used by the Postal Telegraph Department. The magnet *m* has a long pointer, of gilt copper, about

Fig. 11.



5 inches long, fixed at right angles to it, so that its ends point to the zero of the scales placed at each end of it, when the magnetic needle is parallel to the coil.* The magnet is so placed that its central point is at the axis of the coils and also in the same plane with them.

The object of having the needle small, as compared with the diameter of the coil, is to ensure, as far as possible, the magnetic influence of the current on the needle being the same at whatever angle the needle may be with respect to the coil. Theoretically to effect this result, the magnet should be a mere point, but this is of course impossible, and the error is not great when the coil is eight or ten times as large in diameter as the length of the needle. Upon the influence of the coil on the needle being the same, whatever angle the needle takes up with respect to it, depends the truth of the proposition, that the *strengths of currents circulating in the coil are proportional to the tangents of the angles of deviation of the needle.*

As it would be difficult to read off the angular deflection of the needle from the needle itself, on account of the shortness of the latter, a long pointer is attached, which moves over the graduated scales under its extremities.

* Pieces of the instrument are broken away to show the different parts more clearly.

24. One of these scales is graduated to true degrees, and the other to numbers proportional to the tangents of those degrees, so that if we read off two deflections from the degrees scale, the other extremity of the pointer will indicate, approximately, numbers proportional to the tangents of those two degrees of deflection.

Now as the strengths of currents producing certain deflections are proportional to the tangents of the degrees of deflection, if we read off from the degrees scale we must, as we have explained in Chapter I. (§ 11), reduce the degrees to tangents, from a table of tangents,* before working out a formula which has reference to the strengths of currents. If, however, we read off from the tangent scale, no reduction is necessary, and the numbers can be at once inserted in the formula. When time is no object it is better to use the degrees scale, as it is the more accurate of the two, and moreover it is the easier one to read from in consequence of the division marks being all equally wide apart; on the tangent scale the divisions representing the higher numbers are very close together and are difficult to distinguish.

To avoid parallax error, in consequence of the needle being elevated above the scale, a piece of looking-glass is fixed close to the tangent scale, so that when we look at the end of the needle and see that the reflected image of the pointer coincides with the pointer itself, we know that we are looking at the end of the pointer perpendicularly with the scale.

As the instrument is generally only provided with a looking-glass near the tangent scale, it is necessary when reading off from the degrees scale to run the eye along the pointer to the looking-glass end, and see whether the reflected image corresponds with the pointer at that end; if it does, we may be sure that, when we look at the degrees scale, we do so correctly.

25. It will be seen from the figure that the instrument is provided with five terminals, A, B, C, D, and E.

The two terminals D and E are attached to the ends of a thick wire, making two or three turns only round the ring, and having practically no resistance. To the terminals A and B are attached the ends of a finer wire, about No. 24 or 30 B.W.G., making a larger number of turns, and having altogether a resistance of about 25 ohms. A similar wire is also attached to the terminals B and C, so that if we use the terminals A and B, or B and C, we have 25 ohms in circuit; and, if we use terminals A and C, a resistance of 50 ohms is in circuit.

The object of having three different coils is to enable us to work with strong or weak currents; thus, with a strong current,

* Table I.

we should use the terminals D and E, as there being only a few turns of wire the effect on the needle would not be great. With a weaker current we should use the terminals A and B, or B and C, and with a still weaker one the terminals A and C. The proper terminals to use in making any particular measurement are best found by experiment, the different terminals being tried until a convenient deflection is obtained. If it is found that one set of terminals gives too high and another too low a deflection, the battery power must be varied, or resistance inserted in the circuit. Experience only can determine what it is best to do in different experiments, as no satisfactory rule can be laid down.

26. Before using the galvanometer it should be seen that the pointer has not become bent, but stands at right angles to the magnet, and that when suspended it turns freely. On no account should the magnet suspension be oiled, as quite the opposite effect to what is intended will be produced by so doing. Care should be taken that the scale is in its proper position, so that when the two ends of the pointer are over the zero points, the pointer stands at right angles to the coils. The correct setting of the position of the scale with reference to the coil is a mechanical adjustment which when once properly effected is not likely to alter; it is, however, as well to verify its correctness, by means of a set square, before the instrument is brought into general use. The pointer attached to the magnetic needle is very subject to accident, and is often badly adjusted. The correctness of the setting can be verified by placing the galvanometer so that the pointer stands at zero, and then sending a current through the coil first in one direction and then in the other. The deflections on either side of zero in this case should be equal; if they are not, the position of the pointer relative to the needle should be readjusted until the required equality of deflections on either side of zero is obtained. Care should be taken when making this adjustment, to place the instrument clear of any unequally distributed masses of iron, otherwise unequal deflections may be obtained although the pointer and magnet are correctly set.

Angle of Maximum Sensitiveness.

27. In using the tangent galvanometer it is always as well to avoid obtaining either very high or very low deflections. The reason of this is, that any small change in the strength of a current traversing the galvanometer will produce the greatest effect on the needle when the latter stands at some deflection

which is neither very high nor very low. The galvanometer is, in fact, most sensitive when the needle points, under the influence of a current, at that deflection.

Thus, for example, suppose we had a current which produced a deflection of 5°, and this current was increased say by $\frac{1}{10}$th, then the deflection would be increased to 5° 30′, because

$$\tan 5^\circ : \tan 5^\circ\,30' :: 1 : 1\tfrac{1}{10}.$$

Next suppose the needle stood at 80°, and the current was, as before, increased by $\frac{1}{10}$th, then the deflection would be increased to 80° 54′, for

$$\tan 80^\circ : \tan 80^\circ\,54' :: 1 : 1\tfrac{1}{10}.$$

Lastly, let us suppose the needle stood at 43°, then by the increase in the current the deflection would have changed to 45° 43′, for

$$\tan 43^\circ : \tan 45^\circ\,43' :: 1 : 1\tfrac{1}{10}.$$

In the first case then, when the deflection was low, the increase was

$$5^\circ\,30' - 5^\circ = 30';$$

in the second case, when the deflection was high,

$$80^\circ\,54' - 80^\circ = 54';$$

and in the third case, when the deflection was of a medium value,

$$45^\circ\,43' - 43^\circ = 2^\circ\,43'.$$

What, then, is the deflection at which this increase is greatest?

The point to be determined is, what deflection is increased most by any small alteration in the current producing that deflection?

If C be a current giving a deflection of α°, and C_1 a current a little stronger, say, which increases this deflection to β°, we have to find what value given to α°, makes $\beta^\circ - \alpha^\circ$ as large as possible when C and C_1 are very nearly and ultimately equal.

We have

$$C : C_1 :: \tan \alpha^\circ : \tan \beta^\circ,$$

therefore

$$\tan \beta^\circ = \frac{C_1}{C} \tan \alpha^\circ.$$

Now we have to make $\beta^\circ - \alpha^\circ$ a maximum, supposing that the foregoing equation holds good.

Since $\beta^\circ - \alpha^\circ$ is to be a maximum, $\tan(\beta^\circ - \alpha^\circ)$ must also be a maximum, but

$$\tan(\beta^\circ - \alpha^\circ) = \frac{\tan\beta^\circ - \tan\alpha^\circ}{1 + \tan\beta^\circ \tan\alpha^\circ} = \frac{\frac{C_1}{C}\tan\alpha^\circ - \tan\alpha^\circ}{1 + \frac{C_1}{C}\tan^2\alpha^\circ}$$

$$= \frac{\frac{C_1}{C} - 1}{\frac{1}{\tan\alpha^\circ} + \frac{C_1}{C}\tan\alpha^\circ}.$$

We have then to find what value of $\tan\alpha^\circ$ makes this fraction a maximum, and this we shall do by finding what value makes the denominator of the fraction a minimum. Now

$$\frac{1}{\tan\alpha^\circ} + \frac{C_1}{C}\tan\alpha^\circ = \left(\frac{1}{\sqrt{\tan\alpha^\circ}} - \sqrt{\frac{C_1}{C}\tan\alpha^\circ}\right)^2 + 2\sqrt{\frac{C_1}{C}},$$

and this will be a minimum when

$$\frac{1}{\sqrt{\tan\alpha^\circ}} - \sqrt{\frac{C_1}{C}\tan\alpha^\circ} = 0,$$

that is, when

$$1 = \sqrt{\frac{C_1}{C}\tan^2\alpha^\circ}, \quad \text{or,} \quad \tan\alpha^\circ = \sqrt{\frac{C}{C_1}}.$$

but as C_1 and C are ultimately equal, $\frac{C}{C_1}$ becomes equal to 1, therefore,

$$\tan\alpha^\circ = \sqrt{1} = 1 = \tan 45^\circ.$$

28. We see then that in order to make the tangent galvanometer as sensitive as possible, we should obtain the deflection of its needle as near 45° as possible; 45° is in fact the *angle of maximum sensitiveness.*

Every galvanometer has an angle of maximum sensitiveness, although it is not the same in all. It can, however, be found experimentally (see "Calibration of Galvanometers," p. 30), and should be marked on the instrument for future reference.

29. If we require to adjust two currents in two different measurements so that they should be equal in both cases, it is evident that the needle of the galvanometer employed to measure them should in each case show the same deflection. In making the two measurements, we take the deflection obtained by one current as the standard, and then in making the second measurement we adjust the current until the same deflection is obtained. Now the accuracy with which this current can be adjusted depends upon the sensitiveness of the galvanometer to a change in the strength of the current, and we have seen that this sensitiveness is at a maximum when the deflection is 45°. If, therefore, we employ a tangent galvanometer for such a test as that just mentioned, we should endeavour in both measurements to bring the needle to 45°.

30. In what way can the property of the galvanometer be taken advantage of when comparing two deflections?

We must in such a case endeavour to obtain both deflections as near 45° as possible. To do this we should have to get one deflection on one side, and the other deflection on the other side, of 45°. But then the question arises, should we get the deflections at an equal distance on either side, or one closer to the 45° than the other, and if so, should the higher or the lower deflection be the closer of the two?

Now a little consideration will make it clear that if the two deflections in question are taken either near 0° or 90°, they will be much closer together than if they were taken near 45°, for the reason that the tangents of high or low deflections differ more widely from one another than do the tangents of medium deflections. But we have shown that when deflections are high or low, any increase or decrease in the strength of the current producing those deflections has less effect than when the deflections are of a medium value. It is therefore evident that it is most advantageous to get the deflections as wide apart as possible.

Let then $\tan \theta^\circ$ represent the stronger, and $\tan \phi^\circ$ the weaker current, and let one current be n times as strong as the other. We then have to find what values of θ° and ϕ° make

$$\theta^\circ - \phi^\circ$$

a maximum, supposing that

$$\tan \theta^\circ = n \tan \phi^\circ.$$

If in the last investigation we substitute θ° for β°, ϕ° for α°, and n for $\frac{C_1}{C}$, we can see that in order to get the required result

we must make

$$\tan \phi^\circ = \frac{1}{\sqrt{n}},$$

and, since $\tan \theta^\circ = n \tan \phi^\circ$,

$$\tan \theta^\circ = \frac{n}{\sqrt{n}} = \sqrt{n}.$$

If one current strength is to be twice as great as the other, then $n = 2$; consequently,

$$\tan \theta^\circ \doteq \sqrt{2} = 1 \cdot 41421 = \tan 54^\circ\ 44' = \tan 54\tfrac{3}{4}^\circ,$$

and

$$\tan \phi^\circ = \frac{1}{\sqrt{2}} = \cdot 70711 = \tan 35^\circ\ 16' = \tan 35\tfrac{1}{4}^\circ.$$

These then are the deflections that theoretically it is best to obtain in making a test with a tangent galvanometer in which one current is to be twice as strong as the other. But practically we may make the deflections 55° and $35\frac{1}{2}^\circ$, as these are more convenient to adjust to, and $\tan 55^\circ$ is, within $1'$, exactly double $\tan 35\frac{1}{2}^\circ$.

If we examine the theoretical deflections $54^\circ\ 44'$ and $35^\circ\ 16'$ it will be seen that

$$54^\circ\ 44' - 45^\circ = 9^\circ\ 44',$$

and

$$45^\circ - 35^\circ\ 16' = 9^\circ\ 44',$$

or in other words, the angular deflections on either side of 45° are in this case the same. Let us then see whether they are so when n has any value other than 2.

The angular deflection between 45° and θ° will be

$$\theta^\circ - 45^\circ,$$

that between 45° and ϕ°,

$$45^\circ - \phi^\circ,$$

now $\tan(\theta^\circ - 45^\circ) = \dfrac{\tan \theta^\circ - 1}{1 + \tan \theta^\circ}$,

and $\tan(45^\circ - \phi^\circ) = \dfrac{1 - \tan \phi^\circ}{1 + \tan \phi^\circ}$

but we know, since

$$\tan\theta^\circ = \sqrt{n} \text{ and } \tan\phi^\circ = \frac{1}{\sqrt{n}},$$

that

$$\tan\phi^\circ = \frac{1}{\tan\theta^\circ},$$

that is

$$\tan(45^\circ - \phi^\circ) = \frac{1 - \dfrac{1}{\tan\theta^\circ}}{1 + \dfrac{1}{\tan\theta^\circ}} = \frac{\tan\theta^\circ - 1}{1 + \tan\theta^\circ},$$

that is

$$\tan(45^\circ - \phi^\circ) = \tan(\theta^\circ - 45^\circ),$$

or

$$45^\circ - \phi^\circ = \theta^\circ - 45^\circ,$$

showing that these angular deflections *are* the same whatever be the value of n.

This is a very useful fact, as it shows that when we are making a test in which two deflections are involved whose relative values are unknown, we should so adjust the resistances, &c., that the deflections are obtained, as near as possible, at equal distances on either side of 45°.

To sum up, then, we have

Best Conditions for using the Tangent Galvanometer.

31. When a test is made in which only one deflection is concerned, that deflection should be as near 45° as possible.

If there are two deflections to be dealt with, these should be as nearly as possible at equal distances on either side of 45°. If one of these deflections is to be double the other, 55° and $35\frac{1}{2}$° are the most convenient to employ.

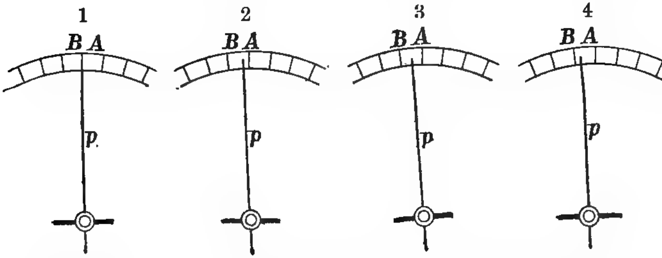
METHOD OF READING GALVANOMETER DEFLECTIONS.

32. The reading of galvanometer deflections requires considerable method, in order that accurate results may be obtained in making measurements.

Let A and B (Fig. 12) be two contiguous division marks on the galvanometer scale. Now, by observation, we can always determine without difficulty whether the pointer lies exactly over A or over B, or whether it lies exactly midway between the two; and further, if it does not occupy either of these exact positions, we can judge without difficulty whether it lies nearest to A or to B. This is equivalent to saying that we can be certain of

the magnitude of the deflection within a *quarter* of a degree. Thus supposing the pointer stood between A and B, but nearer to A than to B, then we should call the deflection "A$\frac{1}{4}$," and supposing the deflection was actually *very* nearly equal to A,

Fig. 12.



Deflection = A. Deflection = A$\frac{1}{4}$. Deflection = A$\frac{1}{2}$. Deflection = A$\frac{3}{4}$.

then A$\frac{1}{4}$ would be a quarter of a division, or degree, too large; if, on the other hand, the deflection was *very* nearly equal to A$\frac{1}{2}$, then A$\frac{1}{4}$ would be a quarter of a division, or degree, too low. In one case the error would be a plus one, and in the other a minus one, but in either case its *maximum* value would be $\frac{1}{4}$ only. We have, in fact, the rule that if A be the smaller of two contiguous deflections A and B, then when the pointer is exactly over A, the deflection should be called "A": if nearer to A than to B, then it should be called "A$\frac{1}{4}$"; if exactly midway between A and B, it should be called "A$\frac{1}{2}$"; and lastly, if the pointer is nearer to B than to A, then the deflection should be called "A$\frac{3}{4}$"; thus, for example, if A and B (Fig. 12) were the 57° and 58° division marks respectively on the scale; then in case 1 the deflection would be taken as 57°, in case 2 the deflection would be taken as 57$\frac{1}{4}$°; and again, in cases 3 and 4 the deflections would be taken as 57$\frac{1}{2}$° and 57$\frac{3}{4}$° respectively. By keeping to these instructions, then, we can be sure of the magnitude of a deflection within $\frac{1}{4}$ of a division or degree.

33. If we are making a measurement with a tangent galvanometer and we read from the *degrees* scale, and if we have two deflections to deal with, one of which is to be a proportional part of the other (usually one-half), then after the first deflection has been observed it has to be reduced to a tangent,* and then the latter being divided, say, by two, the corresponding deflection is ascertained from the tangent table; the resistances, &c., are then adjusted till the required second deflection is as

* Table I.

nearly as possible obtained. If we find that the halved tangent does not exactly correspond to a deflection in the table, then we must take, say, the nearest deflection *below* the exact value, and then take care to adjust so that the deflection of the pointer is a little *above* that angle. Thus suppose the first deflection was 58°, then the tangent of 58° is $1 \cdot 6003$, and $\frac{1 \cdot 6003}{2} = \cdot 8001$; now the nearest number below this in the table is $\cdot 7954$, which is the tangent of $38\frac{1}{2}°$; in adjusting the deflection, therefore, we should take care that we get it rather *more* than $38\frac{1}{2}°$.

Degree of Accuracy attainable in reading Galvanometer Deflections.

34. If the galvanometer scale be so graduated that the number of *divisions* of deflection directly represent the proportionate strengths of the currents producing those deflections, then an error of, say, $\frac{1}{m}$th of a division in d divisions will represent a percentage error, γ, in the strength of the current represented by d, which is given by the proportion

$$\gamma : \tfrac{1}{m} :: 100 : d,$$

or

$$\gamma = \frac{\frac{1}{m} \times 100}{d} \text{ per cent.} \qquad \text{[A]}$$

If, however, the instrument be a tangent galvanometer and the deflection be read from the *degrees* scale, then an error of $\frac{1°}{m}$ in $d°$ will not represent an error of $\frac{\frac{1°}{m} \times 100}{d°}$ per cent.; for in this case we must have the proportion

$$\gamma_0 : \tan d\tfrac{1°}{m} - \tan d° :: 100 : \tan d°,$$

or

$$\gamma_0 = \frac{(\tan d\frac{1°}{m} - \tan d°)\,100}{\tan d°} = \left(\frac{\tan d\frac{1°}{m}}{\tan d°} - 1\right) 100 \text{ per cent.} \qquad \text{[B]}$$

For example.

If the deflection d were 46 *divisions*, then $\frac{1}{4}$ of a division error $(\frac{1}{m})$ would be an error, γ, of

$$\gamma = \frac{\frac{1}{4} \times 100}{46} = \cdot 54 \text{ per cent.}$$

in the current strength represented by the deflection d; but if the deflection were 46°, then $\frac{1}{4}$° error would be an error, γ_o, of

$$\gamma_o = \left(\frac{\tan 46\frac{1}{4}^\circ}{\tan 46^\circ} - 1\right) 100 = \left(\frac{1 \cdot 0446}{1 \cdot 0355} - 1\right) 100 = \cdot 88 \text{ per cent.}$$

in the current strength.

35. In cases where we have two deflections to deal with, one of which, or the tangent of one of which, has to be $\frac{1}{n}$th (usually $\frac{1}{2}$) of the other, then after we have ascertained, as accurately as we can judge, the magnitude of the first deflection, d, the latter (or the tangent of the latter) is divided by n, and then the resistances, &c., in the circuit of the galvanometer are adjusted until the deflection $\frac{d}{n}$ $\left(\text{or the deflection corresponding to } \frac{\tan d}{n}\right)$ is obtained as accurately as possible. Now in adjusting to this latter deflection we are liable to make a plus or minus error of $\frac{1}{m}$th of a division or degree as in the first case, and as $\frac{d}{n}$ $\left(\text{or} \tan \frac{d}{n}\right)$ may itself contain an error due to d being $\frac{1}{m}$th of a division or degree wrong in the first instance, the new deflection may be more than $\frac{1}{m}$th of a division or degree out. What then is the "total possible percentage of error which may exist in the second deflection"?

Now the *absolute* error which may be made in the two deflections must be the same in both cases, viz. $\frac{1}{m}$, but the *percentage* value of the latter will be directly proportional to the value of the deflections; thus a $\frac{1}{4}$ division error in 50 divisions is a $\frac{1}{2}$ per cent. error, but a $\frac{1}{4}$ division error in 25 divisions is a 1 per cent. error; in fact, if γ be the *percentage* error (corresponding to the *absolute* error $\frac{1}{m}$) in d divisions, then $n \gamma$ will be the percentage error (corresponding to the absolute error $\frac{1}{m}$) in $\frac{d}{n}$ divisions. Now if d contains a *percentage* error γ, then $\frac{d}{n}$ must also contain a *percentage* error γ; consequently if we make a percentage error of $n \gamma$ in $\frac{d}{n}$ when d already contains a percentage error γ, then $\frac{d}{n}$ must

contain a total percentage error, Γ, of

$$\Gamma = \gamma + n\gamma = \gamma(1+n)^*$$

or since

$$\gamma = \frac{\frac{1}{m} \times 100}{d}$$

we get

$$\Gamma = \frac{\frac{1}{m} \times 100}{d}(1+n) \qquad [C]$$

(1) *For example.*

If d and $\frac{1}{m}$ were 58 divisions and $\frac{1}{4}$ division, respectively, and further, if the deflection d had to be halved, that is, if $n = 2$, then we should get

$$\Gamma = \frac{\frac{1}{4} \times 100}{58} \times 3 = 1 \cdot 3 \text{ per cent.}$$

If we have to deal with *degrees* of deflection instead of divisions, then in the case of a tangent galvanometer we should have

$$\Gamma_o = \left(\frac{\tan d\frac{1}{m}^\circ}{\tan d^\circ} - 1\right)100 + \left(\frac{\tan d_1\frac{1}{m}^\circ}{\tan d_1^\circ} - 1\right)100 =$$

$$\left(\frac{\tan d\frac{1}{m}^\circ}{\tan d^\circ} + \frac{\tan d_1\frac{1}{m}^\circ}{\tan d_1^\circ} - 2\right)100 \qquad [D]$$

where

$$\tan d_1^\circ = \frac{\tan d^\circ}{n}.$$

(2) *For example.*

If d°, $\frac{1}{m}^\circ$, and n, were 58°, $\frac{1}{4}^\circ$, and 2, respectively, then we should have

$$\tan d_1^\circ = \frac{1 \cdot 6003}{2} = \cdot 8001 \; (= \tan 38\tfrac{1}{2}^\circ),$$

* Strictly speaking this is not absolutely correct, for it assumes that the second percentage should be calculated on $\frac{d}{n}$, whereas it ought to be calculated on $\frac{d + \frac{1}{m}}{n}$; but as $\frac{1}{m}$ is small compared with d, the consequent error is small also.

therefore

$$\Gamma_o = \left(\frac{1 \cdot 6160}{1 \cdot 6003} + \frac{\cdot 8026}{\cdot 7954} - 2\right) 100 = 1 \cdot 7 \text{ per cent.}$$

It may be pointed out that this last example shows the possible percentage of error which may occur when making a halved current test with the tangent galvanometer under the best possible conditions. Practically, therefore, we may say that under no possible conditions could the deflection error in a halved current test be regarded as being less than $1\frac{3}{4}$ per cent. As will be seen when we come to consider such tests, other sources of error are met with which still further reduce the degree of accuracy with which the tests can be made.

36. Although in formulæ [B] and [D] the function of the deflections has been taken as the *tangent*, yet the formulæ apply equally well in cases where the current strengths are proportional to any other function of the deflections.

Calibration of Galvanometers.*

37. The deviations in *degrees* of the needle of a galvanometer which is not of the tangent form are not generally proportional to any simple function of those degrees, yet it is easy to determine the relative values of the deflections in terms of the currents which would produce them, that is, to *calibrate* the scale. In order to do this, it is simply necessary to join up in circuit with the galvanometer, a battery, a set of resistance coils, and also a galvanometer the values of whose deflections are known (a *tangent* galvanometer, for example). This being done, and the galvanometers being set so that their needles are at zero, we insert sufficient resistance in the circuit to reduce the deflection in one of the instruments to 1°, and then by means of a "shunt" (Chapter IV.) we also reduce the deflection of the needle of the second galvanometer to 1°. We now reduce the resistance in the circuit step by step so as to produce deflections of 1°, 2°, 3°, 4°, &c., from the needle of the galvanometer whose scale is required to be calibrated. As each deflection is obtained we observe and note the corresponding deflection on the tangent instrument. When the whole range of the scale (or as much of it as is considered necessary) of the instrument under calibration has been gone through, we can construct a table for use with it by writing down opposite the various degrees of deflection the tangents of the deflections which were obtained on the tangent instrument and which corresponded to

* See also (§ 63), p. 58.

the deflections in question. The table so constructed would be used precisely in the same way as would the table of tangents in the case of a tangent galvanometer, the use including, it may be remarked, the determination of the percentage value of an error in a deflection. It may also be remarked that the *angle of maximum* sensitiveness would be the deflection which was obtained when the needle of the tangent instrument pointed to 45°.

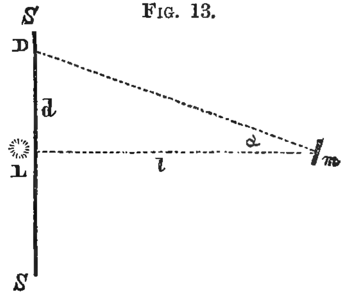
The Thomson Galvanometer.

38. The accuracy with which measurements can be made depends chiefly upon the sensitiveness of the galvanometer employed in making those measurements. The Thomson reflecting galvanometer supplies this requisite sensitiveness, and is the instrument which is almost invariably employed when great accuracy is required, and also when very high resistances have to be measured.

Description.

39. The principle of the galvanometer is that of employing a very light and small magnetic needle, delicately suspended within a large coil of wire, and of magnifying its movements by means of a long index hand of light. This index hand is obtained by throwing a beam of light on a small mirror fixed to the suspended magnetic needle, the ray being reflected back on to a graduated scale. This scale being placed about 3 feet distant from the mirror, it is obvious that a very small angular movement of the mirror will cause the spot of light reflected on the scale to move a considerable distance across it.

Fig. 13.



The needle being very small, and being placed in the centre of a large coil, the tangents of its deflections are approximately directly proportional to the strength of the currents producing them.

In Fig. 13, let L be a lamp which throws a beam upon the mirror m, which has turned through a small angle, and reflected the beam on the scale at D. Let d be the distance through which the beam has moved on the scale from zero point at L, and let l be the distance between the scale and the mirror. Now the angle through which the beam of light turns will be

twice the angle through which the mirror turns. This is clear if we suppose the mirror to have turned through 45°, when the reflected beam will be at 90°, or at right angles to the incident beam. If, then, we call a the angle through which the beam of light turns, $\frac{a}{2}$ will be the angle through which the mirror will have turned. Let $\frac{a_1}{2}$ and $\frac{a_2}{2}$ be the two angles through which the mirror has been turned by two currents, of strengths C_1 and C_2 respectively, then

$$C_1 : C_2 :: \tan\frac{a_1}{2} : \tan\frac{a_2}{2};$$

therefore

$$C_1 : C_2 :: \frac{\sqrt{1+\tan^2 a_1}-1}{\tan a_1} : \frac{\sqrt{1+\tan^2 a_2}-1}{\tan a_2},$$

$\sqrt{1+\tan^2}$ being positive, as the angles are less than 90°.

l being the distance of the scale from the mirror, let d_1 and d_2 be the distances traversed on the scale by the beam of light, then

$$\tan a_1 = \frac{d_1}{l}, \tan a_2 = \frac{d_2}{l}.$$

therefore

$$C_1 : C_2 :: \frac{\sqrt{1+\frac{d_1^2}{l^2}}-1}{\frac{d_1}{l}} : \frac{\sqrt{1+\frac{d_2^2}{l^2}}-1}{\frac{d_2}{l}};$$

therefore

$$C_1 : C_2 :: d_2(\sqrt{l^2+d_1^2}-l) : d_1(\sqrt{l^2+d_2^2}-l);$$

when d_1 and d_2 do not differ largely, then we may take

$$C_1 : C_2 :: d_1 : d_2;$$

but when this is not so the error may be considerable. For instance, suppose $d_1 = 150$, and $d_2 = 300$. According to the last formula this would show that one current is just twice as strong as the other, but by the correct formula, taking $l = 1500$ divisions (which would be about its value), we find that

$$C_1 : C_2 :: 300(\sqrt{1500^2+150^2}-1500) : 150(\sqrt{1500^2+300^2}-1500),$$

that is

$$C_1 : C_2 :: 2244 : 4456,$$

or

$$C_1 : C_2 :: 150 : 297 \cdot 14;$$

so that when extreme accuracy is required we cannot take the strengths of currents as being exactly proportional to the number of divisions of deflection on the scale.

The galvanometer, as usually constructed, consists essentially of a very small magnetic needle, about three-eighths of an inch long, fixed to the back of a small circular mirror, whose diameter is about equal to the length of the magnet. This mirror, which is sometimes a plano-convex lens, of about six feet focus, is suspended from its circumference by a single cocoon fibre devoid of torsion, the magnetic needle being at right angles to the fibre. The mirror is placed in the axis of a large coil of wire, which completely surrounds it, so that the needle is always under the influence of the coil at whatever angle it is deflected to. A beam of light from a lamp placed behind a screen, about three feet distant from the coil, falls on the mirror, and is reflected back on to a graduated scale placed just above the point where the beam emerges from the lamp. The scale is, as we have before said, straight, and is usually graduated to 360 divisions on either side of the zero point.

40. The Thomson galvanometer is made in a variety of forms; Fig. 14 gives a front, and Fig. 15 a side elevation (with glass shade, &c., removed) of one very common pattern.

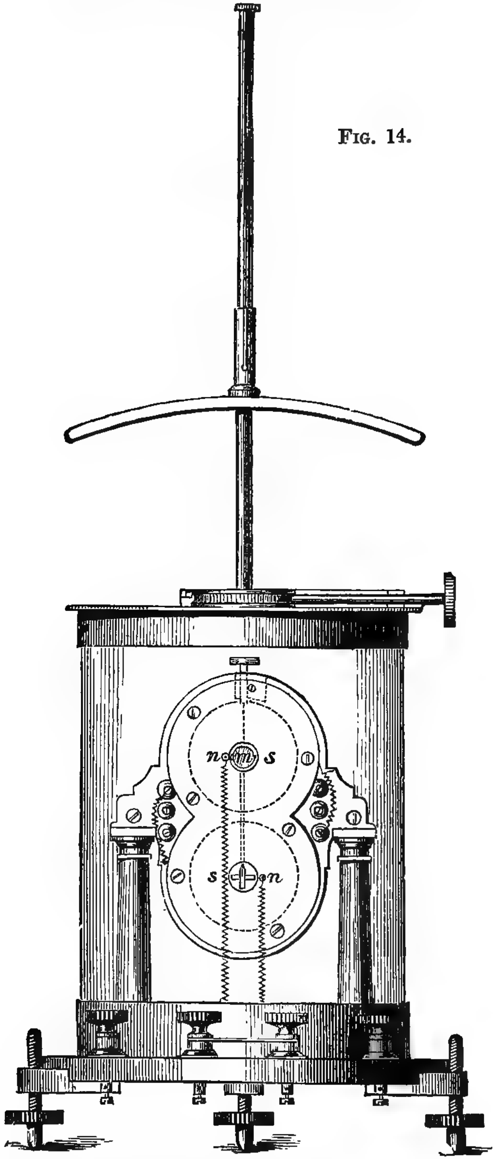
It consists of a base formed of a round plate of ebonite, provided with three levelling screws; two spirit-levels, at right angles to one another, are fixed on the top of this plate, so that the whole instrument can be accurately levelled; sometimes one circular level only is provided, but the double level is much the best arrangement.

From the base rise two brass columns, between which a brass plate is fixed, rounded off at the top and bottom. Against the faces of this plate are fixed the coils (*c*, *c*, *c*, *c*) of the instrument. The brass plate has shallow countersinks on its surface for the faces of the coils to fit into, so that they can be fitted in their correct places without trouble or danger of shifting. Round brass plates press against the outer surfaces of the coils by means of screws, and keep them firmly in their places. There are two round holes in the brass plates coinciding with the centre holes in the coils.

The coils themselves, which are four in number, are wound on bobbins of thin insulating material, the wire being heaped

THOMSON'S REFLECTING GALVANOMETER.

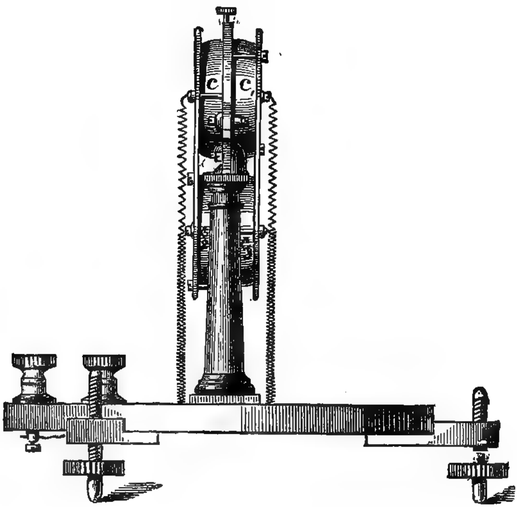
FIG. 14.



Front Elevation. ½ real size.

up towards the cheek of the bobbin which bears against the brass plate. This heaping up is done in accordance with a law of Sir William Thomson, so as to obtain, as far as possible, a maximum effect out of a minimum quantity of wire. The edges of the coils are covered with shellac, so as to protect the wire from injury.

FIG. 15.



Side Elevation. (*Shade removed.*) $\frac{1}{4}$ real size.

Within the holes in the brass plate are placed two little magnets, *n s* and *s n*,* formed of watch-spring highly magnetised; they are connected together by a piece of aluminium wire, so as to form an astatic pair of needles. A small groove is cut in the brass plate, between the upper and the lower hole, for the aluminium wire to hang freely in.

An aluminium fan is fixed at right angles to the lower needle; this fan acts as a damper, and tends to check the oscillations of the needles and to bring them to rest quickly.

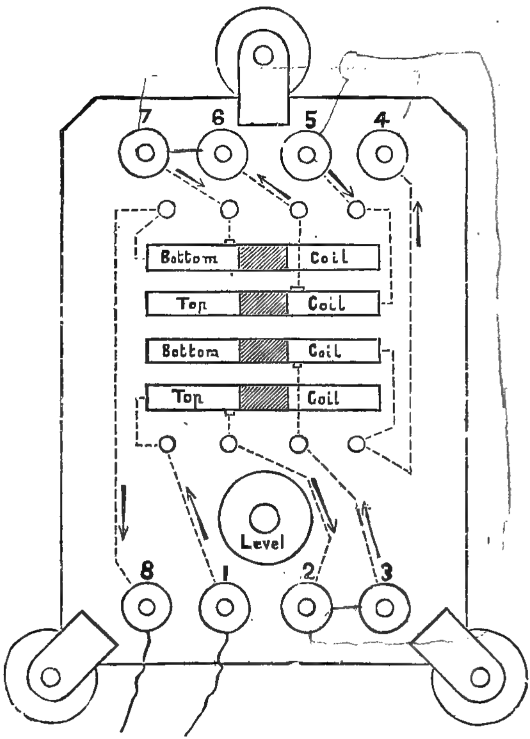
In front of the top needle is fixed the mirror. It is suspended by a fibre attached at its upper end to a small stud, which can be raised or lowered when required. When pressed down as far as it will go the needles rest on the coils, and the tension

* In the more recent instruments it is usual to have several small magnets placed one above the other at a short distance apart, in the place of a single magnet.

being taken off the fibre, there is no danger of breaking it by moving the instrument.

One end of each coil is connected to one of the four terminals in front of the base of the instrument, the other ends being connected to one another through the medium of the small terminals placed midway on either side of the coils.

FIG. 16.



The connections are so made that, when the two middle terminals on the base of the instrument are joined together, the whole four coils are in the circuit of the two outer terminals, so that they all four act on the magnetic needles.

As it is often convenient to be able to couple up the four coils in different ways so as to vary their total resistance, in the

instruments manufactured by the Silvertown Telegraph Works Company the ends of all the four coils are connected to terminals in a manner designed by Messrs. March Webb and R. K. Gray, and shown by Fig. 16. This figure represents the base of one of these instruments. Lines are engraved on the ebonite base to show the routes followed by the various coils. Arrows also are engraved alongside the lines to show the directions in which the currents must flow in order that all the coils may tend to turn the galvanometer needle in the same direction.

There are five possible ways of coupling up *all* the coils together, so as in each case to produce a different resistance. The following will show the various methods:—

I. To obtain total resistance of all the coils in series, connect terminals 2 and 3, 4 and 5, 6 and 7.

II. To obtain $\frac{5}{8}$ resistance, connect terminals 2 and 3, 2 and 5, 7 and 6, 7 and 4.

III. To obtain $\frac{1}{4}$ resistance, connect terminals 2 and 3, 4 and 8, 1 and 5, 6 and 7.

IV. To obtain $\frac{3}{16}$ resistance, connect terminals 2 and 8, 1 and 3, 4 and 5, 6 and 7.

V. To obtain $\frac{1}{16}$ resistance, connect terminals 1 and 3, 3 and 5, 5 and 7, 6 and 8, 4 and 6, 2 and 4.

In each case the leading wires from the galvanometer must be connected to terminals 1 and 8.

Referring again to Fig. 14; over the coils a glass shade is placed, from the middle of the top of which a brass rod rises. A short piece of brass tube slides over this rod, with a weak steel magnet, slightly curved, fixed at right angles to it. This magnet can be slid up or down the rod, or twisted round, as occasion may require. For fine adjustments a tangent screw is provided, which turns the brass rod round, and with it the magnet.

Figs. 17 and 18 show modified forms of the instrument, which, however, in general arrangement are similar to the pattern which has been described.

In the more recent galvanometers manufactured by Messrs. Elliott Brothers, the brass plates, which, in the older instruments, secured the coils in their places, are hinged to the frame, whilst the coils themselves are permanently fixed to the plates; by this arrangement the magnetic needles, with their mirror, fibre-suspension, &c., attachments, can be got at, if required, with the greatest facility. Altogether this improvement is one of the most convenient that has been lately made.

About 5000 or 6000 ohms is usually the total resistance of the coils of these galvanometers.

Fig. 17.

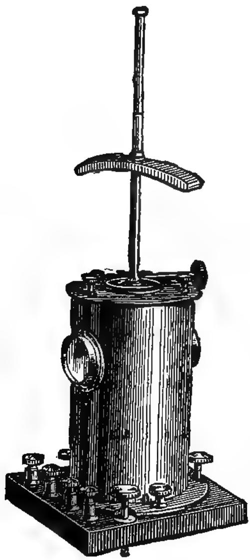


Fig. 18.

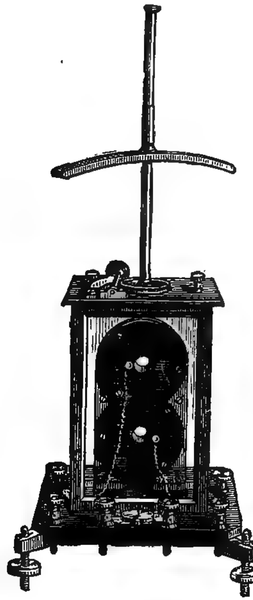


Fig. 19 shows a portable reflecting galvanometer which is very useful, especially for travelling purposes; the three legs are hinged at their junction with the lower part of the coil frame, so that they can be folded together, and thus made to occupy but little space. Owing to the instrument being provided with but two coils (one in front of, and the other behind, the needle) its sensitiveness is not quite so great as the larger instruments with four coils, but for general purposes it is an excellent piece of apparatus.

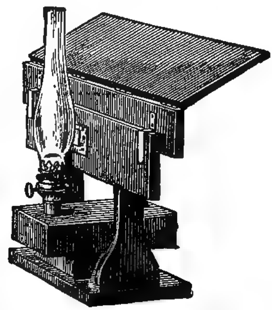
41. We have said that the mirror is sometimes made of a plano-convex lens. This is done so as to obtain a sharp image of the spot of light on the scale. The width of the spot can be regulated by means of a brass slider fixed over the hole in the screen, through which the beam emerges from the lamp.

A much better arrangement than the spot of light is now provided with most instruments. The hole through which the

light emerges is made round, about the size of a sixpence, with a piece of fine platinum wire stretched vertically across its diameter. A lens is placed a little distance in front of this hole, between the scale and galvanometer, so that a round spot of light, with a thin black line across it, is reflected on the scale. This enables readings to be made with great ease, as

Fig. 19.

Fig. 20.



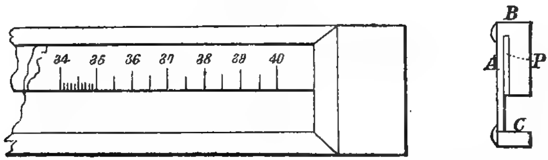
the figures on the scale can be very distinctly seen. (The mirror in this arrangement may be a plane one.) When the spot of light only is used, it is necessary to partially illuminate the scale with a second lamp. The general appearance of a back view of the scale frame with the lamp placed in position, is shown by Fig. 20.

Jacob's Transparent Scale.

42. The position of the ordinary form of scale for the Thomson's galvanometer is to a certain extent inconvenient, especially to near-sighted individuals. Mr. F. Jacob has completely remedied this inconvenience by the arrangement shown in front view and cross section by Fig. 21. In this fig. B is a wooden scale-board with a longitudinal slot, as shown at C; P is the paper scale, cut so that all the division lines reach the lower edge; A is a slip of plane glass with its lower half finely ground from one end of the slip to the other, on the side towards C: the scale is so placed that the lower end of the division lines just touches the ground part of the glass slip. The image of the slit with a fine wire stretched across it is focussed in the ordinary manner on the ground part of the glass, and will of course be clearly seen by the observer on the opposite side of the scale; as the line and

printed divisions are in the same plane, there is no parallax; and a great increase in accuracy of reading the position of the hair line is obtained, owing to the greater ease of observing that two lines coincide when end on to one another, than when superimposed; and further, from the circumstance that the room need

FIG. 21.



not be darkened. The lamp and its slit is placed on one side and reflects the beam of light on to the galvanometer by a mirror or total reflection prism, and by means of two long plane mirrors the actual distance between the galvanometer and scale is reduced, so as to have everything close to the observer's hand. The scale adopted is divided into half millimetres, and it is perfectly easy to read to a quarter of a division, and with a hand magnifying-glass still further. This arrangement has been adopted in the testing-rooms of Messrs. Siemens Brothers and Co., at Woolwich, and gives great satisfaction.

43. In the testing-rooms at the Silvertown Telegraph Works the scales employed are of large dimensions, being about 5 feet long, and are set at a distance of several feet from the galvanometer. By this arrangement a greatly magnified image of the round spot of light with the black line across it is obtained, and the divisions on the scale being of correspondingly large dimensions, the readings can be made with great facility, and with very little fatigue to the eye. The only objection to the arrangement is the space which it necessarily occupies, but as it is not often that many instruments require to be set up in the same room, this need hardly be taken into account.

To set up the Galvanometer.

44. It is essential, before proceeding to set up the instrument for use, to see that the ebonite base is thoroughly dry and clean, so that there may be no leakage from the wires to interfere with the tests taken. Indeed, it is as well to place the galvanometer and the other apparatus to be used on a large sheet of gutta-percha or ebonite, more especially if the room in which the tests are to be made is at all damp. Sometimes little ebonite cups are provided for the levelling screws of the instrument

to stand in, which answer the purpose of insulating very thoroughly.

The instrument should be set up on a very firm table in a basement storey. It is almost useless to test with it in an upper room, as the least vibration sends the spot of light dancing and vibrating to and fro. At all cable works the instrument is placed on a solid brick table built on the earth, so that no vibration can possibly affect it.

A suitable table being chosen, set the galvanometer in any convenient position, and adjust the levelling screws until the bubbles of the level or levels show the instrument to be perfectly level.

Now remove the glass shade, and gently raise the stud at the top of the coils by squeezing the tips of the fingers between the head of the stud and the top of the brass plate in which it runs. If the stud is raised by a direct pull, there is almost a certainty of its coming up with a jerk and breaking the fibre. On no account must the stud be twisted round, except to get rid of any torsion which may exist in the fibre when it has been replaced after becoming broken.

The stud being raised sufficiently high to allow the mirror to swing clear of the coils, replace the glass shade, screw the brass rod with the magnet, on to its top, and set the magnet about half-way up the rod, the poles being placed so as to assist in keeping the magnetic needles north and south.

The scale lamp being lighted, place it in position on the scale stand, the *edge* of the wick being turned towards the brass slider which regulates the width of the beam of light. Having opened the slider to its full extent, the scale and lamp should be placed about 3 feet from the galvanometer, so that it stands parallel with the faces of the coils and so that a line drawn at right angles to the scale from the lamp-hole will pass through the centre of the galvanometer. The reflected beam of light should then fall fairly on the scale. If too high, this may be remedied by propping up the scale, and if too low, by screwing up the levelling screws of the galvanometer. It is easier to prop up the scale than to lower the galvanometer by means of the levelling screws, if the light is too high on the scale.

The spot of light should now be set at the zero point on the scale by turning the regulating magnet by means of the screw; the spot should next be focussed, by advancing or retreating the lamp and scale, until a sharply defined image is obtained on the scale. The width of the slit may then be diminished, by means of the brass slide, until a thin line of light only is obtained on the scale. If the round spot of light with the line across it is used,

the focussing must be made so that the black line of light is sharply defined.

The position of the scale and galvanometer being once obtained, their positions on the table may be marked for future occasions, or, at least, the exact distance of the scale from the galvanometer noted, so that it can be placed right without trouble.

The instrument being now ready for use, if it is not required to be sensitive, place the regulating magnet low down; if, on the contrary, it is required to be sensitive, place it high up.

45. To obtain the *maximum sensitiveness*:—Raise the magnet to the top of the bar, and then turn it half round, so that its poles change places. The magnet will now be opposing the earth's magnetism, and consequently will tend to turn the magnetic needles round. If the magnet is at the top of the rod, the effect of the magnetism of the earth on the magnetic needles will be more powerful than the magnetism of the regulating magnet, and the needles will tend to keep north and south; but by placing the regulating magnet lower down, a point is reached where the earth's magnetism is just counteracted. Under these conditions the needles will stand indifferently in any position. By placing the regulating magnet about an inch higher than the position which gives this exact counteraction, the magnetism of the earth will be just sufficient to keep the magnets north and south, and consequently the spot of light at the zero on the scale, and at the same time leaves them free to be moved by a very slight force. It will be noticed with the regulating magnet in this position, that in order to get the spot of light at the zero point, the magnet must be turned in the opposite direction to that in which it is required that the needles should move.

It is not advisable to adjust the instrument too sensitively, because it is difficult then to keep the spot exactly at zero, as any slight external action may throw it a degree or two out.

46. The presence of iron near the instrument is not prejudicial to its correct working, so long as it remains stationary. The experimenter should, however, remove any keys or knives he may have about him, as they very much affect the galvanometer if he moves about much. These precautions may seem too minute, but as the very object of the Thomson galvanometer is to enable measurements to be made with accuracy, all likely causes of disturbance should be avoided.

47. If the fibre of the instrument by any chance gets broken, the top front plate must be unscrewed, when the coil which it secures can be removed, and the mirror and its appendages got at. Care should be taken, when replacing the fibre, that only

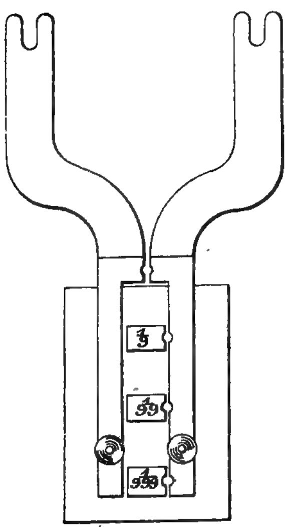
a single thread from the cocoon silk is used, or the sensitiveness of the instrument will be much diminished. The operation requires care, and must be done in a room free from draughts. When the ends of the fibre are passed through their respective holes, and tied, a small drop of shellac varnish may be dropped on them, which will prevent their becoming loose.

It is as well to let the needles remain suspended for a time, so that the fibre may become stretched to its normal length before being used.

The suspending stud should always be pressed down before removing the instruments.

48. A resistance box, containing three *shunts*, is provided with the galvanometer, of the values $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th of the resistance of its coils, which values, as we shall show in the next chapter, enable us to reduce the sensitiveness of the galvanometer to its $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th part respectively.

FIG. 22.



¼ real size

FIG. 23.

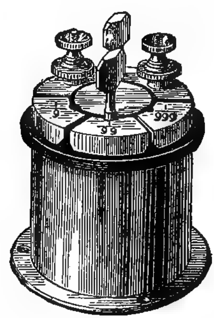


Fig. 22 shows a form of this shunt. By inserting a plug into one or other of the holes, the required shunt is inserted.

The numbers are sometimes marked as $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th, instead of $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th, thereby indicating that the particular shunt reduces the deflections of the needle to that particular fraction, but they have just the same adjustment really in both cases.

The shunts are sometimes enclosed in a round brass box, as shown by Fig. 23, which is perhaps a more portable and elegant fori

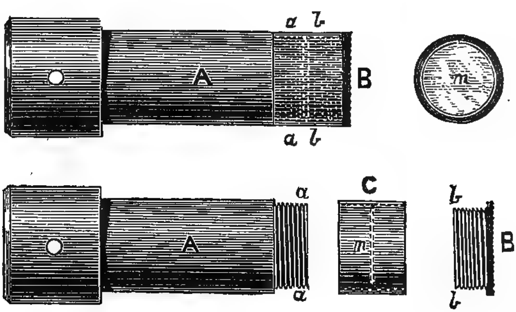
The two broad strips of copper shown in Fig. 22 are used for the purpose of connecting the box with the galvanometer. The blank plug-hole is for the purpose of short-circuiting it, which should always be done when the instrument is not actually in use.

Thomson's Dead-beat Galvanometer.

49. Great inconvenience and loss of time in testing often arise from the needle of the galvanometer not settling down at once to the angle of deflection it should take up when under the influence of a constant current, but oscillating to and fro several times before it finally comes to rest, and again acting in the same way when the current is taken off and the needle returns towards the zero point. The object of the *dead-beat* galvanometer is to avoid these inconvenient oscillations.

Fig. 24 shows the arrangement invented by Sir William Thomson for effecting this object.

Fig. 24.



A is a brass tube, whose end *a a*, which is screwed, is closed by a piece of glass. B is a short piece of tube, which is screwed, and whose end *b b* is similarly closed by a piece of glass. C is a third short piece of tube, into which the ends of A and B screw. The length of this tube is such that when the whole arrangement is united together there is a very small space between the ends *a a* and *b b*; a small air-tight cell in fact is formed.

Hanging midway inside C is a mirror *m*, with a magnetic

needle fixed to it, as in the ordinary Thomson galvanometer. This mirror very nearly fits inside the tube, there being only just room for it to swing freely; it is suspended by a very short fibre.

The space between *a a* and *b b*, although very small, is just sufficient to enable the mirror to turn through an angle large enough to give a good deflection of the spot of light on the scale.

The complete arrangement is inserted in the centre of a single galvanometer coil, so that the mirror occupies the same position that it does in the ordinary galvanometer.

Owing to the air inside the cell being so closely confined, the violent movement of the mirror is checked when it is acted upon by a current passing through the coils, and the consequence is that the mirror, instead of overshooting the mark and then recoiling, turns straight to its proper deflection and stops dead. The same thing takes place when the current is cut off; in this case the spot of light moves back to zero and stops dead at that point.

The suspension fibre being very short, the mirror cannot turn so freely as the one in the ordinary galvanometer; its sensitiveness is therefore not quite so great, but it is sufficiently so for most purposes for which the latter would be used.

The fibre is very easily replaced when broken. One end being attached to the mirror, the other is passed through a small hole in the side of C, and is then drawn sufficiently tight to suspend the mirror inside the tube so that it does not touch the sides, a drop of shellac is then applied to the hole, which closes it and fixes the fibre.

Thomson's Marine Galvanometer.

50. This instrument is specially constructed for use on board ship, where the rolling of the vessel and the constant movement of masses of iron about would render an ordinary reflecting galvanometer quite useless.

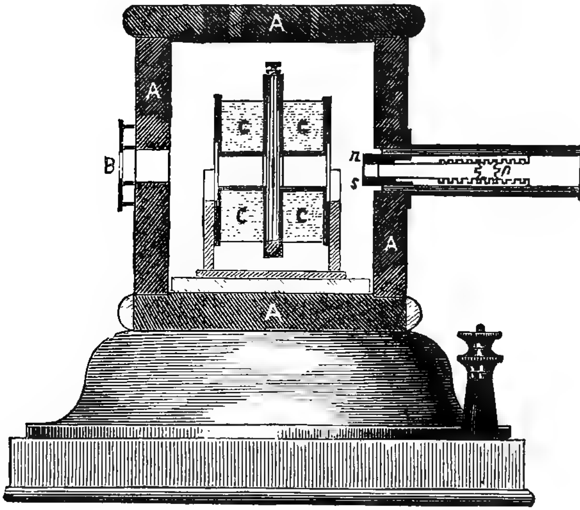
Fig. 25 shows a side view of this instrument, the upper part being drawn in section.

C C C C are the coils, which are similar in form to those employed in the ordinary Thomson's galvanometer; there is, however, but one set, of two coils, instead of two sets as in the latter.

The mirror, with the magnetic needle fixed to its back, is strung on a cocoon fibre in a brass frame. The fibre is fixed at

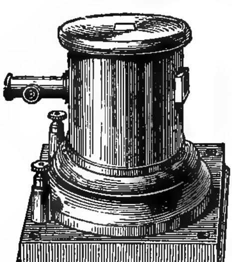
one end, and at the other is attached to a spring, which draws the fibre tight. The frame slides in a groove between the coils, so that it can be drawn out for the purpose of repairing the

FIG. 25.



fibre. A powerful directing horse-shoe magnet (not shown in the figure) embraces the upper parts of the coils, and serves to overpower the directive effect of the earth's magnetism. This latter effect is still further rendered harmless by enclosing the whole system in a massive soft-iron case A A A A, a little window B being left through which the rays of light reflected by the mirror enter and return.

FIG. 26.



For obtaining exact adjustment of the spot of light to zero, two little magnets, *n* and *s*, as broad as the mirror magnet is long, are provided; by turning the pinion *p* these little magnets can be made to advance or retreat, and so act on the mirror magnet to make it turn in one direction or the other, as it is required.

The resistance of this form of galvanometer (which is shown

in general view by Fig. 26) is usually as high as 30,000 or 40,000 ohms.

51. The *angle of maximum sensitiveness* in the Thomson reflecting galvanometers is, it is perhaps unnecessary to say, the largest deflection we can obtain, as the angle of deflection is but a very few *degrees*, and consequently the true maximum angle can never be reached.

FIGURE OF MERIT OF GALVANOMETERS.

52. The "figure of merit" of any galvanometer may best be defined as the *reciprocal* of the amount of current which will produce one division or degree of deflection.* In order to find this current, we have simply to join up the galvanometer in circuit with a battery of a known electromotive force, and a resistance of a known value, and then note the deflection obtained; from this we can easily calculate the current required to produce 1 degree of deflection; thus, for example, if we had a tangent galvanometer which gave a deflection of 50° with a 10-cell Daniell battery, that is, with an electromotive force of 10 volts approximately, there being in circuit a total resistance of 1000 ohms, then the current producing this deflection would be

$$\frac{10}{1000} = \cdot 01 \text{ ampères.}$$

The current which would be required to produce a deflection of 1° would obviously be

$$\cdot 01 \times \frac{\tan 1^\circ}{\tan 50^\circ} = \cdot 01 \times \frac{\cdot 0175}{1 \cdot 198} = \cdot 000146 \text{ ampères;}$$

consequently the figure of merit is $\frac{1}{\cdot 000146}$.

In the case of a Thomson galvanometer, we have simply to divide the current by the deflection obtained with the latter, since the deflections are directly proportional to the currents producing them.

If we require to determine the figure of merit of a galvanometer whose deflections throughout the scale are not proportionate to any ordinary function of the degrees of those

* It is preferable to define the figure of merit as being the *reciprocal* of the current rather than the current itself, inasmuch as by so doing we avoid the apparently contradictory statement that a galvanometer with a *high* figure of merit is one which requires a *low* current to produce a perceptible deflection.

deflections, then it is best to employ a sufficiently low electromotive force and high resistance in circuit to obtain a few degrees of deflection only, and then to divide the current by this number of degrees; for on every galvanometer the first few degrees of deflection are almost exactly proportional to the currents producing them, although the higher deflections are not so.

The "figure of merit" of a galvanometer has a considerable bearing upon the question of the degree of accuracy with which it is possible to make electrical measurements, as will be seen hereafter.

Sensitiveness of Galvanometers.

53. A galvanometer with a high "figure of merit," that is, a galvanometer whose needle will deflect from zero with a very weak current, is not necessarily a highly *sensitive* instrument; by a *sensitive* galvanometer we mean one *whose needle when deflected under the influence of a current will change its deflection perceptibly with a very slight change in the current strength.*

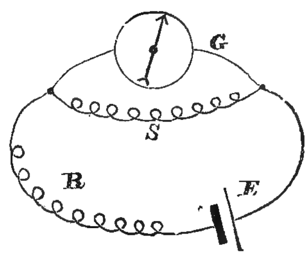
In many tests it is far more important that the galvanometer used be one of great sensitiveness rather than one with a high figure of merit. As a rule it is rarely that an instrument with a *compass* suspended or a pivoted needle is highly sensitive, unless indeed the pivoting is exceptionally good. Practically, it may be taken that for high sensitiveness the needle must be suspended by a fine fibre so that its movements may be perfectly free.

CHAPTER IV.

SHUNTS.

54. In making certain measurements we sometimes find that owing to the sensitiveness of the galvanometer, we are unable to obtain a readable deflection, from the needle being deflected up to the stops. We may reduce this sensitiveness by the insertion of a *Shunt* between the terminals of the instrument. This arrangement is shown by Fig. 27.

Fig. 27.



If it is required to reduce the strength of current which ordinarily passes through the galvanometer to any proportional part of that current, we must calculate, from the resistance of the galvanometer, what the resistance of the shunt should be to effect that purpose.

Now if we call C the current passing through the galvanometer without a shunt, then on introducing the shunt, C will divide between the two resistances, the greater portion of the current going through the smaller resistance, and the smaller portion through the greater. Thus if we suppose the total current, which passes from one terminal of the galvanometer to the other, to consist of $G + S$ parts, then $\frac{G}{G+S}$ of these parts will go through the shunt, and $\frac{S}{G+S}$ parts through the galvanometer; that is to say, the current going through the shunt will be

$$C \frac{G}{G+S},$$

and the current going through the galvanometer,

$$C \frac{S}{G+S}.$$

If, in this last quantity, we put S = G, then current going through galvanometer will be

$$C \frac{G}{G+G} = \frac{C}{2}.$$

Again, if we make $S = \frac{G}{2}$, current going through galvanometer will be

$$C \frac{\frac{G}{2}}{G+\frac{G}{2}} = \frac{C}{3}.$$

Once more, if S be made equal to $\frac{G}{3}$, current going through galvanometer will be

$$C \frac{\frac{G}{3}}{G+\frac{G}{3}} = \frac{C}{4}.$$

Finally, if S be made equal to $\frac{G}{n-1}$, current going through galvanometer will be

$$C \frac{\frac{G}{n-1}}{G+\frac{G}{n-1}} = \frac{C}{n}.$$

From this it is evident, that to reduce the current flowing through the galvanometer to its $\frac{1}{n}$th part, we must insert a shunt whose resistance is $\frac{1}{n-1}$th part of the resistance of the galvanometer.

55. In many galvanometers three shunts are provided,* which enable us to reduce the strength of current flowing through it to its $\frac{1}{10}$th, $\frac{1}{100}$th, or $\frac{1}{1000}$th part. From what has been said, it will be evident that the resistances of the shunts necessary to produce these results will have to be respectively the $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th part of the resistance of the galvanometer. We are

* Page 43.

thus enabled to reduce the sensitiveness of the galvanometer to any one of these three proportions we wish.

56. Suppose now, in making a measurement, we placed a resistance box for a shunt between the terminals of the galvanometer, and then adjusted it until we got a convenient deflection for the purpose we required; what deflection should we get on removing the shunt? Let us call C, as before, the current which passes through the galvanometer when no shunt is inserted, and let C_1 be the current which flows through it when the shunt is inserted, then the current which flows through the shunt will be

$$C - C_1.$$

Now the two currents will flow through the shunt and galvanometer in the inverse proportion of their resistances, that is,

$$C_1 : C - C_1 :: S : G,$$

therefore,

$$C = C_1 \times \frac{G + S}{S}.$$

Or expressed in words, we should say that the current which would flow through the galvanometer, when the shunt was removed, would be $\frac{\textit{Galvanometer} + \textit{Shunt}}{\textit{Shunt}}$ times the strength of the current which flows when the shunt is inserted. This proportion is called the *multiplying power* of the shunt.

57. It will be noticed in a circuit like that shown by Fig. 27 that when a shunt is introduced between the terminals of the galvanometer which reduces its sensitiveness to $\frac{1}{2}$, or a shunt having a resistance equal to that of the galvanometer, it will not exactly halve the current passing through the instrument. If we used a tangent galvanometer, we should find, if the deflection without the shunt was 40 divisions on the tangent scale, the introduction of the shunt would not bring the deflection down to 20, but to some deflection greater than 20. The reason of this is, that the introduction of the shunt reduces the total resistance in the battery circuit, and consequently increases the strength of the current passing out of the battery. It is this increased current, then, which splits between the galvanometer and shunt, and not the original current. To make up for this decreased resistance caused by the introduction of the shunt, it is necessary to add in the battery circuit a *compensating resistance* equal in value to the amount by which the original resistance was reduced. In order to obtain this, we must first consider the law of

The Joint Resistance of two or more Parallel Circuits.

58. If we have several wires whose resistances are R_1, R_2, R_3 .... respectively, then conductivity being the inverse or reciprocal of resistance, their conductivities may be represented by $\frac{1}{R_1}$, $\frac{1}{R_2}$, $\frac{1}{R_3}$ .... Now the joint conductivity of any number of wires is simply the sum of their respective conductivities. Thus, *two* wires of equal conductivities, when joined parallel to one another, will evidently conduct *twice* as well as one of them; and in like manner, *three* wires will conduct *three* times as well as one. Similarly, two wires, one of which has a conductivity of 2, will, when combined with one which has a conductivity of 1, produce a conductivity of $2 + 1$ or 3, for this is simply the same as joining up three wires, each having a conductivity of 1; and so with any number of wires.

Therefore the joint *conductivity* of the several resistances, or multiple arc as it is called, will be $\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$ and conductivity being, as we have said, the reciprocal of resistance, the resistance of the wires will be the reciprocal of this sum, or

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}$$

That is to say, *the joint resistance of any number of wires joined parallel to one another is equal to the reciprocal of the sum of the reciprocals of their respective resistances.*

A particular case of these combinations is that of the joint resistance of two resistances, thus

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2},$$

or, *the joint resistance of two resistances joined parallel to one another is equal to their product, divided by their sum.*

59. Applying the foregoing law, the resistance between the terminals of the galvanometer before the introduction of the shunt being G, that on the introduction of the shunt will be $\frac{G S}{G + S}$. Or, as S is usually made some fractional value of G,

say the $\frac{1}{n-1}$th part (which value would be used in reducing the sensitiveness of the galvanometer to $\frac{1}{n}$th), this combined resistance will be

$$\frac{G\frac{G}{n-1}}{G+\frac{G}{n-1}} = \frac{\frac{G}{n-1}}{1+\frac{1}{n-1}} = \frac{G}{n}. \qquad [1]$$

The resistance therefore to be added in the battery circuit will be

$$G - \frac{G}{n} = G\frac{n-1}{n}. \qquad [2]$$

For example.

It was required to reduce the sensitiveness of a galvanometer, whose resistance was 100 ohms (G), to $\frac{1}{5}$th. What should be the resistance of the shunt and of the compensating resistance?

Resistance of shunt equals

$$100 \times \frac{1}{5-1} = 25 \text{ ohms},$$

and compensating resistance equals

$$100 \times \frac{5-1}{5} = 80 \text{ ohms}.$$

It would be as well if the shunt boxes provided with galvanometers had compensating resistances connected with them, as calculation would be considerably simplified thereby in a large number of measurements.

FIG. 28.

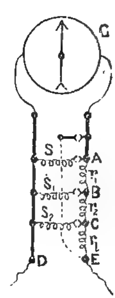


Fig. 28 shows how a set of shunts and compensating resistances may be adapted to any galvanometer; we will consider how their values may be determined.

Let S, S_1, S_2, be the shunts which can be connected to the galvanometer by inserting plugs at A, B, or C.

Let r_1, r_2, r_l, be the compensating resistances, and let

$$r_1 + r_2 + r_l = R_1 \qquad [3]$$
$$r_2 + r_l = R_2. \qquad [4]$$

Now, what we have to do, is to find what values of S, S_1, S_2, and r_1, r_2, $r_{\prime}$, are necessary, so that when a plug is introduced either at A, B, or C, the resistance between D and E shall always be the same, whilst the necessary portion of the current is shunted off from the galvanometer.

Let us first consider the shunt S and the compensating resistance which, in this case, will be R_1.

When the shunts and compensating resistances are not in use, the resistance in circuit is of course G, and this value must always be preserved between D and E.

Let the value of the shunt S required be $\frac{1}{n}$th, then we know (page 50) that the resistance of S necessary to give this, is

$$S = \frac{G}{n - 1},$$

and from [2] that the value of R_1 must be

$$R_1 = G \frac{n - 1}{n}. \qquad [5]$$

We next have to consider what value to give to S_1 and R_2.

Let it be required, by means of these resistances, to reduce the deflection by $\frac{1}{n_1}$th, then the value to be given to S_1 will be

$$S_1 = \frac{G + r_1}{n_1 - 1};$$

to solve which, we must know the value of r_1.

Now, the combined resistance of the shunt and $G + r$ we can see from [1] is

$$\frac{G + r_1}{n_1};$$

therefore the value required to be given to R_2, in order to preserve the resistance between D and E, equal to G, when S_1 is connected, will be

$$R_2 = G - \frac{G + r_1}{n_1},$$

or

$$R_2 + \frac{r_1}{n_1} = G \frac{n_1 - 1}{n_1}; \qquad [6]$$

but from [3], [4], and [5]

$$R_2 + r_1 = R_1 = G\,\frac{n-1}{n}; \qquad [7]$$

therefore, subtracting [7] from [6], we have

$$\frac{r_1}{n_1} - r_1 = G\left(\frac{n_1 - 1}{n_1} - \frac{n-1}{n}\right) = G\,\frac{n_1 - n}{n\,n_1};$$

that is,

$$r_1\,\frac{1 - n_1}{n_1} = G\,\frac{n_1 - n}{n\,n_1},$$

or

$$r_1 = G\,\frac{n - n_1}{n\,(n_1 - 1)};$$

consequently the value of S_1 will be

$$S_1 = G\,\frac{1 + \dfrac{n - n_1}{n\,(n_1 - 1)}}{n_1 - 1} = G\,\frac{(n-1)\,n_1}{n\,(n_1 - 1)^2}\,.$$

In like manner it could be shown that the resistance necessary to give to S_2 and $r_1 + r_2$, to reduce the deflection to its $\frac{1}{n_2}$th part would be

$$S_2 = G\,\frac{(n-1)\,n_2}{n\,(n_2 - 1)^2},$$

and

$$r_1 + r_2 = G\,\frac{n - n_2}{n\,(n_2 - 1)},$$

or

$$r_2 = G\,\frac{n - n_2}{n\,(n_2 - 1)} - r_1.$$

Finally we have from [3] and [5]

$$r_l = R_1 - (r_1 + r_2) = G\,\frac{n-1}{n} - (r_1 + r_2).$$

To summarise then,

$$S = G\,\frac{1}{n-1},$$

$$S_1 = G\frac{(n-1)\,n_1}{n\,(n_1-1)^2},$$

$$S_2 = G\,\frac{(n-1)n_2}{n\,(n_2-1)^2};$$

and for any other shunt S_p

$$S_p = G\frac{(n-1)\,n_p}{n\,(n_p-1)^2}.$$

The compensating resistances *between* the shunts will be

$$r_1 = G\,\frac{n-n_1}{n\,(n_1-1)},$$

$$r_2 = G\,\frac{n-n_2}{n\,(n_2-1)} - r_1;$$

and also we have

$$r_1 + r_2 + \ldots + r_p = G\frac{n-n_p}{n_p-1},$$

or

$$r_p = G\,\frac{n-n_p}{n\,(n_p-1)} - (r_1 + r_2 + \ldots + r_{p-1}).$$

The *last* resistance r_l beyond the last shunt will be

$$r_l = G\frac{n-1}{n} - (r_1 + r_2 + \ldots + r_p).$$

For example.

It was required to provide a galvanometer with $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th shunts, and with corresponding compensating resistances arranged according to Fig. 28. What should be their value?

We have

$$n = 1000,\ n_1 = 100,\ n_2 = 10;$$

therefore,

$$n-1 = 999,\ n_1 - 1 = 99,\ n_2 - 1 = 9.$$

Then

$$S = G\,\frac{1}{999} = G \times \cdot 001001,$$

$$S_1 = G\frac{999 \times 100}{1000 \times 99 \times 99} = G \times \cdot 010193,$$

$$S_2 = G\frac{999 \times 10}{1000 \times 9 \times 9} = G \times \cdot 123333;$$

and

$$r_1 = G\frac{1000 - 100}{1000 \times 99} = G \times \cdot 0090909,$$

$$r_1 + r_2 = G\frac{1000 - 10}{1000 \times 9} = G \times \cdot 11;$$

from which

$$r_2 = G(\cdot 11 - \cdot 0090909) = G \times \cdot 1009091;$$

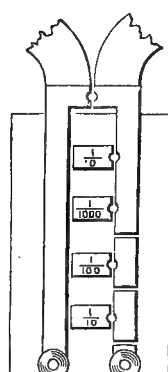
also

$$r_l = G\frac{999}{1000} - (r_1 + r_2) = G(\cdot 999 - \cdot 11) = G \times \cdot 889.$$

If the resistance of the galvanometer, for which these shunts and compensating resistances are to be provided, is 5000 ohms, then

S	$= 5000 \times \cdot 001001$	$=$	$5 \cdot 005$	ohms.
S_1	$= 5000 \times \cdot 010193$	$=$	$50 \cdot 965$	,,
S_2	$= 5000 \times \cdot 123333$	$=$	$616 \cdot 655$	,,
r_1	$= 5000 \times \cdot 0090909$	$=$	$45 \cdot 455$	,,
r_2	$= 5000 \times \cdot 1009091$	$=$	$504 \cdot 545$	,,
r_l	$= 5000 \times \cdot 889$	$=$	$4445 \cdot 000$	,,

Fig. 29.



¼ real size

Fig. 29 shows how an ordinary Thomson galvanometer shunt box would be arranged with compensating resistances.

The plug hole $\frac{1}{0}$, when it has a plug inserted in it, connects the top left-hand brass block to the bottom left-hand block, and so leaves the galvanometer connected to the terminals of the shunt box without any additional resistance in its circuit. The connection between these brass blocks is shown by the dotted line in Fig. 28.

60. The accurate adjustment of ordinary shunts is often a somewhat troublesome operation, in consequence of the numerical values of the resistances of which the shunts are composed not being whole numbers; thus, supposing the resistance of the

galvanometer to be 5000 ohms, then the resistance of the $\frac{1}{10}$th shunt would have to be $5000 \div 9$, or 555·56; and, practically, this could not be adjusted to a greater degree of accuracy than one decimal place. Similarly, the $\frac{1}{100}$th shunt should have a resistance of $5000 \div 99$, or 50·505, and the $\frac{1}{1000}$th shunt a resistance of $5000 \div 999$, or 5·005, both of which numbers are somewhat awkward to adjust exactly.

Now on page 53 (equation [1]) we saw that the combined resistance of the galvanometer and its shunt was $\frac{G}{n}$, consequently to adjust the $\frac{1}{10}$th shunt we may connect it to its galvanometer coil, and adjust it until the joint resistance of the two becomes equal to $5000 \div 10$, or 500 ohms. Similarly, the $\frac{1}{100}$th shunt would be adjusted by connecting it to the galvanometer coil, and adjusting it until the joint resistance was found to be $5000 \div 100$, or 50 ohms; lastly, in like manner we should adjust the $\frac{1}{1000}$th shunt until the combined resistance of the two became $5000 \div 1000$, or 5 ohms.

61. We have shown in a previous chapter that the deflections on the scale of a Thomson galvanometer, except when they are nearly equal, are not directly proportional to the current strengths which produce them, and that to compare them a formula must be used. If we wish to avoid the use of this formula we must adopt some method of avoiding widely different deflections. This we can do by using a variable shunt for the galvanometer, and with it obtaining either equal, or nearly equal, deflections for all measurements made in one set of tests.

The graduated scale of any galvanometer, it should be recollected is not necessarily for the purpose of enabling the strengths of two or more currents to be compared by different deflections, but is also for that of enabling any deflection which may be obtained to be reproduced when required.

62. It is best to obtain as high a deflection as possible, for then not only will a slight variation from the correct resistance of the shunt produce a greater number of degrees of variation from the deflection required, than would be the case if a low deflection was used, but also a higher resistance being required for the shunt, a greater range of adjustment is given to it.

63. By the help of the points we have just considered we can graduate or *calibrate* (§ 37) the scale of a galvanometer. To do this, first calculate from the known resistance of the galvanometer, the resistance of shunts required to reduce the amount of current passing through the galvanometer when no shunt is inserted, to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., the amount passing when a shunt is

inserted, then the resistance of the shunts necessary to reduce the current to

$$\tfrac{1}{2},\ \tfrac{1}{3},\ \tfrac{1}{4},\ \tfrac{1}{5},\ \ldots\ \tfrac{1}{n}\text{th}$$

will, as we have shown, be

$$1,\ \tfrac{1}{2},\ \tfrac{1}{3},\ \tfrac{1}{4},\ \ldots\ \tfrac{1}{n-1}\text{th}$$

of the resistance of the galvanometer. Now, as we have also shown, the insertion of shunts reduces the resistance of the circuit in which the galvanometer is placed; we must therefore also calculate the resistances necessary to be inserted in the circuit in order to compensate for the reduction of resistance which takes place when a shunt is inserted. These resistances will be respectively

$$\tfrac{1}{2},\ \tfrac{2}{3},\ \tfrac{3}{4},\ \tfrac{4}{5},\ \ldots\ \tfrac{n-1}{n}\text{th}$$

of the resistance of the galvanometer.

The shunts and their compensating resistances being calculated, to calibrate the galvanometer we proceed as follows:—

The galvanometer, a resistance box, and a battery are joined up in circuit. The $\frac{1}{2}$ shunt, that is, the shunt equal in resistance to the galvanometer, is then inserted, together with the corresponding compensating resistance in the resistance box. Sufficient resistance is now added in the latter to bring the deflection down to, say, 1°; the shunt and compensating resistance are then removed, and as the resistance in circuit is the same as before, and also the whole of the current passing in the circuit now passes through the galvanometer, the strength of current affecting it is exactly double that which deflected the needle originally; the deflection of the needle, therefore, now represents a strength of current double that of the previous experiment. We next insert the $\frac{1}{3}$ shunt and its compensating resistance, and by again adjusting the resistance coils, obtain a deflection of 1°; on now removing the shunt and compensating resistance we get three times the strength of current passing through the galvanometer: the deflection obtained therefore will represent that strength, and so by inserting

$$\tfrac{1}{4},\ \tfrac{1}{5},\ \ldots\ \tfrac{1}{n}\text{th}$$

shunts one after another, and repeating the process described, we can get the deflections corresponding to strengths of current equal to 1, 2, 3, 4, ... n, and the scale can be marked correspondingly; or these deflections and the corresponding currents producing them can be embodied in a table, so that by referring

to it we can at once see the relative powers of various currents giving different deviations of the needle.

64. By the help of this method of calibrating a galvanometer we can determine its *angle of maximum sensitiveness* (§ 27). All we have to do is to obtain various deflections of the galvanometer needle with various shunts and their corresponding compensating resistances, and in each case to increase the deflection of the needle slightly by reducing the compensating resistances by the same amount; then the required angle will be the one at which the diminution of resistance produced the greatest increase of deflection.

65. It is evident that if, in making a measurement, we want to reduce the deflection of our galvanometer to a readable quantity, we can do so, either by placing a large resistance in its circuit, or by introducing a shunt between its terminals. It is possible also, in certain cases, to produce the same effect by placing a shunt between the poles of the battery, but this is not always advisable, as it interferes with the constancy of the latter.

If the resistance of the battery and galvanometer in a simple circuit be very high it requires a very considerable increase of resistance in the circuit to produce an alteration in the deflection of the galvanometer needle, whereas just the reverse is the case if a shunt be used to produce that effect. This fact is an important one, as it has a considerable bearing upon the accuracy with which measurements can be made.

CHAPTER V.

MEASUREMENT OF GALVANOMETER RESISTANCE.

Half Deflection Method.

66. The simplest method of determining the resistance of a galvanometer is perhaps the one we have already given on page 5 (§ 8). In this method it will be seen we joined up the galvanometer, whose resistance (G) was required, in circuit with a resistance ρ, and a battery of very low resistance, and having obtained a certain deflection we increased ρ to R, so that the current passing in the circuit became halved in strength, the resistance (G) of the galvanometer was then given by the formula

$$G = R - 2\,\rho.$$

If we were measuring the resistance of a *tangent* galvanometer, the deflections obtained should be such that the tangent of one deflection is half the tangent of the other, the precaution against having the deflections too high or too low being duly taken (§ 27).

For example.

With a tangent galvanometer whose resistance (G) was to be determined, and a battery whose resistance was very small, we obtained with a resistance in the resistance box (as the set of resistance coils is sometimes termed) of 10 ohms (ρ) a deflection of 58°, and by increasing the resistance to 120 ohms (R) the deflection was reduced to $38\frac{1}{2}°$ ($\tan 38\frac{1}{2}° = \frac{1}{2} \tan 58°$); what was the resistance of the galvanometer?

$$G = 120 - 2 \times 10 = 100 \text{ ohms.}$$

67. In measuring the resistance of an ordinary galvanometer by this method it would be necessary to know what ratio the deflections bore to the current strengths producing them, so that the resistances may be adjusted accordingly.

A convenient arrangement is to employ a tangent galvanometer of a known low resistance in circuit with the

galvanometer whose resistance is required, and to take the readings from the tangent galvanometer, the resistance then obtained from the formula will evidently be the resistance of the two galvanometers together. If, then, we subtract from this result the known resistance of the tangent galvanometer, we get the resistance we are trying to obtain. If we have not a tangent galvanometer at hand, and if moreover we cannot tell what ratio the deflections bear to the current strengths producing them, we must of course employ a different method of testing.

68. In this, and indeed in all tests, it is important to consider what resistances and battery power should be employed to make the measurements, so that the greatest possible accuracy may be ensured.

If we employ very high resistances to measure a low resistance, a considerable alteration in the former would produce but little alteration in the current flowing through the galvanometer, for the electromotive force being constant, this current, and consequently the galvanometer deflection, is dependent upon the total resistance in the circuit, and an alteration of several units in a large total practically leaves its value the same, but then a few units too much or too little inserted in a formula may make the result appear very much greater or less than its true value. Thus, in the test we have been considering, suppose the battery power had been such that we found it necessary to have the resistance $\rho = 2000$ ohms, and that to halve the deflection we found it necessary to increase ρ to 4100 ohms (R), this would make the resistance of the galvanometer to be, as we saw before,

$$G = 4100 - 2 \times 2000 = 100 \text{ ohms}.$$

Now, practically, if the resistance R had been made 4200 ohms the deflection would have been halved; whatever difference there was would scarcely be appreciable.

If now we work the result out from the formula we get

$$G = 4200 - 2 \times 2000 = 200 \text{ ohms},$$

or double what it ought to be. It is possible indeed that the error might be greater than this. The test, in fact, would be quite useless.

In order to have the best chance of accuracy we should make our resistances as low as possible, for then a small change or error in the latter produces the greatest increase or decrease in the current, and consequently in the deflection of the galvano-

meter needle, and, on the other hand, it produces the smallest error in the value of G, when the latter is worked out from the formula.

In order to make R as low as possible it is evident that we must make ρ as low as possible.

69. What *degree of accuracy* is attainable in making the test? This is dependent upon the "total possible percentage of error which may exist in the second deflection" (§ 35). We have then to consider what error in the value of G the total error in the second deflection will cause.

The error in G must be occasioned by the value of R being obtained incorrectly, this wrong value of R being due to an error made in reading the magnitude of the second deflection. If in the formula

$$G = R - 2\rho$$

we make a mistake of, say, λ'_1 per cent. in R, then the resulting percentage error, λ', in G will be $\lambda' \doteq \lambda'_1 \frac{R}{G}$.

Now the accuracy with which we can adjust R is directly dependent upon the accuracy with which we can adjust (G + R) for the latter is the *total* resistance in the circuit of the galvanometer and therefore any change or error made in the value of the galvanometer deflection (the second deflection) must be in direct proportion to the change or error made in (G + R); consequently if we are liable to make an error of γ' per cent. in the value of the second deflection, and an error of λ'_1 per cent. in R, then we must have

$$\gamma' : \lambda'_1 :: R : G + R$$

or

$$\lambda'_1 = \frac{(G + R)\gamma'}{R};$$

but

$$\lambda' = \lambda'_1 \frac{R}{G}, \quad \text{or,} \quad \lambda'_1 = \lambda' \frac{G}{R},$$

and

$$G = R - 2\rho, \quad \text{or,} \quad R = G + 2\rho,$$

therefore

$$\lambda' \frac{G}{R} = \frac{2(G + \rho)\gamma'}{R};$$

hence

$$\lambda' = 2\left(1 + \frac{\rho}{G}\right)\gamma'. \qquad [A]$$

For example.

In measuring the resistance of the galvanometer in the example given in (§ 66), it was known that the "total possible percentage of error (γ') which could exist in the second deflection" could not exceed $1 \cdot 7$ per cent. (Example 2, § 35). What would be the percentage of accuracy (λ') with which the value of G could be determined?

$$\lambda' = 2\left(1 + \frac{10}{100}\right) 1 \cdot 7 = 3 \cdot 7 \text{ per cent.}$$

A single cell of a battery is the lowest electromotive force that can be practically employed in making the test, but we may find that this one cell gives too low a deflection with the lowest value we can give to ρ, that is 0, and two cells too high a deflection; we should have, therefore to employ two cells and then increase ρ until the proper deflection is obtained. Now on pages 62 and 63 it was pointed out that it is best to make ρ of a low value so that the deviation of the needle from its correct position, when R is not correctly adjusted, may be as great as possible; but equation [A], which represents the relative values of the errors λ' and γ', although it shows that the error λ' is smallest when ρ is as small as possible, at the same time shows that we gain but little by making ρ *very* much smaller than G, for λ' is only twice as great when $\rho = 0$, as it is when $\rho = G$.

70. Practically we may say therefore that the

Best Conditions for making the Test

are to make ρ a fractional value of G; and in the case of a tangent galvanometer the two deflections obtained should be as nearly as possible 55° and $35\frac{1}{2}$°.

Also as regards the

Possible Degree of Accuracy attainable.

If we can determine the value of the deflection of the galvanometer needle to an accuracy of γ' per cent., then we can determine the value of G to an accuracy of $2\left(1 + \frac{\rho}{G}\right) \gamma'$ per cent.

If ρ is very small, then

$$\lambda' = 2\,\gamma';$$

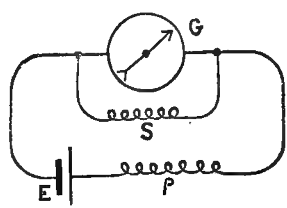
so that even under the best conditions for making the test the accuracy with which the value of G could be determined would be only one-half of the accuracy with which the deflections could be observed.

71. It must be understood that the resistance of the testing battery can only be neglected when it forms a small percentage of the total resistance in its circuit. If, then, the galvanometer to be measured has a low resistance, inasmuch as R will have to be proportionately small, the battery resistance can no longer be ignored without introducing an error; moreover, if R is made small, its range of adjustment becomes very limited. The test, therefore, is not suitable for measuring galvanometers whose resistance consists of a few units only.

EQUAL DEFLECTION METHOD.

72. The theory of this method is as follows:—The galvanometer whose resistance G is required, a resistance ρ, and a battery E of very low resistance, are joined up in circuit, as shown by Fig. 30, a shunt S being between the terminals of the galvanometer; a deflection of the galvanometer needle is produced. Let C be the current flowing out of the battery, then

FIG. 30.



$$C = \frac{E}{\rho + \frac{G\,S}{G + S}}.$$

This current divides into two parts, one part going through S, and the other part through the galvanometer. It does this in the inverse proportion of the resistance of those circuits, the part, C_1, going through the galvanometer being

$$C_1 = \frac{E}{\rho + \frac{G\,S}{G + S}} \times \frac{S}{G + S} = \frac{E\,S}{S(G + \rho) + G\,\rho}.$$

The shunt S is now removed; this causes the deflection of the galvanometer needle to be increased. ρ is now increased to R, so that the deflection becomes the same as it was previous to the removal of the shunt, or in other words, so that the strength

of the current passing through the galvanometer is C_1, then

$$C_1 = \frac{E}{R + G};$$

therefore

$$\frac{E\,S}{S(G + \rho) + G\,\rho} = \frac{E}{R + G}.$$

By dividing both sides by E and multiplying up, we get

$$S\,R + G\,S = G\,S + S\,\rho + G\,\rho;$$

therefore

$$G\,\rho = S\,R - S\,\rho,$$

from which

$$G = S\,\frac{R - \rho}{\rho}.$$

For example.

A galvanometer whose resistance (G) was required, was joined up in circuit with a resistance of 200 ohms (ρ), a shunt of 10 ohms (S) being between the terminals of the galvanometer.

On removing the shunt, it was necessary in order to reduce the increased deflection to what it was originally, to increase ρ to 2200 ohms (R). What was the resistance of the galvanometer?

$$G = 10\,\frac{2200 - 200}{200} = 100 \text{ ohms}.$$

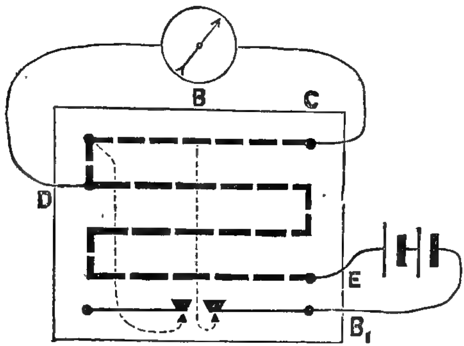
73. In making this test practically, we should proceed thus:—Join up the instruments, as shown by Fig. 31, taking care that the two infinity plugs are firmly in their places. Plug up the three holes between B and C, and remove the necessary plugs between D and B. Next remove plugs from between D and E, so as to introduce the resistance ρ. On the right-hand key being depressed the deflection of the galvanometer needle is obtained. The galvanometer should be gently tapped with the finger in order to see that the needle is properly deflected and is not sticking, as it is very liable to do, especially when a compass suspended needle is used.

The oscillations of the needle may be arrested by a skilful manipulation of the key; slightly raising it when the needle swings under the influence of the current and again depressing it when it recoils.

The needle being steadily deflected, and the precise resist-

ance (ρ) in the box noted, the left-hand infinity plug must be removed, and the resistance between D and E increased until the deflection becomes the same as it was at first, and the resistance (R) being noted, the formula is worked out.

FIG. 31.



74. What are the best values of S and ρ to employ in making a test like this? Should we make S and ρ of low, high, or medium values?

The answer to these queries has an important bearing upon the accuracy with which the test can be made; and as we shall more than once have to consider questions of a similar kind, we shall in the present instance enter at some length into the problem.

There are two quantities whose values we have to determine, viz. S and ρ; let us first consider what S should be, supposing R to be a given quantity and ρ to vary along with S.

If we examine the formula we shall see that if we make S small, then an error of one or two units in the correct value of R will make a much greater difference in the formula than would be the case when there is the same number of units of difference with S large; thus to take a numerical example, suppose we had the following values in the formula :—

$$G = 5\,\frac{500 - 20}{20} = 120 \text{ ohms},$$

and suppose we made R 120 units too large, we should then have

$$G = 5\,\frac{620 - 20}{20} = 150 \text{ ohms};$$

or an error of $150 - 120 = 30$ ohms. Next let us suppose we had the following values:—

$$G = 480\,\frac{500 - 400}{400} = 120 \text{ ohms},$$

and as before let there be an error of 120 units in R, we then have

$$G = 480\,\frac{620 - 400}{400} = 264;$$

or an error of $264 - 120 = 144$ ohms, and if S and ρ had been higher still we should have seen that the error would have been still greater.

To put the case in another way; in the last example let us suppose the error in R had been, not 120 units, but 25 units; that is, make $R = 500 + 25 = 525$; we then find that

$$G = 480\,\frac{525 - 400}{400} = 150 \text{ ohms}.$$

The error in G, in fact, in the former case, where R was 120 units too large, was no greater than it was in the latter case, when the excess in the correct value of R was but 25 units, From this it must be evident that it is highly advantageous to make S as *small* as possible. Let us, however, put the matter in an algebraical form; thus, let λ be the error in G, and let ϕ be the excess in the value of R which causes this error, then we have

$$G + \lambda = S\,\frac{R + \phi - \rho}{\rho} = S\,\frac{R - \rho}{\rho} + S\,\frac{\phi}{\rho},$$

and

$$G = S\,\frac{R - \rho}{\rho}, \quad \text{or,} \quad \rho = \frac{S\,R}{G + S};$$

therefore by subtraction,

$$\lambda = S\,\frac{\phi}{\rho} = S\,\phi \times \frac{G + S}{S\,R} = \frac{\phi\,(G + S)}{R}.$$

From this we see that with a constant error ϕ, made in R, the corresponding constant error λ, made in G, will be as small as possible when S is very small, as indeed we before proved; but we also see that we gain but little by making S a very small fractional value of G, for the error would be only twice as great

with S = G as it would be if S were very nearly = 0. It would not do, however, to make S greater than G, for G + S increases very rapidly by increasing S. Practically, therefore, we may say: make S a fractional value of G.

We have next to determine what is the best value to give to ρ, supposing S to be a fixed quantity.

Now if we put the equation

$$G = S\frac{R - \rho}{\rho}$$

in the form

$$G = S\left(\frac{R}{\rho} - 1\right),$$

we can see that whatever value ρ has, R will have an exactly proportional corresponding value; thus to take the example we first had, viz.:

$$G = 5\,\frac{500 - 20}{20} = 5\left(\frac{500}{20} - 1\right);$$

if in making the test we had made $\rho = 2 \times 20 = 40$, instead of 20, then the value to which R would have required to have been adjusted would have been $2 \times 500 = 1000$, instead of 500. Further, if R had had this value, then an error of 20 units in R would have produced the same error in G as would the 10 units in the first case, when R was 500. At first sight then it might appear that it would not matter what value we gave to ρ. Let us, however, consider in what way the adjustment of R is effected.

The means by which we adjust R is by observing the deflection of the galvanometer needle, and seeing whether we have brought it to the deflection it had when ρ and S were the resistances in the circuit; when this deflection is correct we know that R is correct. But the accuracy with which we can adjust R evidently depends upon the divergence of the needle from its correct position being as large as possible when R is not exactly adjusted, and if this divergence is greater when we alter R from 1000 to 1020 ohms than when we alter it from 500 to 510 ohms, then it is better so to arrange the value of ρ that R shall be 1000 ohms.

Or in other words, if the error in R, corresponding to a constant error in G, produces a greater divergence of the needle from its correct position when R is large than when it is small, then it is better to have R large than small.

Now the current C producing the deflection of the galvanometer needle is

$$C = \frac{E}{R + G},$$

and if we suppose there to be a diminution $-c$, in C, caused by an error ϕ, in R, then we have

$$C - c = \frac{E}{R + \phi + G};$$

or

$$c = C - \frac{E}{R + \phi + G};$$

but we know that

$$C = \frac{E}{R + G};$$

therefore

$$c = \frac{E}{R + G} - \frac{E}{R + \phi + G} = \frac{E\,\phi}{(R + \phi + G)\,(R + G)};$$

or, since ϕ is very small,

$$c = \frac{E\,\phi}{(R + G)^2};$$

c, however, represents the *absolute* change from the correct current and as the latter is itself varied by the value of R, what we require to know is the *relative* change; this will be

$$\frac{c}{C},$$

which equals

$$\frac{E\,\phi}{(R + G)^2} \div \frac{E}{R + G} = \frac{\phi}{R + G}. \qquad [A]$$

But from page 68 we see that the constant error λ, caused in G by an error ϕ in R, is

$$\lambda = \frac{\phi\,(G + S)}{R},$$

or

$$\phi = \frac{\lambda\ R}{G + S};$$

substituting, then, this value of ϕ, we get

$$\frac{c}{C} = \frac{\lambda R}{(G+S)(R+G)} = \frac{\lambda}{(G+S)\left(1+\frac{G}{R}\right)}. \qquad [B]$$

From this equation we see that in order to make c as large as possible, we must make R as large as possible; but it is evident that we increase c very little by making R much larger than G, for the reason we gave when we determined the ratio which S should have to G.

We do not gain, then, anything as regards the sensitiveness of the arrangement by making R very large, but we gain as regards our power of adjusting R, for we can adjust a resistance with a much closer degree of accuracy when it consists of a large number, than when it consists of a small number, of units.

It is therefore advantageous to make R as large as possible.

Since when S, G, and ρ are given values, R must have a value dependent upon them; and since we have determined the value we must give to S, it follows that the value we should give to ρ must be such that R will be as large as possible.

As we cannot make R larger than the resistance we can insert in the resistance box, we must not make ρ so large that R will have to exceed that value.

From the equation

$$G = S\frac{R-\rho}{\rho}$$

we see that

$$\rho = \frac{S}{G+S}R.$$

Theoretically, therefore, we must not make ρ larger than the value we can give to $\frac{S}{G+S}R$.

The highest resistance we can practically give to R is 10,000 ohms; ρ, therefore, must not be larger than $\frac{S}{G+S} \times$ 10,000 ohms. Thus, if we use a shunt whose resistance is $\frac{1}{9}$th the resistance of the galvanometer, we must not make ρ larger than $\frac{1}{10}$th of 10,000, that is 1000, ohms.

Equation [A] shows that the value of c is dependent upon the value of S, and that to make c large we should make S small. We previously proved, however, that there was another reason

why S should be small, consequently we have a double reason why S should have a low value.

75. What *degree of accuracy* is attainable in making the test? This, as in the last test, is dependent upon the value of the deflection error. We have, in fact, to consider what error in the value of G a definite error in reading the deflection of the galvanometer needle will cause.

This we can determine from equation [B]. Let us, then, in this equation substitute *percentages* for absolute values, that is to say, let us have

$$c = \frac{\gamma'}{100} \text{ of C, or, } \frac{c}{C} = \frac{\gamma'}{100}.$$

and

$$\lambda = \frac{\lambda'}{100} \text{ of G};$$

then we get

$$\frac{\gamma'}{100} = \frac{\lambda' \, G \, R}{100 \, (G + S) \, (R + G)},$$

that is to say,

$$\lambda' = \left(1 + \frac{S}{G}\right)\left(1 + \frac{G}{R}\right)\gamma'. \qquad [C]$$

For example.

In measuring the resistance of the galvanometer in the example given on page 66 (§ 72), it was known that the possible error γ' in the current, due to the deflection being incorrect, would not exceed ·88 per cent. (Example § 34). What would be the percentage of accuracy (λ') with which the value of G could be determined?

$$\lambda' = \left(1 + \frac{10}{100}\right)\left(1 + \frac{100}{2200}\right) \cdot 88 = \cdot 96 \text{ per cent.}$$

76. The practical results, then, that we have arrived at from these investigations are, that to obtain the

Best Conditions for making the Test:

First make a rough test to ascertain approximately the value of G. Having done this, insert a shunt (S) between the terminals of G, of a fractional value of the resistance of G.

Next join up ρ in circuit with G and its shunt S, making ρ as large as possible, but not larger than $\frac{S}{G+S}R$; R being the highest resistance that can be obtained.

Insert in the circuit sufficient battery power of low resistance to obtain the deflection of the galvanometer needle as nearly as possible at the *angle of maximum sensitiveness* (§ 27), adjusting ρ, if necessary, so that this angular deflection becomes exact, and note the exact value of ρ.

Now remove the shunt and increase ρ to R, so that the increased deflection becomes the same as it was at first. Note R, and then calculate G from the formula.

Possible Degree of Accuracy attainable.

If we can determine the value of the galvanometer deflection to an accuracy of γ' per cent., then we can determine the value of G to an accuracy (λ') of

$$\lambda' = \left(1 + \frac{S}{G}\right)\left(1 + \frac{G}{R}\right)\gamma' \text{ per cent.}$$

If S is very small, and R very large, then

$$\lambda' = \gamma',$$

so that under the best conditions for making the test, the accuracy with which the value of G could be determined would be the same as the accuracy with which the value of the deflection could be observed.

77. In the practical execution of the test, inasmuch as there are only three resistances between D and B (Fig. 31) our choice of a shunt is limited from this source, but these three will usually be sufficient for most purposes.

78. The method we have described of making the test may be modified by making S or ρ the adjustable resistances instead of R, but in either of these cases it can be shown, by an investigation precisely similar to the one we have made, that the proper values of the resistances should be those we have indicated.

FAHIE'S METHOD.

79. If in the last test we make S the adjustable resistance, and make R = 2 ρ, we get

$$G = S\frac{R-\rho}{\rho} = S\frac{2\rho - \rho}{\rho} = S,$$

that is, the resistance of the shunt will be the resistance of the galvanometer.

80. The connections for making the test with the set of resistances shown by Fig. 31 would have to be so arranged that the resistances between D and E form the shunt, and those between D and C the resistances ρ and R. This arrangement, however, in consequence of there being so few plugs between D and C, is not a satisfactory one, as some difficulty would probably be found in adjusting the battery power and resistance R so as to obtain the deflection of maximum sensitiveness. With two sets of resistance coils, however, the test can easily be made.

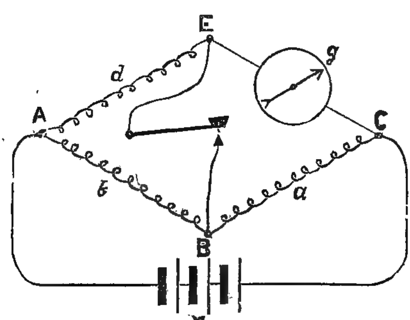
As in the previous method, it is best to make the resistance R as high as possible, for then any small change in the value of S produces the greatest movement of the galvanometer needle. The *possible degree of accuracy attainable* is the same as in the last test.

In order that satisfactory results may be obtained in the foregoing tests, it is necessary that the galvanometer be a *sensitive* one (§ 53), otherwise even a moderate degree of accuracy cannot be assured.

Thomson's Method.

81. Join up the galvanometer g with resistances a, b, and d, and a battery of electromotive force E and resistance r, as shown by Fig. 32, and let a key be inserted between the points E

Fig. 32.



and B, so that by its depression these points can be connected together.

First, let us suppose the key to be up and the points con-

sequently disconnected. The current C_1 flowing through the galvanometer will then be

$$C_1 = \frac{E}{r + \frac{(a+b)(d+g)}{a+b+d+g}} \times \frac{a+b}{a+b+d+g}$$

$$= \frac{E(a+b)}{r(a+b+d+g)+(a+b)(d+g)}. \qquad [1]$$

Next, suppose the key to be depressed and the points E and B thereby to be connected together, then the current (C_2) flowing through the galvanometer will be

$$C_2 = \frac{E}{r + \frac{b\,d}{b+d} + \frac{a\,g}{a+g}} \times \frac{a}{a+g} =$$

$$\frac{E\,a\,(b+d)}{r\,(a+g)(b+d) + a\,g\,(b+d) + b\,d\,(a+g)}. \qquad [2]$$

Further, let us suppose the adjustment of the resistances to be such that

$$C_1 = C_2,$$

we then get

$$\frac{E\,(a+b)}{r\,(a+b+d+g) + (a+b)(d+g)} =$$

$$\frac{E\,a\,(b+d)}{r\,(a+g)(b+d) + a\,g\,(b+d) + b\,d\,(a+g)}; \qquad [3]$$

by multiplying up and arranging the quantities we get

$$r\,[(a+b+g)(b+d)\,a + b\,g\,(b+d)] + b\,g\,(a+b)\,d + [d\,(b+g) + b\,g]\,(a+b)\,a = r\,[(a+b+g)(b+d)\,a + a\,d\,(b+d)] + a\,d\,(a+b)\,d + [d\,(b+g) + b\,g]\,(a+b)\,a;$$

therefore

$$b\,g\,[r\,(b+d) + (a+b)\,d] = a\,d\,[r\,(b+d) + (a+b)\,d];$$

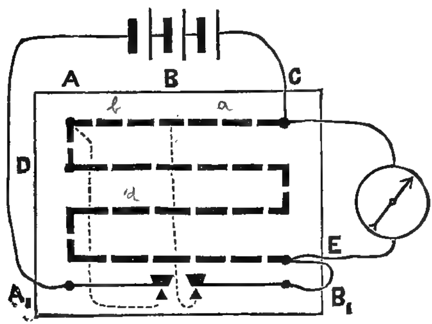
that is,

$$a\,d = b\,g, \quad \text{or,} \quad g = \frac{a\,d}{b}.$$

A great advantage of this test is the fact of its being entirely independent of the battery resistance. It is also very easily made, as must be evident.

In making the test practically, the connections would be made as shown by Fig. 33. The terminals E and B_1 would be joined by a short piece of thick wire. The other connections are obvious.

FIG. 33.



The left-hand key (which is not shown in the theoretical figure) being first depressed and then kept permanently down, the right-hand key must be alternately depressed and raised, the resistance d, that is the resistance between A and E, being at the same time adjusted until the deflection of the galvanometer needle remains the same whether the key is up or down.

82. We will now determine the best arrangement of resistances for making the test.

What we have to do is to suppose that in the equation

$$g = \frac{a\,d}{b}$$

there is a small but constant error in g, caused by a corresponding error in one of the other quantities, let us say d, and then find what values of d and, say, a, will cause the alteration of the deflection of the galvanometer needle produced on raising and depressing the key, to be as large as possible.

Let λ be the difference between the exact value of g and the value given it by the formula when we have d too large, and let the increased value of d be d_1.

We then have

$$g + \lambda = \frac{a\,d_1}{b};$$

therefore

$$a\, d_1 = b\, g + b\, \lambda.$$

We next have to determine what the alteration in the strength of the current passing through the galvanometer, produced by raising and depressing the key, is equal to.

If in either equation [1] or equation [2] we put $b\, g$ equal to $a\, d$, or b equal to $\frac{a\, d}{g}$, then the resulting equation will give the current, C, which would flow through the galvanometer when the adjustment is exact; by doing this we get

$$C = \frac{E\, a}{r\,(g + a) + a\,(d + g)}.$$

When the adjustment is not exact, the currents produced on raising and depressing the key will be obtained by equations [1] and [2], and the difference between these two currents relative to the current produced when exact equilibrium is obtained will give the relative current producing the alteration in the deflection of the galvanometer needle; hence we find

$$\frac{C_2 - C_1}{C} =$$

$$\frac{r\,(g + a) + a\,(d + g)}{a}\left\{\frac{a\,(d_1 + b)}{r\,(d_1 + b)\,(g + a) + d_1\, b\,(g + a) + g\, a\,(d_1 + b)} - \frac{b + a}{r\,(d_1 + g + b + a) + (d_1 + g)\,(b + a)}\right\} =$$

$$\frac{(a\, d_1 - b\, g)\,\{r\,(d_1 + b) + d_1\,(b + a)\}\,\{r\,(a + g) + a\,(d + g)\}}{a\{r(d_1 + g + b + a) + (d_1 + g)\,(b + a)\}\,\{r(d_1 + b)(g + a) + d_1 b(g + a) + g a(d_1 + b)\}};$$

but since $a\, d_1$ is very nearly equal to $b\, g$, we may without sensible error put $a\, d_1 = a\, d = b\, g$, or $b = \frac{a\, d}{g}$ except where differences are concerned; in which case we get

$$\frac{C_2 - C_1}{C} = \frac{g\,(a\, d_1 - b\, g)}{a\,(a + g)\,(d + g)};$$

and since $a\, d_1 = b\, g + b\;\lambda$, and $\frac{g}{a} = \frac{d}{b}$, we get

$$\therefore\; \frac{C_2 - C_1}{C} = \frac{\lambda\, d}{(a + g)\,(d + g)} = \frac{\lambda}{(a + g)\left(1 + \frac{g}{d}\right)}. \qquad \text{[A]}$$

From this it is evident that, in order to make $\frac{C_2 - C_1}{C}$ as large as possible, we must make d as large, and a as small, as possible. It is evident also that, as regards increasing $\frac{C_2 - C_1}{C}$, it is useless making d very much larger, or a very much smaller, than g. If we make d about ten times as large, and a ten times as small, as g, we shall have good conditions for ensuring accuracy, though as regards our power of adjustment, it would be advantageous to make d larger still if possible.

From the equation

$$b\,g = a\,d$$

we see that g being a fixed quantity, and a as small as possible, we can make d as large as we like by making b as large as possible.

83. It may be pointed out, that when a is small and d and b large, we have the battery connecting the junction of the two greater with the junction of the two lesser resistances.

84. What *degree of accuracy* is attainable in making the test? This we can determine from equation [A]. Let us then, in the latter, substitute *percentages* for *absolute* values, that is, let

$$C_2 - C_1 = \frac{\gamma'}{100} \text{ of } C, \quad \text{or,} \quad \frac{C_2 - C_1}{C} = \frac{\gamma'}{100}$$

and let

$$\lambda = \frac{\lambda'}{100} \text{ of } g;$$

then we get

$$\frac{\gamma'}{100} = \frac{\lambda' g}{100\,(a + g)\left(1 + \frac{g}{d}\right)},$$

that is,

$$\lambda' = \left(1 + \frac{a}{g}\right)\left(1 + \frac{g}{d}\right)\gamma'.$$

For example.

In measuring the resistance of a galvanometer by the foregoing method, the values of a, b, and d were 10, 100, and 300 ohms respectively. What was the resistance of the galvanometer, and what was the possible degree of accuracy attainable? The smallest change in the value of the galvanometer deflection which it was possible to observe,* was ·88 per cent. (§ 34).

* This is synonymous with "the degree of accuracy with which the value of the galvanometer deflection can be read" (page 27).

$$g = \frac{10 \times 300}{100} = 30 \text{ ohms.}$$

$$\lambda' = \left(1 + \frac{10}{30}\right)\left(1 + \frac{30}{300}\right) \cdot 88 = 1 \cdot 3 \text{ per cent.}$$

To sum up, we have

Best Conditions for making the Test.

85. Make a not greater than $\frac{1}{10}$th of g, and make b not less than ten times as great as g, and preferably as much higher than g as possible, but not of such a high value that d, when exactly adjusted, has to exceed all the resistance we can insert between D and E (Fig. 33).

Adjust d approximately, and then if necessary adjust the battery power, so that the final deflection is as possible that of maximum sensitiveness, and then, having exactly adjusted d, calculate g from the formula.

Possible Degree of Accuracy attainable.

If we can read the galvanometer deflection to an accuracy of γ' per cent., then we can determine the value of g to an accuracy (λ') of

$$\lambda' = \left(1 + \frac{a}{g}\right)\left(1 + \frac{g}{d}\right) \gamma' \text{ per cent.}$$

If a is small and d large, then we get

$$\lambda' = \gamma',$$

so that under the best conditions for making the test, the accuracy with which the value of G could be determined would be the same as the accuracy with which the value of a change in the deflection could be observed.

86. In the practical execution of the test with the set of resistance coils shown by Fig. 33, the lowest value we could give to a would be 10 units, unless we improvised a resistance of less value, which it might be necessary to do.

THOMSON'S METHOD WITH A SLIDE WIRE RESISTANCE.

87. The foregoing test is sometimes made by having $a + b$, a slide wire resistance (§ 17) d being a fixed resistance; in this case the slide would be moved along between A and C, until the

point is found at which the depression and raising of the key makes no alteration in the permanent deflection of the galvanometer needle.

As in the equation

$$r = \frac{a\,d}{b},$$

$\frac{a}{b}$ is merely the ratio of the resistances into which the total resistance $a + b$ is divided, and as the resistances are directly proportional to the lengths of the wire on either side of the slide, it is sufficient for a and b to be expressed in terms of the divisions into which the length of wire is divided.

Now as the total length, k, of the slide wire is constant, that is as

$$a + b = k, \quad \text{or,} \quad b = k - a,$$

therefore we must have

$$g = d\left(\frac{a}{k - a}\right).$$

k is usually divided into 1000 divisions, hence

$$g = d\left(\frac{a}{1000 - a}\right).$$

For example.

In the foregoing test, equilibrium was produced when d was 1 ohm, and a, 450 divisions; what was the resistance of the galvanometer?

$$g = 1\left(\frac{450}{1000 - 450}\right) = \frac{450}{550} = \cdot 85 \text{ ohms.}$$

88. *The best conditions for making the test* in the case where a slide wire is used, are generally similar to those in the previous case, that is to say, we should require to have a small and d large.

Now, the total resistance of $a + b$ in the case of a slide wire would, under most conditions met with in actual practice, be small compared with g; consequently a would be small also. For this reason, therefore, it would not signify what were the relative values of a and b in making the test. But in order to make d large, it is obvious that a must be small compared with b; thus, if d is to be 10 times g, then a must be 10 times b. In the first case, when the test was made by adjusting d, it was pointed out that although there is an advantage in making d

as large as possible, in so far that by so doing the range of adjustment is made large, yet as regards the general sensitiveness of the whole arrangement, there is little, if any, advantage in making d greater than about 10 times g. In the case of the slide wire, where d is not the adjustable resistance, there is an actual disadvantage in making d excessively large, for the reason that, if we do so, we make a correspondingly smaller than b, and when this is the case, the error in g (when worked out by the formula $g = \frac{a\,d}{b}$) produced by the value of a being, say, 1 scale division out, becomes comparatively large. Thus, if the slide wire scale were graduated into 1000 divisions (which is usually the case), it is clear that if the slider stood at, say, the "10" division mark on the scale, then an alteration or a mistake of 1 division would mean a change of 10 per cent. in the value of a, whilst if the slider stood at "100," then a change of 1 division would only mean a 1 per cent. change in the value of a. The change in a corresponding to a movement of 1 division, would be obviously less if the slider were near the centre of the scale, that is, near the "500" division mark, but in this case the increase in the range of adjustment would be more than compensated for by the reduced sensitiveness of the arrangement. The *possible degree of accuracy attainable* in making the test would be as follows:—

Let there be an error λ in g, caused by the slider being δ divisions out of correct adjustment, then we have

$$g + \lambda = d\left(\frac{a + \delta}{1000 - (a + \delta)}\right)$$

or

$$\lambda = d\left(\frac{a + \delta}{1000 - (a + \delta)}\right) - g = d\left[\frac{a + d}{1000 - (a + d)} - \frac{a}{1000 - a}\right]$$

$$= \frac{d\,1000\,\delta}{(1000 - a)^2}$$

since δ is very small.

If we put percentages instead of absolute values, that is to say, if we have

$$\lambda = \frac{\lambda'}{100} \text{ of } g = \frac{\lambda'}{100} \times d\left(\frac{a}{1000 - a}\right),$$

then we get

$$\lambda' = \frac{100000\delta}{a\,(1000 - a)} \text{ per cent.}$$

If the galvanometer is sufficiently sensitive to enable the position of the slider to be determined to an accuracy of 1 division, then $\delta = 1$.

For example.

In the last example, what would be the degree of accuracy, λ', with which the value of g could be obtained, supposing that the position of the slider could be determined to an accuracy of 1 division (δ)?

$$\lambda' = \frac{100000 \times 1}{450\,(1000 - 450)} = \cdot 40 \text{ per cent.}$$

89. The facility and accuracy with which all the foregoing tests (except the half deflection test) can be made may be greatly increased by the following device:—Instead of making the test with the galvanometer needle brought to the "angle of maximum sensitiveness" (§ 28), make it with the needle brought approximately to zero by means of a powerful permanent magnet set near the instrument. Under these conditions the galvanometer needle will be highly sensitive to any small change in the current strength.

90. In the case of Thomson's test with the slide wire, if the test is made by using a permanent magnet in the manner described, it is best to make d of a higher value than would otherwise be the case; for then, since the slider would have to be set near the centre of the wire, a greater range of adjustment is given to it, for 5 divisions near the centre portion of the wire (500 division mark) is equivalent to only 1 division near the 100 division mark. It is true that the arrangement is not quite so sensitive as when the slider has to be set towards the end of the scale; but still if *sufficient* sensitiveness be obtained, the small loss is more than compensated for by the advantage gained in having an increased range on the scale.

91. In order that satisfactory results may be obtained in the foregoing tests, it is necessary that the galvanometer be "sensitive" (§ 53), otherwise even a moderate degree of accuracy cannot be assured.

Diminished Deflection Direct Method.

92. This method, which has been generally described in Chapter I. (§ 5), is as follows:—The galvanometer G, a battery of low resistance, and a resistance ρ, are joined up in simple circuit; the deflection obtained is noted. Let this deflection be

due to a current C_1, then calling E the electromotive force of the battery, we have

$$C_1 = \frac{E}{G + \rho}, \text{ or, } C_1 G + C_1 \rho = E.$$

The resistance ρ is now increased to R, so that a new deflection due to a current, C_2, is produced; then we have

$$C_2 = \frac{E}{G + R}, \text{ or, } C_2 G + C_2 R = E;$$

hence

$$C_1 G + C_1 \rho = C_2 G + C_2 R,$$

or

$$G (C_1 - C_2) = C_2 R - C_1 \rho,$$

therefore

$$G = \frac{C_2 R - C_1 \rho}{C_1 - C_2}. \qquad [A]$$

In the case of a tangent galvanometer, if the deflections, D and d, are read from the *tangent* scale, then those deflections can be directly substituted for the quantities C_1, C_2, for

$$D : d :: C_1 : C_2;$$

in this case, then, we have

$$G = \frac{d R - D \rho}{D - d}. \qquad [B]$$

(1.) *For example.*

With a tangent galvanometer whose resistance G was required, and a battery of very small resistance, we obtained with a resistance of 10 ohms (ρ) in the circuit, a deflection of 60 divisions (D) on the tangent scale of the instrument; when the resistance was increased to 230 ohms (R) the deflection was reduced to 20 divisions (d); what was the resistance of the galvanometer?

$$G = \frac{20 \times 230 - 60 \times 10}{60 - 20} = 100 \text{ ohms}.$$

If the readings are made from the *degrees* scale, then we must substitute the tangents of the deflections for the deflections themselves; the formula then becomes

$$G = \frac{\tan d^\circ R - \tan D^\circ \rho}{\tan D^\circ - \tan d^\circ}. \qquad [C]$$

(2.) *For example.*

In a measurement similar to the foregoing, the readings were made from the *degrees* scale of the instrument, and deflections of 50° (D°) and $21\frac{3}{4}°$ ($d°$) respectively were obtained with resistances of 10 ohms (ρ) and 229 ohms (R) in the circuit. What was the resistance of the galvanometer?

$$\tan 50° = 1{\cdot}1918, \text{ and, } \tan 21\tfrac{3}{4}° = {\cdot}3990,$$

therefore

$$G = \frac{{\cdot}3990 \times 229 - 1{\cdot}1918 \times 10}{1{\cdot}1918 - {\cdot}3990} = 100 \text{ ohms.}$$

93. If in equations [B] and [C] we have $\rho = 0$, that is to say, if we make the test by having at first no resistance in the circuit except that of the galvanometer itself, then we get

$$G = R\frac{d}{D - d} \qquad [D]$$

and

$$G = R\frac{\tan d°}{\tan D° - \tan d°}. \qquad [E]$$

94. What are the "Best conditions for making the test," and, what is the "Possible degree of accuracy attainable?" There are two points to be considered in the first question; one is—what value should ρ have? and the other—what should be the relative values of C_1 and C_2?

Now we are liable to make an error in reading the value of C_1, or an error in reading the value of C_2, or again we may make errors both in C_1 and C_2, but inasmuch as the result of two errors would, of course, be greater than one only, it is advisable to make the test under conditions which ensure the result of the double error being as small as possible. Let us, therefore, in equation [A] suppose that there is a small error, c_2, in C_2, and a small error, c_1, in C_1, the error c_2 being plus and c_1 minus, so that the resulting total error in G is as great as possible; also let λ be this total error, that is, let us have

$$G + \lambda = \frac{(C_2 + c_2)R - (C_1 - c_1)\rho}{(C_1 - c_1) - (C_2 + c_2)},$$

or

$$\lambda = \frac{(C_2 + c_2)R - (C_1 - c_1)\rho}{(C_1 - c_1) - (C_2 + c_2)} - G;$$

but

$$G = \frac{C_2 R - C_1 \rho}{C_1 - C_2}, \quad \text{or,} \quad R = \frac{G(C_1 - C_2) + C_1 \rho}{C_2}.$$

If we insert this value of R in the above equation, and multiply up, cancel, etc., then we get

$$\lambda = \frac{(C_1 c_2 + C_2 c_1)}{C_2 [(C_1 - c_1) - (C_2 + c_2)]} (G + \rho);$$

or, since c_1 and c_2 are very small, we may say

$$\lambda = \frac{C_1 c_2 + C_2 c_1}{C_2 (C_1 - C_2)} (G + \rho). \qquad \text{[F]}$$

From this equation we can see that if C_1 and C_2 have fixed values, then λ varies directly as $G + \rho$, consequently in order to make λ as small as possible, we must make ρ as small as possible; but we can also see that there is no great advantage in making ρ *very* much smaller than G.

We have next to consider what the relative values of C_1 and C_2 should be, ρ being taken as constant. In order to do this, we must assume C_1 to be constant, and then determine what value C_2 should have. We have then in equation [F] to find what value of C_2 makes λ as small as possible; to do this we require to make

$$\frac{C_1 c_2 + C_2 c_1}{C_2 (C_1 - C_2)}$$

as small as possible by variation of C_2.

Now

$$\frac{C_1 c_2 + C_2 c_1}{C_2 (C_1 - C_2)} = \frac{c_2}{C_1} \left[\frac{C_1 - C_2}{C_2} + \frac{C^2 (\kappa + 1)}{C_1 - C_2} + \kappa + 2 \right]$$

where $\kappa = \dfrac{c_1}{c_2}$; and since $\dfrac{c_2}{C_1}$ is constant, what we have to do is to make

$$\left[\frac{C_1 - C_2}{C_2} + \frac{C_2 (\kappa + 1)}{C_1 - C_2} + \kappa + 2 \right]$$

as small as possible.

Now

$$\left[\frac{C_1 - C_2}{C_2} + \frac{C_2 (\kappa + 1)}{C_1 - C_2} + \kappa + 2 \right] = \frac{C_1 - C_2}{C_2} \left[1 - \frac{C_2 \sqrt{\kappa + 1}}{C_1 - C_2} \right]^2$$
$$+ 2 \sqrt{\kappa + 1} + \kappa + 2,$$

and in order to make the latter as small as possible we must make $1 - \frac{C_2 \sqrt{\kappa + 1}}{C_1 - C_2}$ as small as possible, that is to say, we must make it equal to 0, therefore

$$1 - \frac{C_2 \sqrt{\kappa + 1}}{C_1 - C_2} = 0, \quad \text{or,} \quad C_1 - C_2 = C_2 \sqrt{\kappa + 1},$$

from which we get

$$C_2 (\sqrt{\kappa + 1} + 1) = C_1, \quad \text{or,} \quad C_2 = \frac{C_1}{\sqrt{\kappa + 1} + 1}. \quad [G]$$

The greatest possible value which λ could have would be that which would result when both the errors c_1 and c_2 existed, these two errors being of equal value, or rather c_2 being as large as c_1. If the deflections are read in *divisions*, then c_1 and c_2 would be equal; but if the deflections are read in *degrees*, then c_1 will be larger than c_2, in proportion as C_2 is smaller than C_1. In the case where the greatest possible error can exist, that is, when $c_2 = c_1$, or $\kappa = 1$, then we have

$$C_2 = \frac{C_1}{\sqrt{2} + 1} = \frac{C_1}{2 \cdot 4142}.$$

Practically we may make

$$C_2 = \frac{C_1}{3};$$

for although this does not give the exact minimum value to λ, yet the difference between it and the actual minimum is very small, thus if

$$C_2 = \frac{C_1}{2 \cdot 4142},$$

then from equation [F] we get

$$\lambda = c_1 \frac{C_1 + \frac{C_1}{2 \cdot 4142}}{\frac{C_1}{2 \cdot 4142}\left(C_1 - \frac{C_1}{2 \cdot 4142}\right)} (G + \rho) = \frac{c_1}{C_1} 5 \cdot 828 \, (G + \rho);$$

but if

$$C_2 = \frac{C_1}{3},$$

then

$$\lambda = c_1 \frac{C_1 + \frac{C_1}{3}}{\frac{C_1}{3}\left(C_1 - \frac{C_1}{3}\right)} (G + \rho) = \frac{c_1}{C_1} 6\cdot000 (G + \rho);$$

that is to say, the errors would be as

$$6\cdot000 \text{ to } 5\cdot828,$$

a difference which is of no practical importance.

If the readings were made from the *degrees* scale of a tangent galvanometer, then the error c_1 would be larger than the error c_2, in which case it would be actually an advantage to make C_2 equal to $\frac{C_1}{3}$ in preference to making it equal to $\frac{C_1}{2\cdot4142}$; thus, if c_1 were, say, 3 times as large as c_2, then the best value to give to C_2 would be

$$C_2 = \frac{C_1}{\sqrt{3+1}+1} = \frac{C_1}{3}.$$

The rule that C_2 should approximately equal $\frac{C_1}{3}$ may therefore be taken as the one which would enable satisfactory results to be obtained under all conditions. If the deflections, D, d, are read in *divisions*, then we must have

$$d = \frac{D}{3}$$

approximately. But if the deflections are in *degrees*, and we read from a tangent galvanometer, then we must have

$$\tan d^\circ = \frac{\tan D^\circ}{3}$$

approximately.

95. We have next to consider what is the "Possible degree of accuracy attainable" when ρ and G have any particular values; this we can ascertain from equation [F]. Let us, then, in this equation put percentages for absolute values, that is to say, let us have

$$\lambda = \frac{\lambda'}{100} \text{ of } G, \quad \text{or,} \quad \lambda' = \frac{100\,\lambda}{G},$$

then we get

$$\lambda' = \frac{(C_1 c_2 + C_2 c_1)\,100}{C_2\,(C_1 - C_2)}\left(1 + \frac{\rho}{G}\right). \qquad [H]$$

If the deflections are read in *divisions*, then the errors in both must be of the same absolute values; let each of these values be $\frac{1}{m}$th of a division, then we must have

$$\lambda' = \frac{\frac{1}{m}\,(D + d)\,100}{d\,(D - d)}\left(1 + \frac{\rho}{G}\right). \qquad [I]$$

For example.

In example (1) (§ 92) what would be the degree of accuracy with which the test could be made? The deflections could be read to an accuracy of $\frac{1}{4}$ of a division.

$$\lambda' = \frac{\frac{1}{4}\,(60 + 20)\,100}{20\,(60 - 20)}\left(1 + \frac{10}{100}\right) = 2 \cdot 8 \text{ per cent.}$$

If the deflections are read in *degrees* from a tangent galvanometer, then we must have

$$\lambda' = \frac{(\tan D^\circ\,\delta_2 + \tan d^\circ\,\delta_1)\,100}{\tan d^\circ\,(\tan D^\circ - \tan d^\circ)}\left(1 + \frac{\rho}{G}\right) \text{ per cent.}$$

where δ_1 and δ_2 are of the respective values

$$\delta_1 = \tan D\tfrac{1}{m}^\circ - \tan D, \quad \text{and,} \quad \delta_2 = \tan d\tfrac{1}{m}^\circ - \tan d^\circ,$$

$\frac{1}{m}^\circ$ being the possible error in the deflections.

For example.

In example (2) (§ 92) what would be the degree of accuracy with which the test could be made? The deflections could be read to an accuracy of $\frac{1}{4}^\circ$.

$$\delta_1 = \tan 50\tfrac{1}{4}^\circ - \tan 50^\circ = \cdot 0106,$$

and

$$\delta_2 = \tan 22^\circ - \tan 21\tfrac{3}{4}^\circ = \cdot 0050\,;$$

therefore

$$\lambda' = \frac{(1 \cdot 1918 \times \cdot 0050 + \cdot 3990 \times \cdot 0106)\,100}{\cdot 3990\,(1 \cdot 1918 - \cdot 3990)}\left(1 + \frac{10}{100}\right) =$$

$3 \cdot 6$ per cent.

To sum up, then, we have

Best Conditions for making the Test.

96. Make ρ as small as possible.

Make R of such a value that when the deflections, D, d, are in *divisions*, then

$$d = \frac{D}{3}$$

approximately; and when the deflections are in *degrees* on a tangent galvanometer, then

$$\tan d^\circ = \frac{\tan D^\circ}{3}$$

approximately.

Possible Degree of Accuracy attainable.

If the deflections are in *divisions*, and if we can read their value to an accuracy of $\frac{1}{m}$th of a division, then we can determine the value of G to an accuracy, λ', of

$$\lambda' = \frac{\frac{1}{m}(D + d)\,100}{d\,(D - d)}\left(1 + \frac{\rho}{G}\right) \text{ per cent.}$$

If the deflections are in *degrees* on a tangent galvanometer, then if we can read their value to an accuracy of $\frac{1}{m}$th of a degree, we can determine the value of G to an accuracy, λ', of

$$\lambda' = \frac{(\tan D^\circ\,\delta_2 + \tan d^\circ\,\delta_1)\,100}{\tan d^\circ\,(\tan D^\circ - \tan d^\circ)}\left(1 + \frac{\rho}{G}\right) \text{ per cent.}$$

where

$$\delta_1 = \tan D^{\frac{1}{m}^\circ} - \tan D^\circ, \quad \text{and,} \quad \delta_2 = \tan d^{\frac{1}{m}^\circ} - \tan d^\circ.$$

Diminished Deflection Shunt Method.

97. Referring to Fig. 34, this method is as follows:—

The galvanometer G, whose resistance is to be determined, is joined up with a resistance R, a battery E, and a shunt S_1; the deflection obtained is noted; let this deflection be due to a current C_1, then (§ 72) we have

$$C_1 = \frac{E\,S_1}{G\,(S_1 + R) + S_1\,R},$$

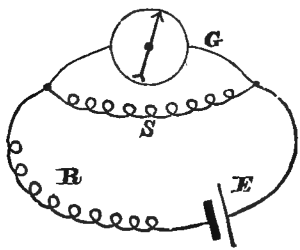
or

$$\frac{C_1 G (S_1 + R) + C_1 S_1 R}{S_1} = E.$$

The resistance of the shunt is now reduced to S_2, so that the galvanometer deflection is also reduced; let this new deflection be due to a current C_2, then we must have

$$\frac{C_2 G (S_2 + R) + C_2 S_2 R}{S_2} = E;$$

FIG. 34.



therefore

$$\frac{C_2 G (S_2 + R) + C_2 S_2 R}{S_2} = \frac{C_1 G (S_1 + R) + C_1 S_1 R}{S_1},$$

that is

$$G [C_2 S_1 (S_2 + R) - C_1 S_2 (S_1 + R)] = S_1 S_2 R (C_1 - C_2),$$

from which we get

$$G = \frac{S_1 S_2 R (C_1 - C_2)}{C_2 S_1 (S_2 + R) - C_1 S_2 (S_1 + R)},$$

or

$$G = \frac{C_1 - C_2}{C_2 \left(\frac{1}{S_2} + \frac{1}{R}\right) - C_1 \left(\frac{1}{S_1} + \frac{1}{R}\right)}. \qquad [A]$$

In the case of a tangent galvanometer, if the deflections, D and d, are read from the *tangent* scale, then we should have

$$G = \frac{D - d}{d \left(\frac{1}{S_2} + \frac{1}{R}\right) - D \left(\frac{1}{S_1} + \frac{1}{R}\right)}. \qquad [B]$$

(1.) *For example.*

With a tangent galvanometer whose resistance, G, was required, and a battery of very small resistance, we obtained with a shunt of 200 ohms (S_1), a deflection of 60 divisions (D) on the tangent scale of the instrument; when the shunt was reduced to 25 ohms (S_2), the deflection was reduced to 20 divisions (d). The resistance, R, was 400 ohms. What was the resistance of the galvanometer?

$$G = \frac{60 - 20}{20\left(\frac{1}{25} + \frac{1}{400}\right) - 60\left(\frac{1}{200} + \frac{1}{400}\right)} = 100 \text{ ohms.}$$

If the deflections are read in *degrees*, then in equation [B] we must substitute tan D° and tan d° for D and d respectively; we then get

$$G = \frac{\tan D^\circ - \tan d^\circ}{\tan d^\circ\left(\frac{1}{S_2} + \frac{1}{R}\right) - \tan D^\circ\left(\frac{1}{S_1} + \frac{1}{R}\right)}. \qquad [C]$$

(2.) *For example.*

In a measurement similar to the foregoing the readings were made from the *degrees* scale of the instrument, and deflections of 50° (D°) and $21\frac{3}{4}$° (d°) respectively were obtained. The values of S_1, S_2, and R were 200, 25, and 380 ohms respectively. What was the resistance, G, of the galvanometer?

$$\tan 50^\circ = 1{\cdot}1918,\ \tan 21\tfrac{3}{4}^\circ = {\cdot}3990,$$

therefore

$$G = \frac{1.1918 - {\cdot}3990}{{\cdot}3990\left(\frac{1}{25} + \frac{1}{380}\right) - 1{\cdot}1918\left(\frac{1}{200} + \frac{1}{380}\right)} = 100 \text{ ohms.}$$

98. If we make the test by having no shunt inserted when the first deflection is observed, that is to say, if we have $S_1 = \infty$, or, $\frac{1}{S_1} = 0$, then equation [B] becomes

$$G = \frac{D - d}{d\left(\frac{1}{S_2} + \frac{1}{R}\right) - \frac{D}{R}}, \qquad [D]$$

and equation [C]

$$G = \frac{\tan D^\circ - \tan d^\circ}{\tan d^\circ \left(\frac{1}{S_2} + \frac{1}{R}\right) - \frac{\tan D^\circ}{R}}. \qquad [E]$$

Further still, if we make R a very high resistance, that is, if in equations [D] and [E] we make $\frac{1}{R} = 0$, then we get the simplifications

$$G = S_2 \left(\frac{D}{d} - 1\right) \qquad [F]$$

and

$$G = S_2 \left(\frac{\tan D^\circ}{\tan d^\circ} - 1\right). \qquad [G]$$

99. In order to determine the "Best conditions for making the test," and also the "Possible degree of accuracy attainable," let us write equation [A] in the form,

$$\frac{1}{G} = \frac{C_2\left(\frac{1}{S_2} + \frac{1}{R}\right) - C_1\left(\frac{1}{S_1} + \frac{1}{R}\right)}{C_1 - C_2}.$$

Now this equation is similar in form to equation [B] (§ 92) in the last test (Diminished deflection direct method), the only difference being that we have $\frac{1}{G}$ instead of G, and $\left(\frac{1}{S_2} + \frac{1}{R}\right)$ and $\left(\frac{1}{S_1} + \frac{1}{R}\right)$ instead of R and ρ, respectively; and inasmuch as an λ' per cent. error in $\frac{1}{G}$ is an λ' per cent. error in G (though of the opposite sign), we can see that the value of λ' must be expressed by an equation of the same form as equation [H] (§ 95), that is to say, we must have

$$\lambda' = \frac{(C_1 c_2 + C_2 c_1)\,100}{C_2 (C_1 - C_2)} \left[1 + G\left(\frac{1}{S_1} + \frac{1}{R}\right)\right] \text{ per cent.} \qquad [H]$$

We can see, therefore, from the investigations in the last test that we must have

Best Conditions for making the Test.

100. Make S_1 and R as large as possible * (§ 96).

Make S_2 of such a value that when the deflections, D and d, are in *divisions*, then

$$d = \frac{D}{3}$$

approximately; and when the deflections are in *degrees* on a tangent galvanometer, then

$$\tan d^\circ = \frac{\tan D^\circ}{3}$$

approximately.

Possible Degree of Accuracy attainable.

If the deflections are in *divisions*, and if we can read their value to an accuracy of $\frac{1}{m}$th of a division, then we can determine the value of G to an accuracy, λ', of

$$\lambda' = \frac{\frac{1}{m}(D + d)\,100}{d\,(D - d)}\left[1 + G\left(\frac{1}{S_1} + \frac{1}{R}\right)\right] \text{ per cent.}$$

If the deflections are in *degrees* on a tangent galvanometer, then if we can read their value to an accuracy of $\frac{1}{m}$th of a *degree*, we can determine the value of G to an accuracy, λ', of

$$\lambda' = \frac{(\tan D^\circ\,\delta_2 + \tan d^\circ\,\delta_1)\,100}{\tan d^\circ\,(\tan D^\circ - \tan d^\circ)}\left[1 + G\left(\frac{1}{S_1} + \frac{1}{R}\right)\right] \text{ per cent.}$$

where

$$\delta_1 = \tan D\tfrac{1}{m}^\circ - \tan D^\circ, \quad \text{and,} \quad \delta_2 = \tan d\tfrac{1}{m}^\circ - \tan d^\circ.$$

101. It may be remarked, that in the foregoing methods unless the galvanometer under measurement has a high degree of "sensitiveness" (§ 53), then even a moderate degree of accuracy in making the test cannot be assured.

* The investigations in the case of the last test prove that we should make $\left(\frac{1}{S_1} + \frac{1}{R}\right)$ as *small* as possible; this, of course, is equivalent to making S_1 and R as *large* as possible.

CHAPTER VI.

MEASUREMENT OF THE INTERNAL RESISTANCE OF BATTERIES.

Half Deflection Method.

102. On page 5 a formula is given for determining the resistance r of a battery, viz.:—

$$r = R - (2\rho + G),$$

where G is the resistance of the galvanometer employed to make the test, ρ a resistance which gave a certain current through the galvanometer, and R a larger resistance which caused the strength of this current to be halved.

As this, though a simple, is a very good test, and is one which is very frequently made use of, a numerical example may prove of value.

For example.

With a galvanometer whose resistance was 100 ohms (G), and a battery whose resistance (r) was to be determined, we obtained with a resistance in the resistance box of 150 ohms (ρ), a deflection representing a current of a certain strength, and on increasing ρ to 600 ohms (R), we obtained a deflection which showed the current strength to be halved. What was the resistance of the battery?

$$r = 600 - (2 \times 150 + 100) = 200 \text{ ohms}.$$

To avoid mistakes, it should be carefully observed that in working out the formula we "*First double the smaller resistance; to the result add the resistance of the galvanometer, and deduct this total from the greater resistance.*"

103. A very common method of making this test is to employ a galvanometer of practically no resistance, and to take the first deflection with no resistance in the circuit except that of the battery itself. In this case $(2\rho + G) = 0$, so that

$$r = R$$

or the added resistance is the resistance of the battery.

104. If we compare the first method (§ 102) with the test for determining the resistance of a galvanometer described on page 51 (§ 66), we can see that the two are almost identical. In the one case we determine the resistance of the galvanometer, and in the other we determine the resistance of the battery plus the galvanometer, and then from the result deduct the value of the galvanometer. This being so, we can see that the

Best Conditions for making the Test

are obtained by making $\rho + G$ a fractional value of r; to do which we should require a galvanometer of low resistance.

As regards the *possible degree of accuracy attainable*, we can see from the galvanometer test referred to, that

$$\lambda' = 2\left(1 + \frac{G}{r}\right)\gamma';$$

that is to say:—

Possible Degree of Accuracy attainable.

If we can be certain of the value of the galvanometer deflection to an accuracy of γ' per cent., then we can be certain of the accuracy of the value of r within $2\left(1 + \frac{G}{r}\right)\gamma'$ per cent.

Or if we employ a galvanometer of low resistance, then we can be certain of the accuracy of the value of r within $2\,\gamma'$ per cent.

If the galvanometer deflection be too high, i.e., above about 55° (page 25, § 31), with the lowest value we can give to ρ, then the galvanometer must be reduced in sensitiveness by being shunted, and the value of G in the formula will then be the combined resistance of the galvanometer and shunt, that is, the product of the two divided by their sum.

Thomson's Method.

105. Fig. 35 shows the theoretical, and Fig. 36 the practical methods of making this test.

The theory of the method is as follows: The galvanometer G, a resistance (Res.), and the battery whose resistance r is required, are joined up in simple circuit with a shunt S between the poles of the battery; a deflection of the galvanometer needle is produced with a resistance ρ in the resistance box. The shunt is now removed; this causes the deflection to become larger;

ρ is then increased until the deflection becomes the same as it was at first. Let the new resistance be R, and let E be the electromotive force of the battery and C the current passing through the galvanometer.

FIG. 35.

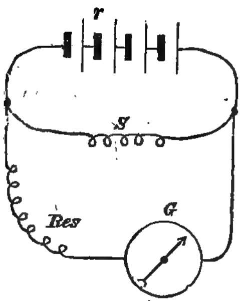
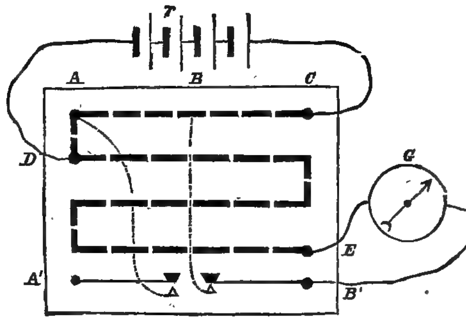


FIG. 36.



In the first case we have

$$C = \frac{E}{r + \frac{S(\rho + G)}{S + \rho + G}} \times \frac{S}{S + \rho + G}$$

$$= \frac{E\,S}{r(S + \rho + G) + S(\rho + G)},$$

and in the second case

$$C = \frac{E}{r + R + G};$$

therefore

$$\frac{E}{r + R + G} = \frac{E\,S}{r(S + \rho + G) + S(\rho + G)}.$$

By multiplying up and cancelling,

$$r(\rho + G) = S(R - \rho),$$

or

$$r = S\,\frac{R - \rho}{\rho + G}.$$

For example.

A battery whose resistance (r) was required, was joined up in circuit with a resistance of 200 ohms (ρ) and a galvanometer of 100 ohms (G), a shunt of 10 ohms (S) being between the poles of the battery.

On removing the shunt it was necessary, in order to reduce the increased deflection to what it was originally, to increase ρ to 3200 ohms (R). What was the resistance of the battery?

$$r = 10 \frac{3200 - 200}{200 + 100} = 100 \text{ ohms.}$$

106. The investigation for determining the best resistances to employ in making this test would be conducted in precisely the same manner as that given on page 67, *et seq.* For the equation

$$r = S \frac{R - \rho}{\rho + G}$$

is the same as

$$r = S \frac{(R + G) - (\rho + G)}{\rho + G},$$

which is the same kind of equation as the one in the test we have referred to, viz.:—

$$G = S \frac{R - \rho}{\rho};$$

and as in this case we proved that S was to be as small and R as large as possible, so from the preceding equation we should prove that S should be as small, and R + G as large, as possible. In order, therefore, to obtain the

Best Conditions for making the Test,

107. First make a rough test to ascertain approximately what is the value of r. Having done this, insert a shunt (S) between the poles of the battery, of less resistance than r.

Next join up ρ in circuit with G, with the battery, and with its shunt S, making $\rho + G$ not larger than $\frac{S}{G}(G + R)$; R being the highest resistance that can be inserted in the circuit.

The galvanometer needle being obtained at the angle of maximum sensitiveness, note the value of ρ.

Now remove the shunt and increase ρ to R, so that the

increased deflection becomes the same as it was at first. Note R and calculate r from the formula.

Possible Degree of Accuracy attainable.

From the galvanometer test referred to, we can see that if we can determine the value of the galvanometer deflection to an accuracy of γ' per cent., then we can determine the accuracy of r to an accuracy of

$$\left(1 + \frac{S}{r}\right)\left(1 + \frac{r}{R + G}\right)\gamma' \text{ per cent.}$$

108. As we cannot in this test vary the resistance of the galvanometer so as to obtain the deflection at the angle of maximum sensitiveness, we must, if the deflection be too high with the highest resistances we can put in the circuit, reduce its sensitiveness by means of a shunt between its terminals, the value of G in the formula will then be the combined resistance of the galvanometer and its shunt.

The constancy of a battery being much impaired by its being on a circuit of low resistance, it is not advisable to reduce the deflection of the galvanometer by making S very small. In fact S, although it should be lower than the resistance of the battery, should not, in this test, be made lower than we can help. Thus, if the resistance of the battery were about 200 ohms, it would be preferable to make S 100 rather than 10 ohms. Should the deflection of the galvanometer needle be too low, the only thing to be done is to use another which has a higher figure of merit.

109. A Thomson galvanometer answers very well for tests like this, as its figure of merit can always be made sufficiently low by placing a shunt made of a short piece of wire between its terminals.

110. If we could manage to adjust ρ in the first place so that together with G it equals S, we get the simplified formula

$$r = S\,\frac{R - \rho}{S} = R - \rho\,;$$

or again, if we commence with no other resistance in the galvanometer circuit beyond that of the galvanometer itself, we get the simplification

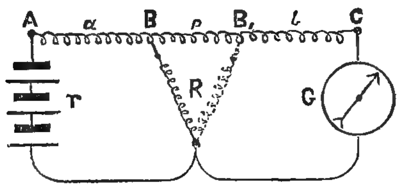
$$r = S\,\frac{R}{G}\cdot$$

If we arrange the tests, however, so as to use these simplified formulæ, we are obliged to employ an arrangement of resistances which would not be at all advisable if we wish for accuracy, and it is very questionable whether any advantage is gained by adopting a simplification of a formula, in itself simple, at the expense of accurate testing.

Siemens' Method.

111. Fig. 37 shows the arrangement of resistances, &c., for determining the resistance of a battery by Siemens' method.

Fig. 37.



A C is a resistance on the slide principle (§ 17), R a resistance connected to the junction of the galvanometer G and the battery whose resistance r is required. The other end of R is connected to the slider B.

Now it will be found that if B be moved towards A or towards C from a certain point midway between A and C, the current flowing through the galvanometer will be increased.

It follows from this that if we put B near A and obtain a certain deflection, we can also obtain this same deflection by sliding B to a point near C.

Let B and B_1 be these points, and let a be the resistance between A and B, b the resistance between B_1 and C, and ρ the resistance between B and B_1. Also let E be the electromotive force of the battery, and r its resistance, and let C be the current deflecting the galvanometer needle.

Now when the slider is at B

$$C = \frac{E}{r + a + \dfrac{R(\rho + b + G)}{R + \rho + b + G}} \times \frac{R}{R + \rho + b + G}$$

$$= \frac{E\,R}{(r + a)(R + \rho + b + G) + R(\rho + b + G)},$$

and when the slider is at B_1

$$C = \frac{E\,R}{(r + a + \rho)(R + b + G) + R(b + G)};$$

therefore

$$(r + a)\ (R + \rho + b + G) + R(\rho + b + G)$$
$$= (r + a + \rho)\ (R + b + G) + R(b + G);$$

therefore

$$(r + a)\ \rho + R\,\rho = \rho\,(R + b + G);$$

from which

$$r + a = b + G$$

or

$$r = G + b - a.$$

In making this test, then, what we have to do, is to note what are the values of A B (a) and B_1 C (b) when the same deflections are obtained on the galvanometer, then from these values and the resistance of the galvanometer we can determine the resistance of the battery.

112. Another way of making the test is to find the point between A and C which gives the least deflection; then a and b will be the resistances on either side of this point.

113. Let us now consider what are the "Best conditions for making the test." The points to be considered are, what are the best resistances to make R and A C, and also, at what point should we place the slider to commence with, that is, should we place it near one of the ends of A C or at some point nearer the middle of the latter?

From the equation

$$r = G + b - a$$

it is clear that any error made in b or a, will make an exactly corresponding error in r; in considering the problem, therefore, we have simply to determine what arrangement of resistances, &c., will cause any slight error in a or b, that is any slight movement of the slider, to make the greatest possible alteration in the current, that is in the deflection of the galvanometer needle.

Let us suppose the slider was at B for the first observation, and let us suppose that when the slider was at that point, a current C flowed through the galvanometer, and that when the slider was moved to B_1 the current was also C. Further, when the slider was moved a distance λ beyond B towards, say, A, let us suppose the current was increased to $C + c$.

We have then to determine what arrangement of resistances, &c., will make $\frac{c}{C}$ as large as possible.

Now

$$C = \frac{E\,R}{(r+a)\,(R+\rho+b+G)+R\,(\rho+b+G)},$$

and we know that

$$r+a=b+G;$$

consequently

$$C = \frac{E\,R}{(r+a)\,(R+\rho+r+a)+R\,(\rho+r+a)},$$

and by putting $a-\lambda$ for a, and $\rho+\lambda$ for ρ, we get

$$C+c = \frac{E\,R}{(r+a-\lambda)\,(R+\rho+r+a)+R\,(\rho+r+a)} = C_1,$$

or,

$$c = C_1 - C;$$

therefore

$$\frac{c}{C} = \frac{C_1}{C} - 1;$$

therefore

$$\frac{c}{C} = \frac{\lambda\,(R+\rho+r+a)}{(r+a-\lambda)\,(R+\rho+r+a)+R\,(\rho+r+a)};$$

or, since λ is a very small quantity, we may say

$$\frac{c}{C} = \frac{\lambda\,(R+\rho+r+a)}{(r+a)\,(R+\rho+r+a)+R\,(\rho+r+a)}, \quad [A]$$

or

$$\frac{c}{C} = \frac{\lambda}{r+a+\dfrac{R\,(\rho+r+a)}{R+(\rho+r+a)}}. \quad [B]$$

We will first determine at what point the slider should be placed to commence with.

Now if we show at what point it should be placed near A, we determine the point at which it should be placed near C, for $r+a$ must equal $G+b$. What we have to do then is to determine the best value to give to a.

To do this we must suppose the resistance A C to be constant, or since r and G are naturally constants, we must have

$$r + a + \rho + b + G;$$

that is,

$$r + a + \rho + r + a,$$

equal to a constant, say, K; therefore

$$\rho + r + a = K - (r + a),$$

therefore, by equation [A], we get

$$\frac{c}{C} = \frac{\lambda\,(R + K - (r + a))}{(r + a)\,(R + K - (r + a)) + R\,(K - (r + a))}$$

$$= \frac{\lambda\,(R + K - (r + a))}{(r + a)\,(K - (r + a)) + R\,K}.$$

From this we see that the smaller we make $(r + a)$ the larger will be the numerator of the fraction. Also if $r + a$ be less than $\frac{K}{2}$ (which it must be in the test), the smaller we make it the smaller will be the denominator of the fraction;* con-

* This may be proved as follows:—

$$(r + a)\,(K - (r + a)) = (r + a)\,K - (r + a)^2 = \frac{K^2}{4} - \left((r + a) - \frac{K}{2}\right)^2.$$

If in the latter expression we make

$$r + a = \frac{K}{2},$$

then

$$\left((r + a) - \frac{K}{2}\right)^2 = 0,$$

which makes the expression as small as possible.

But if we make $r + a$ either larger *or* smaller than $\frac{K}{2}$, then $\left((r + a) - \frac{K}{2}\right)^2$ does not equal 0, but it has a plus value which increases in proportion as we make either $(r + a)$ larger than $\frac{K}{2}$, or $\frac{K}{2}$ larger than $(r + a)$; for although $\left((r + a) - \frac{K}{2}\right)$ in one case will have a positive, and in the other case a negative, value, still $\left((r + a) - \frac{K}{2}\right)^2$ is positive in both cases.

If, therefore, we make $(r + a)$ smaller than $\frac{K}{2}$, the value of the expression referred to, and consequently the value of $(r + a)\,(K - (r + a))$, will increase in proportion.

sequently the smaller we make $(r + a)$, and therefore a, the larger will $\frac{c}{C}$ be.

It is best, therefore, to place the slider to commence with as near to one end of A C as possible.

Next we have to determine what value we should give to A C. This we shall do if we determine what value ρ should have. If we write equation [B] in the form

$$\frac{c}{C} = \frac{\lambda}{r + a + \cfrac{1}{\cfrac{1}{R} + \cfrac{1}{\rho + r + a}}}.$$

we can see that r, a, and R being constant, $\frac{c}{C}$ is made as large as possible by making ρ as small as possible; but we can also see that there is but little use in making ρ much smaller than $r + a$, or, as a ought to be small, in making it much smaller than r.

Lastly we have to find what value it is best to give to R; this we can also determine from the last equation. We can see from the latter that, r, a, and ρ, being constant quantities, $\frac{c}{C}$ is made as large as possible by making R as small as possible; but we can also see that we gain but very little by making R much smaller than $r + a$, or, as a ought to be small, by making it smaller than r. Actually of course we could not make R extremely small for the reason that the battery and galvanometer would then be practically short circuited and a readable deflection could not be obtained.

Since
$$r + a = G + b,$$
a can only be made small by having G small; it is therefore best to have a galvanometer of as low a resistance as possible, or rather of a resistance not exceeding r.

We proved that the slider should be as near one end of A C as possible. The end we can place it nearest to must evidently be the end to which the greatest resistance is connected; therefore, whichever value of r or G happens to be the greatest, at the end to which that larger value is connected should the slider be placed, to commence with.

In order to determine the "percentage of accuracy attainable" we must in equation [B] put percentages λ' and γ' for the absolute values λ and c, that is to say, we must have

$$\lambda = \frac{\lambda'}{100} \text{ of } r. \quad \text{and.} \quad c = \frac{\gamma'}{100} \text{ of } C,$$

in which case we get

$$\lambda' = \left[r + a + \frac{R(\rho + r + a)}{R + \rho + r + a}\right]\frac{\gamma'}{r} \text{ per cent.}$$

To summarise the results, then, we have

Best Conditions for making the Test.

114. The slider at commencing should be as near as possible to that end of A C to which is connected the greatest of the values r and G. The value of A C should be not less than the value of the greater of the two quantities r and G. R should be lower than the greater of the two quantities r and G.

The galvanometer resistance should not exceed r, and the deflection should be obtained at the angle of maximum sensitiveness. This can be done by varying R; but inasmuch as the latter should be lower than r, it is desirable to use a galvanometer of such sensitiveness that R can be made sufficiently small without reducing the deflection too low.

Possible Degree of Accuracy attainable.

If we can be certain of the galvanometer deflection to an accuracy of γ' per cent., then we can be certain of the value of r to an accuracy, λ', of

$$\lambda' = \left[r + a + \frac{R(\rho + r + a)}{R + \rho + r + a}\right]\frac{\gamma'}{r} \text{ per cent.}$$

If R, a, and ρ are very small compared with r, then we get

$$\lambda' = \gamma'.$$

115. As in previous tests, we should first determine the value of r roughly and then more exactly with the resistances properly arranged.

116. We have hitherto supposed A C to be a *slide resistance*, but it is not absolutely necessary that it should be so; the test can very well be made in the following manner:—

Referring to the figure, and supposing r to be greater than G, let the resistances ρ and b be ordinary ones and both capable of variation, and let the resistance a be done away with.

Having connected R to B, that is, to the pole A of the battery, plug up all the resistance in b and adjust ρ and R till the deflection of maximum sensitiveness is obtained on the galvano-

meter. Care must be taken that the adjustment of ρ and R is so made that R is less and ρ greater than G. If the galvanometer has a sufficiently high figure of merit, there will be no difficulty in doing this.

Next shift the connection of R from B to B_1 and proceed to adjust b and ρ until the original deflection is reproduced, the adjustment being made in such a manner that the same resistance is plugged up in ρ that is unplugged in b; then

$$r = G + b.$$

It must be noted that of the two quantities G and r the one which has the greatest resistance must be connected to ρ at B. In the case we have considered we have supposed that r was the larger quantity, but if G had been the larger of the two the position of G and r would have had to have been reversed, and the resistance of r would have been given by the formula

$$r = G - b.$$

The *modus operandi* of the test would, however, be precisely the same in the two cases.

Special sets of resistance coils are evidently necessary to make this test, as it cannot be made with the ordinary set (Fig. 6, page 13) alone.

Mance's Method.

117. This test is of a very similar nature to Thomson's method of determining the resistance of a galvanometer given on page 74. Fig. 38 shows the theoretical method of making the test.

In the theoretical figure, a, b, and d are resistances, g a galvanometer, and E a battery whose resistance r is required.

A key is inserted between the junctions of a with b and d with r. By depressing this key the junctions are connected together.

Let us first suppose the key to be up, then the current C_1 flowing through the galvanometer will be

$$C_1 = \frac{E}{r + d + \dfrac{(a+b)\,g}{a+b+g}} \times \frac{a+b}{a+b+g}$$

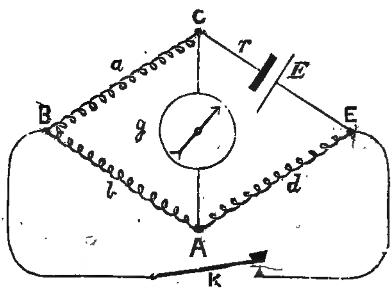
$$= \frac{E\,(a+b)}{g\,(a+b+d+r) + (a+b)\,(d+r)}. \qquad [1]$$

Next suppose the key to be pressed down; then the current C_2 flowing through the galvanometer will be

$$C_2 = \frac{E}{r + \frac{\left(\frac{b\,d}{b+d} + g\right) a}{\frac{b\,d}{b+d} + g + a}} \times \frac{a}{\frac{b\,d}{b+d} + g + a}$$

$$= \frac{E\,(b+d)\,a}{g\,(a+r)\,(b+d) + b\,d\,(a+r) + a\,r\,(b+d)}. \qquad [2]$$

FIG. 38.



Now if the resistances be adjusted so that the deflection of the galvanometer needle remains the same whether the key is depressed or not, then equations [1] and [2] are equal; that is

$$\frac{E\,(a+b)}{g\,(a+b+d+r) + (a+b)\,(d+r)}$$

$$= \frac{E\,(b+d)\,a}{g\,(a+r)\,(b+d) + b\,d\,(a+r) + a\,r\,(b+d)}.$$

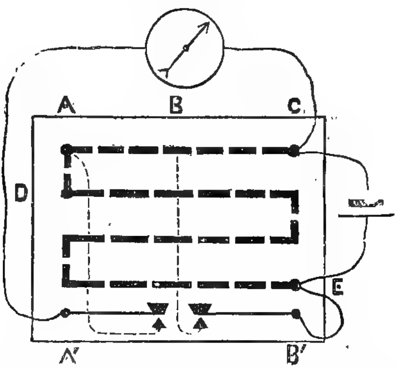
Now if we refer to "Thomson's galvanometer resistance test" on page 75, we can see that this equation is similar to equation [3] on that page, with the exception that r and g are interchanged. It must therefore be obvious, by the same development of the equation as that given on the page referred to, that

$$r = \frac{a\,d}{b}.$$

118. The great advantage of this test is that the electromotive force of the battery need only be constant during the very short interval of time occupied in depressing and raising the key.

119. In making the test practically the connections would be made as shown by Fig. 39. Terminals E and B′ would be joined by a short piece of thick wire; the other connections are obvious.

FIG. 39.



The left-hand key puts the galvanometer on; it must be depressed and held permanently down, and the right-hand key then alternately depressed and raised and the resistance d, that is the resistance between A and E, at the same time adjusted until the deflection of the galvanometer needle remains the same whether the key is up or down.

120. Again referring to Thomson's galvanometer resistance test; it must be clear, by substituting r for g in the equations, that to obtain the

Best Conditions for making the Test,

Make a as low as possible and b as high as possible, but not so high that d when exactly adjusted would exceed all the resistance we could insert between D and E (see Fig. 39).

Adjust d approximately and then, if necessary, adjust the resistance of the galvanometer shunt (which it will be necessary to employ) so that the final deflection is as nearly as possible that of maximum sensitiveness, and then, having exactly adjusted d, calculate r from the formula.

Possible Degree of Accuracy attainable.

If we can determine the value of the galvanometer deflection to an accuracy of γ' per cent., then we can be certain of the value of r to an accuracy of $\left(1 + \frac{a}{r}\right)\left(1 + \frac{r}{d}\right)\gamma'$ per cent.

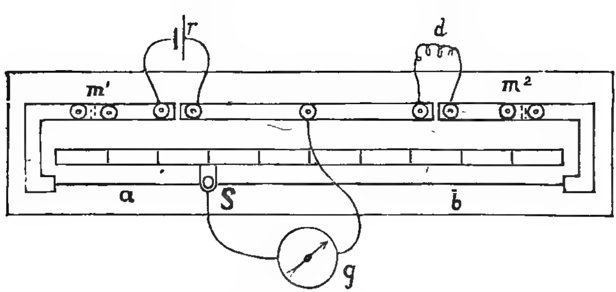
121. In the practical execution of the test with the set of resistance coils shown by Fig. 39, the lowest value we could give to a would be 10 units, unless we improvised a resistance of less value, which it might be necessary to do.

Mance's Method with the Slide Wire Bridge.

122. Mance's test is sometimes made by having $a + b$ a slide wire resistance, d being a fixed resistance; in this case the slider would be moved along between A and C until the point is found at which the depression or raising of the key makes no alteration in the deflection of the galvanometer needle.

For practically executing the test the apparatus known as the "Slide Wire" or "Metre Bridge" may be used. This apparatus, which is shown by Fig. 40, is described in Chapter

Fig. 40.



VIII. (The Wheatstone Bridge). The slide wire, $a + b$, which is 1 metre long, is stretched upon an oblong board (forming the base of the instrument) parallel to a metre scale divided throughout its whole length into millimetres, and so placed that its two ends are as nearly as possible opposite to divisions 0 and 1000 respectively of the scale. The ends of the wire are soldered to a broad, thick copper band, which passes round each end of the graduated scale, and runs parallel

to it on the side opposite to the wire. This band is interrupted by four gaps, at m_1, r, d, and m_2. On each side of these gaps, and also at B, C, and E, are terminals. In making the test under consideration, the gaps, m_1, and m_2, are closed by thick copper straps. The slider S makes contact with the slide wire by the depression of a knob on S.

The battery, r, a resistance, d, and a galvanometer, g, being joined up as shown, the slider S is moved along the scale, the knob being depressed at intervals, until the point is reached at which the depression makes no change in the permanent deflection of the galvanometer needle. When this is the case, then, as in Thomson's galvanometer test (§ 87), we have

$$r = d\left(\frac{a}{1000 - a}\right).$$

For example.

In the foregoing test, equilibrium was produced when d was 1 ohm, and a, 450 divisions; what was the resistance, r, of the battery?

$$r = 1\frac{450}{1000 - 450} = \frac{450}{550} = \cdot 85 \text{ ohms.}$$

123. The *best conditions for making the test* are similar to those required for "Thomson's galvanometer test" (§ 88), namely, we should make d larger than r, but not greater than about 10 times r.

As a rule the complete slide wire bridge is furnished with but four resistance coils of 1 ohm each, so that the choice of a resistance to insert in d is limited, and it may not be possible to follow out the rule of "making d about 10 times as large as r." In this case the possibility of an accurate measurement becomes proportionately reduced below the highest possible standard, so that on the one hand a cell whose resistance is much less than one-tenth of an ohm, or, on the other hand, a cell whose resistance exceeds 4 ohms, cannot be measured with the highest possible accuracy.

Strictly speaking (as has been pointed out) in order to ensure accuracy it is necessary that the resistance of the portion of the slide wire, a, be less than the resistance of the battery to be measured; but as the resistance of the whole length of the wire will not exceed one-tenth of an ohm, the resistance of the length, a, will practically be less than the resistance of the battery, unless, of course, this resistance is extremely low.

The *possible degree of accuracy attainable* we can see from Thomson's galvanometer test (§ 89) must be given by the equation

$$\lambda' = \frac{100000\,\delta}{a\,(1000 - a)} \text{ per cent.}$$

where δ is the degree of accuracy in divisions to which the slider, S, can be adjusted. If we can adjust to an accuracy of 1 division, then $\delta = 1$.

For example.

In the last example, what would be the degree of accuracy, λ', with which the value of r could be obtained, supposing that the position of the slider could be determined to an accuracy of 1 division (δ)?

$$\lambda' = \frac{100000 \times 1}{450\,(1000 - 450)} = \cdot 40 \text{ per cent.}$$

124. The facility and accuracy with which all the foregoing tests (except the half-deflection test) can be made may be greatly increased by the following arrangement: Use a galvanometer with a high "figure of merit" (§ 52), and instead of making the test with the needle brought to the "angle of maximum sensitiveness" (§ 27), make it with the needle brought approximately to zero by means of a powerful permanent magnet set near the instrument; under these conditions the galvanometer needle will be highly sensitive to any small change in the current strength.

Another arrangement which may be very conveniently adopted is to employ a galvanometer with a high "figure of merit," and wound with two wires. One of these wires would be joined in circuit with the battery under test, &c., in the usual way; the other would be connected in circuit with a small battery and a set of resistance coils, the connections being so made that the currents through the two coils oppose one another. When the deflection due to the battery under test is obtained, the second battery and resistance coils are connected up, and then this battery is adjusted until the needle is brought to zero as nearly as possible. The test is then made, as in the case where a permanent magnet is used.

125. In the case of Mance's test with the slide-wire bridge, if the test is made either by using a permanent magnet in the way described, or by using a galvanometer wound with a double wire, it is best to make d as nearly equal to the resistance of the battery as possible (it should not be made less), as in this case, since the slider, S, will have to be set near the centre of

the scale, a greater range of adjustment is given to it, for 5 divisions near the centre portion of the scale (500 division mark) are equivalent to only 1 division near the 100 division mark. It is true the arrangement is not quite so sensitive as when the slider has to be set towards the end of the scale; but still, if we can employ a galvanometer with a high figure of merit, this small loss of sensitiveness is more than compensated for by the increased range which can be obtained on the scale.

126. In order that satisfactory results may be obtained in any of the foregoing tests, it is very necessary that the galvanometer used be a "sensitive" one (§ 53), otherwise even a moderate degree of accuracy cannot be assured.

Diminished Deflection Direct Method.

127. This method, which has been generally described in Chapter I. (§ 6), is as follows:—

The battery whose resistance, r, is required, a galvanometer of resistance, G, and a resistance, ρ, are joined up in simple circuit; the deflection obtained is noted. Let this deflection be due to a current, C_1, then calling E the electromotive force of the battery, we have

$$C_1 = \frac{E}{r + G + \rho}, \quad \text{or,} \quad C_1 (r + G) + C_1 \rho = E.$$

The resistance, ρ, is now increased to R, so that a new deflection due to a current, C_2, is produced, then we have

$$C_2 = \frac{E}{r + G + R}, \quad \text{or,} \quad C_2 (r + G) + C_2 R = E;$$

hence

$$C_1 (r + G) + C_1 \rho = C_2 (r + G) + C_2 R,$$

or

$$(r + G)(C_1 - C_2) = C_2 R - C_1 \rho;$$

therefore

$$r + G = \frac{C_2 R - C_1 \rho}{C_1 - C_2},$$

that is

$$r = \frac{C_2 R - C_1 \rho}{C_1 - C_2} - G. \qquad [A]$$

If a tangent galvanometer is employed for making the test, then if the deflections, D, and d, are read from the *tangent*

scale of the instrument, those deflections can be directly substituted for the quantities, C_1, C_2, for

$$D : d :: C_1 : C_2;$$

in this case, then, we have

$$r = \frac{d\,R - D\,\rho}{D - d} - \cancel{G}. \qquad [B]$$

(1.) *For example.*

With a tangent galvanometer whose resistance was 10 ohms (G), and a battery whose resistance, r, was required, a deflection of 60 divisions (D) on the tangent scale of the instrument was obtained, when a resistance of 10 ohms (ρ) was in circuit; when the latter resistance was increased to 230 ohms (R) the deflection was reduced to 20 divisions (d). What was the resistance of the battery?

$$\cancel{r} = \frac{20 \times 230 - 60 \times 10}{60 - 20} - \cancel{10} = 90 \text{ ohms.}$$

If the readings are made from the *degrees* scale, then we must substitute the tangents of the deflections for the deflections themselves; the formula then becomes

$$r = \frac{\tan d^\circ\, R - \tan D^\circ\, \rho}{\tan D^\circ - \tan d^\circ} - G. \qquad [C]$$

(2.) *For example.*

In a measurement similar to the foregoing the readings were made from the *degrees* scale of the galvanometer, and deflections of 50° (D°) and $21\frac{3}{4}$° (d°) respectively were obtained with resistances of 10 ohms (ρ) and 229 ohms (R) in the circuit. The resistance of the galvanometer was 10 ohms (G). What was the resistance, r, of the battery?

$$\tan 50^\circ = 1{\cdot}1918, \quad \text{and,} \quad \tan 21\tfrac{3}{4}^\circ = {\cdot}3990,$$

therefore

$$r = \frac{{\cdot}3990 \times 229 - 1{\cdot}1918 \times 10}{1{\cdot}1918 - {\cdot}3990} - 10 = 90 \text{ ohms.}$$

128. If in equations [B] and [C] we have $\rho = 0$, that is to say, if we make the test by having at first no resistance in the

circuit except that of the galvanometer and the battery itself, then we get

$$r = R\frac{d}{D - d} - G \qquad [D]$$

and

$$r = R\frac{\tan d}{\tan D - \tan d} - G. \qquad [E]$$

129. In order to determine the "Best conditions for making the test," and also the "Possible degree of accuracy attainable," let us write equation [A] in the form

$$r = \frac{C_2(R + G) - C_1(\rho + G)}{C_1 - C_2}.$$

Now this equation is similar to equation [B] (page 83) in the "Diminished deflection direct method" of determining the resistance of a galvanometer, except that in the latter method we have the quantities R and ρ in the place of the quantities $(R + G)$ and $(\rho + G)$; consequently we can at once see from the investigation in the test referred to that we must have—

Best Conditions for making the Test.

130. Make ρ as small as possible.

Make R of such a value that when the deflections, D, d, are in *divisions*, then

$$d = \frac{D}{3}$$

approximately; and when the deflections are in *degrees* on a tangent galvanometer, then

$$\tan d^\circ = \frac{\tan D^\circ}{3}$$

approximately.

Possible Degree of Accuracy attainable.

If the deflections are in *divisions*, and if we can read their value to an accuracy of $\frac{1}{m}$th of a division, then we can determine the value of r to an accuracy, λ', of

$$\lambda' = \frac{\frac{1}{m}(D + d)\,100}{d\,(D - d)}\left(1 + \frac{\rho + G}{r}\right) \text{ per cent.}$$

If the deflections are in *degrees* on a tangent galvanometer, then if we can read their value to an accuracy of $\frac{1}{m}$th of a degree, we can determine the value of G to an accuracy, λ', of

$$\lambda' = \frac{(\tan D^\circ \,\delta_2 - \tan d^\circ \,\delta_1)\,100}{\tan d^\circ\,(\tan D^\circ - \tan d^\circ)}\left(1 + \frac{\rho + G}{r}\right) \text{ per cent.}$$

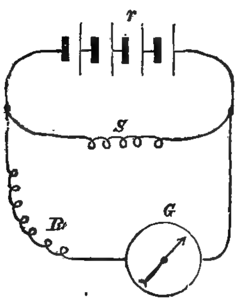
where

$$\delta_1 = \tan D^{\frac{1}{m}^\circ} - \tan D^\circ, \quad \text{and,} \quad \delta_2 = \tan d^{\frac{1}{m}^\circ} - \tan d^\circ.$$

Diminished Deflection Shunt Method.

131. This method is shown by Fig. 41. The battery, r, whose resistance is to be determined, is joined up in circuit with a resistance, R, a galvanometer, G, and a shunt, S_1; the deflection obtained is noted; let this deflection be due to a current C_1, then calling E the electromotive force of the battery, we have (page 96)

Fig. 41.



$$C_1 = \frac{E\,S_1}{r\,(S_1 + R + G) + S_1\,(R + G)},$$

or

$$\frac{C_1\,r\,(S_1 + R + G) + C_1\,S_1\,(R + G)}{S_1} = E.$$

The resistance of the shunt is now reduced to S_2, so that the galvanometer deflection is also reduced; let this new deflection be due to a current C_2, then we must have

$$\frac{C_2\,r\,(S_2 + R + G) + C_2\,S_2\,(R + G)}{S_2} = E;$$

therefore

$$\frac{C_2 r(S_2 + R + G) + C_2 S_2\,(R + G)}{S_2} = \frac{C_1 r(S_1 + R + G) + C_1 S_1\,(R + G)}{S_1},$$

that is,

$$r\,[C_2 S_1\,(S_2 + R + G) - C_1 S_2\,(S_1 + R + G)] = S_1 S_2\,(R + G)\,(C_1 - C_2),$$

from which we get

$$r = \frac{S_1\,S_2\,(R + G)\,(C_1 - C_2)}{C_2\,S_1\,(S_2 + R + G) - C_1\,S_2\,(S_1 + R + G)},$$

or

$$r = \frac{C_1 - C_2}{C_2\left(\frac{1}{S_2} + \frac{1}{R + G}\right) - C_1\left(\frac{1}{S_1} + \frac{1}{R + G}\right)}. \qquad [A]$$

In the case of a tangent galvanometer, if the deflections, D and d, are read from the *tangent* scale, then we should have

$$r = \frac{D - d}{d\left(\frac{1}{S_2} + \frac{1}{R + G}\right) - D\left(\frac{1}{S_1} + \frac{1}{R + G}\right)}. \qquad [B]$$

(1.) *For example.*

With a tangent galvanometer whose resistance was 10 ohms (G), and a battery whose resistance, r, was required, we obtained with a shunt of 200 ohms (S_1), a deflection of 60 divisions (D) on the tangent scale of the instrument; when the shunt was reduced to 25 ohms (S_2) the deflection was reduced to 20 divisions (d). The resistance, R, was 710 ohms. What was the resistance of the battery?

$$r = \frac{60 - 20}{20\left(\frac{1}{25} + \frac{1}{710 + 10}\right) - 60\left(\frac{1}{200} + \frac{1}{710 + 10}\right)} = 90 \text{ ohms.}$$

If the deflections are read in *degrees*, then in equation [B] we must substitute tan D° and tan d° for D and d respectively, we then get

$$r = \frac{\tan D^\circ - \tan d^\circ}{\tan d^\circ\left(\frac{1}{S_2} + \frac{1}{R + G}\right) - \tan D^\circ\left(\frac{1}{S_2} + \frac{1}{R + G}\right)} \qquad [C]$$

(2.) *For example.*

In a measurement similar to the foregoing the readings were made from the *degrees* scale of the galvanometer, and deflections of 50° (D°) and $21\frac{3}{4}$° (d°), respectively, were obtained. The values of S_1, S_2, R, and G were 200, 25, 655, and 10 ohms, respectively. What was the resistance, r, of the battery?

$$\tan 50^\circ = 1 \cdot 1918, \quad \tan 21\tfrac{3}{4}^\circ = \cdot 3990.$$

therefore

$$r = \frac{1 \cdot 1918 - \cdot 3990}{\cdot 3990\left(\frac{1}{25} + \frac{1}{655 + 10}\right) - 1 \cdot 1918\left(\frac{1}{200} + \frac{1}{655 + 10}\right)} = 90 \text{ ohms.}$$

132. If we make the test by having no shunt inserted when the first deflection is observed, that is to say, if we have $S_1 = \infty$, or, $\frac{1}{S_1} = 0$, then equation [B] becomes

$$r = \frac{D - d}{d\left(\frac{1}{S_2} + \frac{1}{R + G}\right) - \frac{D}{R + G}} \qquad [D]$$

and equation [C]

$$r = \frac{\tan D^\circ - \tan d^\circ}{\tan d^\circ\left(\frac{1}{S_2} + \frac{1}{R + G}\right) - \frac{\tan D^\circ}{R + G}} \qquad [E]$$

Further still, if we make R a very high resistance, that is, if in equations [D] and [E] we make $\frac{1}{R + G} = 0$, then we get the simplifications

$$r = S_2\left(\frac{D}{d} - 1\right) \qquad [F]$$

and

$$r = S_2\left(\frac{\tan D^\circ}{\tan d^\circ} - 1\right). \qquad [G]$$

133. If we refer to the "Diminished deflection shunt method" of determining the resistance of a "galvanometer" we can see that equation [A] (page 90) in that test is almost precisely similar to equation [A] (page 115) of the present test, the only difference being that in the latter we have $\frac{1}{R + G}$ in the place of $\frac{1}{R}$, consequently we must have—

Best Conditions for making the Test.

Make S_1 and R as large as possible.

Make S_2 of such a value that when the deflections, D, d, are in *divisions*, then

$$d = \frac{D}{3}$$

approximately; and when the deflections are in *degrees* on a tangent galvanometer, then

$$\tan d^\circ = \frac{\tan D^\circ}{3}$$

approximately.

Possible Degree of Accuracy attainable.

If the deflections are in *divisions*, and if we can read their value to an accuracy of $\frac{1}{m}$th of a division, then we can determine the value of r to an accuracy, λ', of

$$\lambda' = \frac{\frac{1}{m}\,(D + d)\,100}{d\,(D - d)}\left[1 + r\left(\frac{1}{S_1} + \frac{1}{R + G}\right)\right] \text{ per cent.}$$

If the deflections are in *degrees* on a tangent galvanometer, then if we can read their value to an accuracy of $\frac{1}{m}$th of a *degree*, we can determine the value of r to an accuracy, λ', of

$$\lambda' = \frac{(\tan D^\circ\,\delta_2 + \tan d^\circ\,\delta_1)\,100}{\tan d^\circ\,(\tan D^\circ - \tan d^\circ)}\left[1 + r\left(\frac{1}{S_1} + \frac{1}{R + G}\right)\right] \text{per cent.}$$

where

$$\delta_1 = \tan D\tfrac{1}{m}^\circ - \tan D^\circ, \text{ and, } \delta_2 = \tan d\tfrac{1}{m}^\circ - \tan d^\circ.$$

134. In all the foregoing tests it is very necessary that the galvanometer used be a highly sensitive one (page 48), otherwise even a moderate degree of accuracy cannot be obtained.

135. Other methods of measuring the resistance of batteries will be referred to hereafter (see Index); these methods involve principles which have not yet been discussed.

CHAPTER VII.

MEASUREMENT OF THE ELECTROMOTIVE FORCE OF BATTERIES.

136. The methods of measuring or comparing the electromotive forces of batteries are perhaps more numerous than any other class of measurements.

Although no absolute standard of the unit of electromotive force (the *volt*) exists, yet there are several standards of known value with which comparisons may be made.

STANDARD CELLS.

WHEATSTONE'S STANDARD CELL.

137. This consists of an outer vessel containing a saturated solution of sulphate of copper; into this is placed a porous cell about 2 inches high, containing mercury with a few scraps of zinc dissolved in it; a cylinder of copper is placed in the copper solution, and connection is made with the zinc amalgam by a copper-wire dipping into it.

These cells, although not suitable for continued use, can be relied upon to give a perfectly constant current for half an hour or so, in fact, for quite a sufficient time to enable any ordinary tests to be made; also the electromotive forces of any two of such cells may practically be relied upon as being equal.

The porous tubes in these cells, after use, should be thrown into nitric acid for a short time, so as to dissolve any copper which may have become deposited in their pores; they must next be washed in water, and will then be ready for use again. The amalgam can be used over and over again.

The electromotive force of a Wheatstone cell is approximately 1·079 volts.

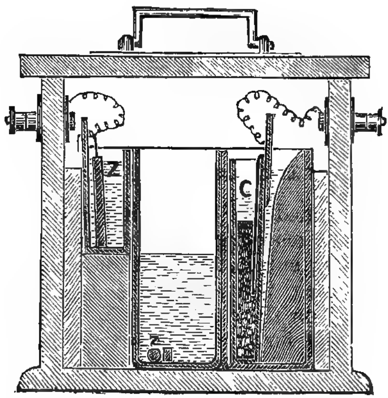
POST OFFICE STANDARD CELL.

138. A sectional view of this cell is shown by Fig. 42.

The cell is formed with three chambers; in the left-hand one is placed a zinc plate, Z, immersed in water, and in the right-hand one is placed a flat, porous pot, C, containing a copper

plate and sulphate of copper crystals, the pot being kept immersed in water. The two chambers are called "idle" cells, as the zinc plate and porous pot are kept in them when the cell is not in use.

FIG. 42.



The centre chamber contains a semi-saturated solution of sulphate of zinc, and a piece of zinc rod, z, the latter lying in a small compartment at the bottom of the chamber.

When the cell is required for use, the zinc plate and the porous pot and its contents are removed from their idle compartments and placed in the centre chamber; the cell is then ready for work. When the cell is no longer required for use, the zinc plate and porous pot are again placed in their respective idle chambers, and whilst the cell is at rest any sulphate of copper solution which may have become mingled with the sulphate of zinc solution in the centre chamber, has its copper decomposed and deposited on the zinc rod; thus the solution is always kept clear.

When in thoroughly good condition this form of cell has an electromotive force of 1·079 volts approximately, but if it is in daily use the power is practically a little less than this; in the Postal Telegraph Department the value is assumed to be 1·07 volts as being generally nearer the actual value.

Although the foregoing cell will last for a considerable time without attention, yet it must not be imagined (as often seems to be the case) that it will keep up its power for an indefinite period. After a certain time, to be judged by experience, all its

constituent parts should be thoroughly cleaned, the zinc plate and rod scraped, &c.

139. The Wheatstone and Post Office standard cells, although they cannot be relied upon for extreme accuracy, are sufficiently correct for most purposes, and they have the advantage (which is common to all batteries on the Daniell principle) of not losing their power materially when worked through a low resistance.

Practically, upon an emergency any form of Daniell cell may be used as a standard, the zinc plate being immersed in a semi-saturated solution of sulphate of zinc, and the electromotive force being taken as 1·079 volts.

CLARK'S STANDARD CELL.

140. A cell is formed by employing pure mercury as the negative element, the mercury being covered with a paste made by boiling mercurous sulphate in a thoroughly saturated solution of zinc sulphate; the positive element consists of pure distilled zinc resting on the paste.

The best method of forming this battery is to dissolve pure zinc sulphate to saturation in boiling distilled water. When cool, the solution is poured off from the crystals and mixed to a thick paste with mercurous sulphate, which is again boiled to drive off any air; this paste is then poured on to the surface of the mercury, previously heated in a suitable glass cell; a piece of pure zinc is then suspended in the paste, and the vessel may be advantageously sealed up with melted paraffin wax. Contact with the mercury may be made by means of a platinum wire passing down a glass tube, cemented to the inside of the cell, and dipping below the surface of the mercury, or more conveniently by a small external glass tube blown on to the cell, and opening into it close to the bottom. The mercurous sulphate can be obtained commercially; but it may be prepared by dissolving pure mercury in excess in hot sulphuric acid at a temperature below boiling point. The salt, which is a nearly insoluble white powder, should be well washed in distilled water, and care should be taken to obtain it free from the mercuric sulphate (persulphate), the presence of which may be known by the mixture turning yellowish on the addition of water. The careful washing of the salt is a matter of essential importance, as the presence of any free acid, or of persulphate, produces a considerable change in the electromotive force of the cell.

These standard cells are known to give a perfectly uniform electromotive force of 1·457 volts for over a year, provided they

are not worked through a low resistance. It is necessary, therefore, in employing them to take care that they are only used in circuits of very high resistance, or for charging a condenser, or are balanced by a second battery, as in Clark's electromotive force test (page 158).

DE LA RUE'S CHLORIDE OF SILVER CELL.

141. The Chloride of silver cells of Mr. Warren de la Rue are said to be remarkably well adapted for standard elements. They will bear a considerable amount of agitation without their electromotive force being varied.

FIG. 43.

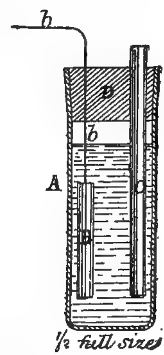


Fig. 43 shows one of these cells. A is a glass vessel closed by a stopper of paraffin wax. The positive element consists of a cylindrical rod *c* of chemically pure zinc. The negative element is a cylinder B of chloride of silver, having a silver electrode *b* cast into it. This cylinder is usually enclosed in a bag of thin parchment paper. The solution for charging the cell is made by dissolving 23 grammes of pure salammoniac in one litre of water.

The electromotive force of the chloride of silver cell is 1·068 volts.

As in the case of Clark's cells, the De la Rue battery when used as a standard must not be worked through a low resistance.

ELECTROMOTIVE FORCE MEASUREMENTS.

142. To measure the electromotive force of a battery, we have to compare it with a standard of one or more cells, and having thus ascertained the relative values of the two, the electromotive force of the battery, in volts, is obtained by an ordinary proportion sum.

For example.

The relative electromotive forces of a battery and 3 standard Daniell cells was found to be as 1·25 to 1; what was the electromotive force, in volts, of the battery?

$$1{\cdot}25 : 1 :: 3 \times 1{\cdot}079 : x;$$

therefore

$$x = \frac{1 \times 3 \times 1{\cdot}079}{1{\cdot}25} = 2{\cdot}59 \text{ volts.}$$

Equal Resistance Method.

143. Let there be two batteries, whose electromotive forces E_1 and E_2 are to be compared. Join up battery E_1 with a tangent galvanometer and resistance in simple circuit, as shown by Fig. 36 (page 96). All the plugs between A and C being inserted, the infinity plug between A and D being removed, and the connections being made, depress the right-hand key, and remove a sufficient number of plugs from between D and E to obtain a convenient deflection on, say, the tangent scale of the galvanometer. Note this deflection—let it be d_1 divisions; and also note the *total* resistance (R) in circuit—that is, the resistance between D and E, plus the resistance of the galvanometer, plus the resistance of the battery (which must be determined beforehand). Now remove battery E_1 and insert battery E_2 in its place, and if this battery has a different resistance to E_1, readjust between D and E so that total resistance in circuit is the same as it was at first. Again note the deflection of the galvanometer needle—let it be d_2 divisions. Then if C_1 be the current producing the deflection d_1, and C_2 the current producing the deflection d_2, we must have by Ohm's law (page 1),

$$C_1 = \frac{E_1}{R}, \quad \text{and,} \quad C_2 = \frac{E_2}{R},$$

therefore

$$E_1 : E_2 :: C_1 : C_2,$$

or since d_1 and d_2 are directly proportional to C_1 and C_2, we must have

$$E_1 : E_2 :: d_1 : d_2.$$

For example.

With a tangent galvanometer, whose resistance was 100 ohms, and battery E_1, whose resistance was 70 ohms, we obtained, with a resistance of 1830 ohms (total, $100 + 70 + 1830 = 2000$), in the resistance box, a deflection of 50 divisions on the tangent scale of the galvanometer; and with battery E_2, whose resistance was 50 ohms, we obtained, with a resistance of 1850 ohms (total, $100 + 50 + 1850 = 2000$, as before), in the resistance box, a deflection of 40 divisions; then

$$E_1 : E_2 :: 50 : 40,$$
$$\text{or as } 1\cdot25 : 1.$$

If the deflections are read on the *degrees* scale of the tangent galvanometer, then d_1 and d_2 must be the *tangents* of the deflections.

In cases where the resistances of the batteries whose electromotive forces are to be compared are very small, we may, by using a very high resistance, practically regard the total resistance in circuit as being the same, whatever battery we use. The deflections then obtained with any number of different batteries will represent their comparative electromotive forces. The galvanometer will, in this case, of course have to be one with a high figure of merit.

144. The "Best conditions for making the test," and the "Possible degree of accuracy attainable," are almost obvious; they are

Best Conditions for making the Test.

Make the resistances in the circuits as high as possible.

Possible Degree of Accuracy attainable.

If we can be certain of the value of the two deflections to accuracies of δ'_1 and δ'_2 per cent. respectively, then we can be certain of the relative values of the two electromotive forces to an accuracy of $\delta'_1 + \delta'_2$ per cent.

Equal Deflection Method.

145. Join up as in last method, and having noted the deflection and total resistance in circuit (R_1) with battery E_1, remove it and insert battery E_2 in its place. Now readjust resistance between D and E, until the deflection of the galvanometer needle becomes the same as it was at first. Note the resistance in circuit (R_2); then calling C the current,

$$C = \frac{E_1}{R_1}, \text{ and, } C = \frac{E_2}{R_2},$$

that is,

$$E_1 : E_2 :: R_1 : R_2,$$

or the electromotive forces of the batteries are directly as the total resistances that are in circuit with the respective batteries.

For example.

With a galvanometer whose resistance was 100 ohms, and a battery E_1 whose resistance was 50 ohms, we obtained, with a

resistance of 2350 ohms (total, $100 + 50 + 2350 = 2500$), in the resistance box, a deflection of 40°; and with a battery E_2, whose resistance was 70 ohms, it was necessary, in order to bring the galvanometer needle again to 40°, to have a resistance of 1830 ohms (total, $100 + 70 + 1830 = 2000$), in the resistance box; then

$$E_1 : E_2 :: 2500 : 2000,$$
$$\text{or as } 5 : 4.$$

An advantage in this test lies in the fact that it can be made with a galvanometer the relative values of whose deflections are unknown.

The

Best Conditions for making the Test

and the

Possible Degree of Accuracy attainable

are the same as in the last test.

Wiedemann's Method.

146. In Fig. 36 (page 96) join the zinc pole of battery E_1 to D, as shown, and the other pole to the zinc pole of battery E_2, whose other pole in turn is to be joined to C. Adjust the resistance so as to obtain a high deflection on the tangent scale of the galvanometer—let the current producing this deflection be C; then

$$C = \frac{E_1 + E_2}{R}$$

where R is the total resistance in the circuit. Now reverse battery E_2 (the weaker one) so that the two oppose one another, —we shall then get a smaller deflection due to a current $C_{,}$; then

$$C_{,} = \frac{E_1 - E_2}{R}.$$

From these two equations we get

$$E_1 C - E_2 C = E_1 C_{,} + E_2 C_{,},$$

that is,

$$E_1 : E_2 :: C + C_{,} : C - C_{,},$$

or, substituting deflections d, $d_{,}$, for current strengths C, $C_{,}$,

$$E_1 : E_2 :: d + d_{,} : d - d_{,}.$$

For example.

Two batteries E_1 and E_2 being joined up together in simple circuit, we obtained, by adjusting the resistance in resistance box, a deflection of 72 divisions (d) on the tangent scale of the galvanometer; and with the same resistance in circuit we obtained, on reversing battery E_2, a deflection of 8 divisions ($d_,$); then

$$E_1 : E_2 :: 72 + 8 : 72 - 8,$$
$$\text{or as } 80 : 64,$$
$$\text{that is, as } 1 \cdot 25 : 1.$$

If the deflections are read on the *degrees* scale of a tangent galvanometer, then d and $d_,$ must be the *tangents* of the deflections.

147. In order to make the test as accurately as possible under the last conditions, the resistance in the circuit should be so adjusted that the two deflections make approximately equal angles on opposite sides of 45° (§ 31, page 25). The more resistance it is possible to place in the circuit of the batteries the better, since the tendency of the latter to polarise is thereby reduced to a minimum.

148. Wiedemann's method is a very satisfactory one since it is absolutely independent of the resistance of the two batteries, thus one battery might have a resistance of a fraction of an ohm only and the other a resistance of several thousand ohms, yet this will in no way affect the correctness of the results, but to avoid errors due to polarisation it is necessary with some batteries to include several thousand ohms in the circuit; if the galvanometer used be one with a high figure of merit this can always be done.

149. The "Possible degree of accuracy attainable" in making the test is greatly dependent upon the relative values of the two electromotive forces. Let us first suppose that the deflections are read in *divisions*, and let us suppose that there is a possible error δ in both deflections. Now if we take both errors to be of similar signs, then we should have a total absolute error of $2\,\delta$ in the quantity $(d + d_,)$, but if one error were plus and the other minus, then we should have a total absolute error of $2\,\delta$ in the quantity $(d - d_,)$. But the latter quantity must be smaller than $(d + d_,)$, therefore an absolute error $2\,\delta$ in its value must represent a greater percentage error in the relative values of E_1 and E_2 than would be the case if the same absolute error were in $(d + d_,)$. As we must assume the resultant error to be the

greatest *possible*, we must therefore take the error $2\,\delta$ to be in the quantity $(d - d_{\prime})$.

Let, then, λ be the error in the relative values of E_1 and E_2, that is in $\frac{E_1}{E_2}$, caused by, say, an error δ in d, and an error $-\delta$ in $d_{\prime}$, then we have

$$\frac{E_1}{E_2} - \lambda = \frac{(d + \delta) + (d_{\prime} - \delta)}{(d + \delta) - (d_{\prime} - \delta)} = \frac{d + d_{\prime}}{d - d_{\prime} + 2\,\delta},$$

therefore

$$\lambda = \frac{E_1}{E_2} - \frac{d + d_{\prime}}{d - d_{\prime} + 2\,\delta} = \frac{d + d_{\prime}}{d - d_{\prime}} - \frac{d + d_{\prime}}{d - d_{\prime} + 2\,\delta}$$

$$= \frac{2\,\delta\,(d + d_{\prime})}{(d - d_{\prime})\,(d - d_{\prime} + 2\,\delta)},$$

or since $2\,\delta$ is very small we may say

$$\lambda = \frac{2\,\delta\,(d + d_{\prime})}{(d - d_{\prime})^2}.$$

If we put the percentage for the absolute value of λ, that is if we have

$$\lambda = \frac{\lambda'}{100} \text{ of } \frac{E_1}{E_2} = \frac{\lambda'}{100} \times \frac{d + d_{\prime}}{d - d_{\prime}},$$

then we get

$$\frac{\lambda'}{100} \times \frac{d + d_{\prime}}{d - d_{\prime}} = \frac{2\,\delta\,(d + d_{\prime})}{(d - d_{\prime})^2},$$

that is to say

$$\lambda' = \frac{2\,\delta\,100}{d - d_{\prime}}. \qquad \text{[A]}$$

For example.

In the example given on page 125 the deflections could each be read to an accuracy of $\frac{1}{4}$ of a division; what was the degree of accuracy with which the value of $\frac{E_1}{E_2}$ could be determined?

$$\lambda' = \frac{2 \times \frac{1}{4} \times 100}{72 - 8} = \cdot 78 \text{ per cent.}$$

If $d_{,}$ is small compared with d, then

$$\lambda' = \frac{2\,\delta}{d}.$$

We can see from equation [A] that unless $d_{,}$ is small compared with d, the accuracy with which the test can be made will be but small; for if $d_{,}$ approaches in value to d, then $d - d_{,}$ becomes very small, that is λ' becomes large. In order that $d_{,}$ may be as much smaller than d as possible, E_1 and E_2 must be as nearly equal as possible; the test therefore will not be a satisfactory one unless such is the case.

If $d_{,}$ is small compared with d, then

$$\lambda = \frac{2\,\delta\,100}{d},$$

or if we put the percentage instead of the absolute value of δ, that is if we have

$$\delta = \frac{\delta'}{100} \text{ of } d,$$

then we get

$$\lambda' = 2\,\delta',$$

so that under the best conditions for making the test the accuracy with which the value of $\frac{E_1}{E_2}$ could be determined would be but one-half the accuracy with which the higher deflection could be observed.

150. To determine the degree of accuracy attainable in the case where the readings are made from the *degrees* scale of a tangent galvanometer, we must in the preceding investigation substitute tangents for divisions of deflection. Thus we have

$$\frac{E_1}{E_2} - \lambda = \frac{\tan(d^\circ + \delta^\circ) + \tan(d_{,}^\circ + \delta^\circ)}{\tan(d^\circ + \delta^\circ) - \tan(d_{,}^\circ + \delta^\circ)},$$

or

$$\lambda = \frac{\tan d^\circ + \tan d_{,}^\circ}{\tan d^\circ - \tan d_{,}^\circ} - \frac{\tan(d^\circ + \delta^\circ) + \tan(d_{,}^\circ - \delta^\circ)}{\tan(d^\circ + \delta^\circ) - \tan(d_{,}^\circ - \delta^\circ)}.$$

If in this equation we put

$$\tan (d^\circ + \delta^\circ) = \frac{\tan d^\circ + \tan \delta^\circ}{1 - d^\circ \tan \delta^\circ}$$

and

$$\tan (d^\circ - \delta^\circ) = \frac{\tan d^\circ - \tan \delta^\circ}{1 + \tan d^\circ \tan \delta^\circ},$$

we get

$$\lambda = \frac{2 \tan \delta^\circ [(\tan d^\circ + \tan d_{,}^\circ)(1 + \tan d^\circ \tan d_{,}^\circ) + X]}{(\tan d^\circ - \tan d_{,}^\circ)(\tan d^\circ - \tan d_{,}^\circ + Y)}$$

where X and Y are a number of factors of $\tan \delta^\circ$. But since $\tan \delta^\circ$ is very small, we may put X and Y equal to 0, in which case we have

$$\lambda = \frac{2 \tan \delta^\circ (\tan d^\circ + \tan d_{,}^\circ)}{\tan d^\circ - \tan d_{,}^\circ} \times \frac{1 + \tan d^\circ \tan d_{,}^\circ}{\tan d^\circ - \tan d_{,}^\circ}$$

$$= \frac{2 \tan \delta^\circ (\tan d^\circ + \tan d_{,}^\circ)}{\tan d^\circ - \tan d_{,}^\circ} \times \frac{1}{\tan (d^\circ - d_{,}^\circ)}.$$

If we put the percentage for the absolute value of λ, that is, if if we have

$$\lambda = \frac{\lambda'}{100} \text{ of } \frac{E_1}{E_2} = \frac{\lambda'}{100} \times \frac{\tan d^\circ + \tan d_{,}^\circ}{\tan d^\circ - \tan d_{,}^\circ},$$

then we get

$$\lambda' = \frac{2 \tan \delta^\circ\, 100}{\tan (d^\circ - d_{,}^\circ)}. \qquad \text{[B]}$$

For example.

In comparing the electromotive forces of two batteries by Wiedemann's method, the deflections obtained on the *degrees* scale of a tangent galvanometer were 71° and 18° respectively; what were the relative electromotive forces of the batteries, and what would have been the degree of accuracy with which the value of $\frac{E_1}{E_2}$ could be determined? The value of the deflections could be read to an accuracy of $\frac{1}{4}$°.

$$E_1 : E_2 :: \tan 71^\circ + \tan 18^\circ : \tan 71^\circ - \tan 18^\circ,$$

or as

$$2 \cdot 9042 + \cdot 3249 \text{ to } 2 \cdot 9042 - \cdot 3249,$$

that is, as

$$1 \cdot 25 \text{ to } 1;$$

also

$$\lambda' = \frac{2 \times \tan \frac{1}{4}^\circ \times 100}{\tan (71^\circ - 18^\circ)} = \frac{2 \times \cdot 4363}{1 \cdot 3270} = \cdot 65 \text{ per cent.}$$

Like equation [A] (page 126), equation [B] shows that unless $d_{\prime}^\circ$ is small compared with d°, the test cannot be made with a high degree of accuracy.

To sum up, then, we have

Best Conditions for making the Test.

151. To obtain satisfactory results, E_1 and E_2 should be as nearly as possible equal.

As much resistance should be included in the circuit as possible.

If the readings are made on the *degrees* scale of a tangent galvanometer, then the resistance in circuit should be so adjusted that the deflections, as nearly as possible, make equal angles on opposite sides of 45°.

Possible Degree of Accuracy attainable.

When the readings are in *divisions*, then

$$\text{Percentage of accuracy} = \frac{\frac{2}{m}\,100}{d - d_{\prime}}$$

where $\frac{1}{m}$ is the smallest fraction of a division to which the deflections can be read.

When the readings are in *degrees* on a tangent galvanometer, then

$$\text{Percentage of accuracy} = \frac{2 \tan \frac{1^\circ}{m}\,100}{\tan (d^\circ - d_{\prime}^\circ)}$$

where $\frac{1^\circ}{m}$ is the smallest fraction of a degree to which the deflections can be read.

WHEATSTONE'S METHOD.

152. The most elegant method of comparing the electromotive forces of batteries is that of the late Sir Charles Wheatstone.

Battery E_1 is joined up in simple circuit with a galvanometer and a resistance; a deflection of α° is obtained. The resistance is now increased by ρ_1, so that a new deflection, β°, is produced.

Battery E_2 is next joined up in the place of E_1, and the resistance in circuit is adjusted until the deflection obtained is α°, as at first. The resistance is now increased by ρ_2, so that the deflection is reduced to β°, as in the first instance.

Now from the "Equal resistance method" (page 122), we see that the total resistances, R_1 and R_2, in circuit, which were required in the two cases to bring the deflections to α°, must be in direct proportion to the electromotive forces, E_1, E_2, of the two batteries. Also the total resistances, $R_1 + \rho_1$, and $R_2 + \rho_2$, in circuit which were required in the two cases to bring the deflections to β°, must be in direct proportion to the electromotive forces, E_1, E_2.

We therefore have

$$E_1 : E_2 :: R_1 : R_2,$$

or

$$E_1 R_2 = E_2 R_1,$$

and

$$E_1 : E_2 :: R_1 + \rho_1 : R_2 + \rho_2,$$

or

$$E_1 R_2 + E_1 \rho_2 = E_2 R_1 + E_2 \rho_1 = E_1 R_2 + E_2 \rho_1;$$

that is

$$E_1 \rho_2 = E_2 \rho_1,$$

or

$$E_1 : E_2 :: \rho_1 : \rho_2,$$

In fact, the electromotive forces of the batteries are directly proportional to the added resistances which, in both cases, were required to bring the deflections of the galvanometer needle from α° down to β°.

For example.

With a galvanometer and battery E_1 we obtained, with a resistance of 1950 ohms in the resistance box, a deflection of 54°, and by adding 2000 ohms (ρ_1), a deflection of 34°. Battery E_2 being inserted in the place of E_1, a resistance of 1650 ohms was inserted in the resistance box, which brought the galvanometer needle to 54° as at first, and by adding 1600 ohms (ρ_2), the deflection was reduced to 34° as in the first instance; then

$$E_1 : E_2 :: 2000 : 1600,$$
$$\text{or as } 1{\cdot}25 : 1.$$

153. In this and the preceding tests we have supposed that the electromotive forces of any *two* batteries were being compared, but it must be evident that by noting the deflections, resistances added, &c., as the case may be, with any number of batteries, their electromotive forces may all be compared.

154. We will now proceed to determine the "Best conditions for making the foregoing test."

There are two points to be determined: first, what should be the resistances in circuit when observing the first deflections, and second, what proportion should the added resistances bear to the original resistances?

When the test is executed, there are two or more sets of observations made, viz., one for each battery. But it will be found, on examination, that the proportion between the electromotive forces, the original resistances, and the added resistances, is the same for every set; consequently, we have only to determine what relative values these quantities should have in any one set. Those in the others will be in the same proportion.

It will be convenient to consider first what proportion the added resistance should bear to the original resistance. For this purpose we will suppose ρ_1 to be the former resistance.

Now ρ_1 represents the electromotive force of the battery, and therefore in order that the test may be made as accurately as possible, it is necessary that we should be able to adjust or determine the value of ρ_1 as accurately as possible. In order to obtain the required value of ρ_1, we first adjust R_1 so as to obtain the deflection $\alpha°$, and then we increase R_1 by ρ_1 so as to obtain the deflection $\beta°$; consequently, the accuracy with which we can obtain ρ_1 must be dependent upon the accuracy with which we can read both the deflections, $\alpha°$ and $\beta°$.

Let, then, the first deflection ($\alpha°$) be due to a current, C_1, then we have

$$C_1 = \frac{E_1}{R_1}, \quad \text{or,} \quad C_1 R_1 = E_1.$$

When the current is reduced to C_2 by the addition of ρ_1, then we get

$$C_2 = \frac{E_1}{R_1 + \rho_1}, \quad \text{or,} \quad C_2 R_1 + C_2 \rho_1 = E_1;$$

therefore

$$C_2 R_1 + C_2 \rho_1 = C_1 R_1,$$

or

$$\rho_1 = R_1 \left(\frac{C_1}{C_2} - 1\right).$$

Now this equation is identical with equation [F] (page 92) in the "Diminished deflection shunt method" of determining the resistance of a galvanometer; consequently, we can see from the investigations there given, that ρ_1 would be most accurately obtained if

$$C_2 = \frac{C_1}{3}$$

approximately; but when this is the case

$$\rho_1 = R_1 \left(\frac{C_1}{\frac{C_1}{3}} - 1 \right) = 2\, R_1\,;$$

that is to say, the added resistance should be about double the original resistance.

As regards the "Possible degree of accuracy attainable," we can see from equation [H] (page 92) in the test before referred to, that the percentage of accuracy, λ', attainable must be

$$\lambda' = \frac{(C_1\, c_2 + C_2\, c_1)\, 100^*}{C_2\,(C_1 - C_2)} \text{ per cent.}$$

As it is the relative electromotive forces of *two* batteries which have to be determined, that is to say, the value of $\frac{E_1}{E_2}$, the percentage of accuracy with which the test can be made will be double the above.

As regards the value for the original resistance there is little to be said. It does not affect the accuracy of the test, except in so far as the power of adjustment is concerned; this is evidently made as favourable as possible by making the resistance as high as convenient.

We must have therefore

Best Conditions for making the Test.

155. When making the observations with the first battery, make the original resistance as high as convenient, and make the added resistance as nearly as possible double this.

* The expression $\left[1 + G\left(\frac{1}{S_1} + \frac{1}{R}\right)\right]$ in the equation referred to [H] (page 92) becomes equal to 1 when S_1 and R are very high; this must be the case when equation [B] (page 90) becomes simplified into equation [F] (page 92).

Possible Degree of Accuracy attainable.

When the readings are in *divisions*, then

$$\text{Percentage of accuracy} = \frac{\frac{1}{m}\,(D - d)\,200}{d\,(D - d)}$$

where $\frac{1}{m}$ is the smallest fraction of a division to which the deflections can be read.

If the deflections are in *degrees* on a tangent galvanometer, then if we can read their values to an accuracy of $\frac{1}{m}$th of a *degree*, we have

$$\text{Percentage of accuracy} = \frac{(\tan D^\circ\,\delta_2 + \tan d^\circ\,\delta_1)\,200}{\tan d^\circ\,(\tan D^\circ - \tan d^\circ)}$$

where

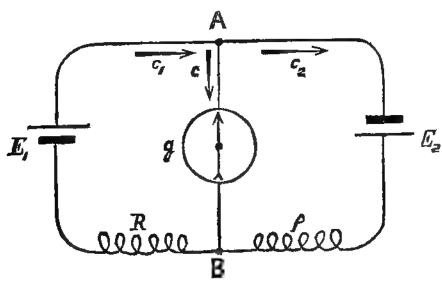
$$\delta_1 = \tan D\tfrac{1}{m}^\circ - \tan D^\circ, \text{ and, } \delta_2 = \tan d\,\tfrac{1}{m}^\circ - \tan d^\circ.$$

156. Wheatstone's test can be made with any form of galvanometer, as it is not necessary that the values of the deflections in terms of the currents producing them be known, except for the determination of the "Percentage of accuracy attainable." If, however, the galvanometer be "calibrated" (page 30), this percentage can be determined.

Lumsden's Method.

157. This is an excellent method of determining the comparative electromotive forces of batteries. The principle of the arrangement is shown by Fig. 44.

Fig. 44.



First Method.

The two batteries E_1, E_2 are joined up with their opposite poles connected together, and with resistances R, ρ in their circuit. A galvanometer g is connected between the points A, B.

One of the resistances, say ρ, being fixed, the other, R, is adjusted until no deflection is observed on the galvanometer. When this is the case we get the proportion

$$\frac{E_1}{E_2} = \frac{R}{\rho}.$$

158. In order to understand why this is the case, let us examine the theory of the method; this may be explained by the help of Kirchoff's two laws,* viz. :—

1. *The algebraical sum of the current strengths in all the wires which meet in a point is equal to nothing.*

2. *The algebraical sum of all the products of the current strengths and resistances in all the wires forming an enclosed figure, equals the algebraical sum of all the electromotive forces in the circuit.*

159. Supposing, at first, equilibrium not to be produced, then we have the following equations connecting the various current strengths, resistances, and electromotive forces :—

$$c_1 - c - c_2 = 0. \qquad [1]$$

$$R\,c_1 + g\,c - E_1 = 0. \qquad [2]$$

$$\rho\,c_2 - g\,c - E_2 = 0. \qquad [3]$$

From equation [1] we get, $c_1 = c + c_2$;

therefore

$$R\,(c + c_2) + g\,c - E_1 = 0.$$

From equation [3] we get

$$c_2 = \frac{E_2 + g\,c}{\rho};$$

therefore

$$R\left(c + \frac{E_2 + g\,c}{\rho}\right) + g\,c - E_1 = 0;$$

therefore

$$R\,\rho\,c + R\,E_2 + R\,g\,c + \rho\,g\,c - \rho\,E_1 = 0;$$

therefore

$$c = \frac{\rho\,E_1 - R\,E_2}{g\,(R + \rho) + R\,\rho}. \qquad [4]$$

If in this equation we put

$$c = 0,$$

then

$$\rho\,E_1 - R\,E_2 = 0,$$

* For the proof of these laws see Appendix.

$$\frac{E_1}{E_2} = \frac{R}{\rho};$$

that is,

$$E_1 : E_2 :: R : \rho.$$

160. Let us consider what are the "Best conditions for making the test." What we have to determine is, what are the best values to give to R and ρ? Now, since E_1 and E_2 are definite quantities, the value given to R (supposing this to be the adjustable resistance) will be determined by the value given to ρ; we must therefore determine the value to give to the latter.

The greater the accuracy with which we can adjust R, the greater will be the accuracy with which we can determine the value of $\frac{E_1}{E_2}$, that is, the relative values of E_1 and E_2. But the accuracy with which we can adjust R depends upon its range of adjustment being as great as possible, and this can only be the case when it has as high a value as possible. Thus, if R were 100 units, we could only adjust it to an accuracy of 1 unit in 100, or 1 per cent.; but if R were 10,000, then 1 unit in 10,000 represents an adjustment of $\frac{1}{100}$ per cent. But it is no use making R 10,000, unless a change of 1 unit in its value produces a perceptible deflection of the galvanometer needle.

The best value therefore to give to R *is the highest one in which a change of* 1 *unit from its correct value produces a perceptible deflection* of the galvanometer needle. Since R is dependent upon the value given to ρ, what we require to know is the highest value to give to the latter quantity.

Equation [4] shows the current, c, obtained through the galvanometer when equilibrium is not produced. If in this equation we put R − 1 in the numerator instead of R and then put

$$R\,E_2 = \rho\,E_1, \text{ or, } R = \frac{\rho\,E_1}{E_2},$$

we shall get the current corresponding to the change of 1 unit in the correct value of R_1. Thus

$$c = \frac{\rho\,E_1 - (R-1)\,E_2}{g\,(R+\rho) + R\,\rho} = \frac{E_2}{g\left(\frac{\rho\,E_1}{E_2} + \rho\right) + \frac{\rho\,E_1}{E_2}\,\rho}$$

$$= \frac{E_2}{\rho\,\frac{E_1}{E}\left[g\left(1 + \frac{E_2}{E_1}\right) + \rho\right]}, \qquad \text{[A]}$$

or

$$\rho\left[g\left(1+\frac{E_2}{E_1}\right)+\rho\right]=\frac{E_2^{\,2}}{E_1\,c}. \qquad [B]$$

Practically, the minimum readable deflection of a Thomson galvanometer (which is the best to employ in a test of this kind) is one division, and the reciprocal of the current producing this deflection is the *figure of merit* of the instrument (page 47). If, therefore, in the last equation we put for c the reciprocal of the figure of merit of the galvanometer, we can determine the highest value which can be given to ρ, E_1 and E_2 both being in *volts*.

If we wish to get the exact value of ρ, we can do so by solving the quadratic equation; but, practically, we only require to get a rough idea of what the value of ρ may be, and this we may obtain by giving different values to ρ, and trying which of them nearly satisfies the equation.

For example.

Two batteries, whose electromotive forces E_1 and E_2 were known to be of the approximate values of 2 : 1 (E_2 being 1 *volt*), were to be tested by the foregoing method with a Thomson galvanometer whose resistance was 5000 ohms (g) and figure of merit 1,000,000,000: What was the highest value that could be given to ρ?

$$\rho\,[5000\,(1+\tfrac{1}{2})+\rho]=1{,}000{,}000{,}000\times\tfrac{1}{2},$$

or

$$\rho\,[7500+\rho]=500{,}000{,}000.$$

From this we can see that if we make $\rho = 19{,}000$ we shall be very nearly right, for

$$19{,}000\,[7500+19{,}000]=503{,}500{,}000.$$

With this value of ρ, the value which R would have when adjusted, would be

$$R=\rho\,\frac{E_1}{E_2}=19{,}000\times\frac{2}{1}=38{,}000,$$

and with this value we could obtain a degree of accuracy equal to

$$\frac{1}{38{,}000}\times 100=\cdot 0026 \text{ per cent.}$$

Having then ascertained the value to give to ρ, suppose we actually made it 19,000, and further, we found that in order to get equilibrium as nearly as possible, we had to adjust R to 36,250 ohms, then the relative values of E_1 and E_2 would be

$$E_1 : E_2 :: 36{,}250 : 19{,}000,$$

or

$$:: 1{\cdot}9089 : 1,$$

and we know this is correct within $\cdot 0026$ per cent.

From equation [A] (page 135) we can see that c is greatest when E_2 is larger than E_1. It is therefore best to so arrange the test that the resistance to be adjusted is the one in circuit with the strongest of the two batteries. Also we can see that the more the batteries differ in electromotive force the better, as the greater will be the value of ρ.

Second Method.

161. In the example we have taken we have supposed the resistances of the batteries to have been so low that their values could practically be neglected in comparison with the high resistances R, ρ, which we were able to put in circuit. If, however, the batteries consist of a great number of cells of high resistance, and also the galvanometer be not a highly sensitive one, and consequently R and ρ have to be proportionately small, then we can no longer ignore the resistances of the batteries, and these must either be added on to R and ρ or eliminated in the following manner.

Suppose the resistances of E_1 and E_2 to be r_1 and r_2 respectively, then when equilibrium is produced we have

$$E_1 : E_2 :: R + r_1 : \rho + r_2,$$

or

$$E_1 r_2 - E_2 r_1 = E_2 R - E_1 \rho. \qquad [1]$$

Now if we decrease ρ to ρ_1 and again obtain balance by decreasing R to R_1, we get a second proportion, viz.—

$$E_1 r_2 - E_2 r_1 = E_2 R_1 - E_1 \rho_1. \qquad [2]$$

By subtracting [2] from [1] we get

$$E_2 R - E_2 R_1 - E_1 \rho + E_1 \rho_1 = 0,$$

or

$$E_2 (R - R_1) = E_1 (\rho - \rho_1);$$

that is

$$\frac{E_1}{E_2} = \frac{R - R_1}{\rho - \rho_1}, \qquad [A]$$

or

$$E_1 : E_2 :: R - R_1 : \rho - \rho_1,$$

a proportion in which differences of resistance alone appear. In fact $(R - R_1)$ and $(\rho - \rho_1)$ are merely the resistances which we subtracted from R and ρ, in order to get equilibrium a second time.

For example.

Two batteries whose electromotive forces E_1 and E_2 were to be compared, were joined up in circuit with a galvanometer and two resistances as shown by Fig. 44, the resistance ρ being 500 ohms; in order to obtain equilibrium R was adjusted to 1050 ohms; ρ was then decreased to 300 ohms (ρ_1), and in order to again obtain equilibrium, R had to be reduced to 630 ohms (R_1). What were the comparative electromotive forces of the batteries?

$$E_1 : E_2 :: 1050 - 630 : 500 - 300$$
$$:: \quad 420 \quad : \quad 200$$

or

as 2·1 to 1.

162. The question now arises what are the best values to give to R_1 and ρ_1, or rather to ρ_1, for the value given to the latter will determine the value given to R_1.

In order to work out the problem let us suppose, in the equation

$$\frac{E_1}{E_2} = \frac{R - R_1}{\rho - \rho_1},$$

there is a small error λ in $\frac{E_1}{E_2}$ caused by a definite error $-\phi$ in R_1, that is, let

$$\frac{E_1}{E_2} + \lambda = \frac{R - (R_1 - \phi)}{\rho - \rho_1} = \frac{R - R_1}{\rho - \rho_1} + \frac{\phi}{\rho - \rho_1}. \qquad [B]$$

By subtracting [A] from [B] we get

$$\lambda = \frac{\phi}{\rho - \rho_1}.$$

This shows that with the definite error ϕ, λ is as small as possible when ρ_1 is as small as possible. λ would be very great if ρ_1 approached in value to ρ, but it would be small when ρ_1 is about equal to $\frac{\rho}{2}$, and but little less if ρ_1 is made very much smaller still. Although, therefore, we should make ρ_1 small, there is but little advantage in making it very much smaller than $\frac{\rho}{2}$; in fact, there is an actual disadvantage, for when ρ_1 is very small, R_1 is proportionately small and its range of adjustment is correspondingly limited.

From equation [A] (page 135) we can see that in the present case the currents flowing through the galvanometer when equilibrium is not established, in consequence of R and R_1 being each 1 unit out of adjustment, are

$$c_1 = \frac{E_2}{(\rho + r_2)\frac{E_1}{E_2}\left[g\left(1 + \frac{E_2}{E_1}\right) + \rho + r_2\right]}$$

and

$$c_2 = \frac{E_2}{(\rho_1 + r_2)\frac{E_1}{E_2}\left[g\left(1 + \frac{E_2}{E_1}\right) + \rho_1 + r_2\right]}$$

respectively; and from these equations it is evident that if c_1 is a perceptible deflection when R is 1 unit out, c_2 will be a still more perceptible deflection when R_1 is 1 unit out, since R_1 must be smaller than R; consequently the value we give to R_1 will not be limited by any considerations with regard to a perceptible deflection being obtained.

As in the first test, c_1 and c_2 are both greatest when E_1 is larger than E_2, the batteries should therefore be so arranged that this is the case.

With regard to the *Possible degree of accuracy attainable* with this test, we can see first of all that R cannot be adjusted quite so accurately as in the case where the resistance of the batteries was negligible; we can, however, ascertain the exact degree attainable by putting $\rho + r_2$ instead of ρ in equation [B] (page 136). Thus to take the example given on page 136, suppose the battery E_2 had a resistance of 5000 ohms (r_2) approximately, then we should have

$$(\rho + 5000)\,[5000\,(1 + \tfrac{1}{2}) + \rho + 5000] = 1{,}000{,}000{,}000 \times \tfrac{1}{2},$$

or

$$(\rho + 5000)\,[12{,}500 + \rho] = 500{,}000{,}000.$$

If in this equation we make $\rho = 14{,}000$, we get

$$(14{,}000 + 5000)\,[12{,}500 + 14{,}000] = 503{,}500{,}000,$$

which is close to the correct value. In other words, if ρ does not exceed 14,000 ohms, we can be sure of the value of R within 1 unit.

The degree of accuracy with which we can determine the value of $\frac{E_1}{E_2}$ from the equation

$$\frac{E_1}{E_2} = \frac{R - R_1}{\rho - \rho_1}$$

depends upon the degree of accuracy with which we can adjust both R and R_1, and as the errors in either of them may be either + or −, the greatest possible total error is that which will be produced by a + error in R, and a − error in R_1, or *vice versâ*. Let these errors be both 1 unit and let the corresponding error in $\frac{E_1}{E_2}$ be λ, then we have

$$\frac{E_1}{E_2} + \lambda = \frac{R + 1 - (R^1 - 1)}{\rho - \rho_1} = \frac{R - R_1}{\rho - \rho_1} + \frac{2}{\rho - \rho_1},$$

and

$$\frac{E_1}{E_2} = \frac{R - R_1}{\rho - \rho_1};$$

therefore

$$\lambda = \frac{2}{\rho - \rho_1}.$$

Since we require to know what *percentage* (λ') of error this represents, we have

$$\lambda = \frac{\lambda'}{100} \text{ of } \frac{E_1}{E_2},$$

or

$$\lambda' = 100\,\lambda\,\frac{E_1}{E_2} = \frac{200}{\rho - \rho_1} \cdot \frac{E_2}{E_1}. \qquad [C]$$

To take the example we have just considered, we see that the possible percentage of accuracy attainable, supposing ρ_1 to equal $\frac{\rho}{2}$, is

$$\lambda' = \frac{200}{14{,}000 - 7000} \times \tfrac{1}{2} = \cdot 014 \text{ per cent.}$$

163. With a Thomson galvanometer of ordinary sensitiveness it is evident from the foregoing investigation, that if we have two batteries, one E_2 having an electromotive force of 1 volt or more, and E_1 an electromotive force of twice that value or more, we can without difficulty determine their relative electromotive forces to an accuracy of, at least, ·015 per cent.; and if the resistance of the batteries be very low we can be certain of the accuracy within, say, ·003 per cent.

164. It is possible to get a still greater accuracy by employing a set of resistance coils adjustable to $\frac{1}{10}$th or $\frac{1}{100}$th of a unit, for in this case we can make both R and R_1 low without losing the range of adjustment, whilst by making these quantities low we increase the value of the galvanometer deflection when exact adjustment is not obtained; this is only the case, however, when the resistances of the batteries and of the galvanometer are low.

We can easily determine to what extent the degree of accuracy is increased by using submultiples of the units; first by ascertaining from equation [B] (page 136) what value ρ can have, $\frac{E_2^2}{E_1\,c}$ being divided by 10 if R is adjustable to $\frac{1}{10}$ths, and by 100 if R is adjustable to $\frac{1}{100}$ths; and second by working out the value of λ' from equation [C] (page 140) which gives the required percentage of accuracy.

Of course when great accuracy is required the test must be made by the method in which the resistance of the battery is eliminated; it is no use making the test by the first method, since the accuracy attainable by having R adjustable to $\frac{1}{10}$th or $\frac{1}{100}$th of an ohm is more than counterbalanced by the error produced by not taking into account the resistance of the battery.

To summarise the results we have obtained, we have

Best Conditions for making the Test.

First Method.

165. First make a rough test to ascertain the approximate values of E_1 and E_2, then make ρ of such a value that

$$\rho\left[g\left(1 + \frac{E_2}{E_1}\right) + \rho\right] = \frac{E_2^2}{E_1\,c}$$

approximately, c being the reciprocal of the figure of merit of the galvanometer, and E_1 the stronger of the two batteries, E_1 and E_2 being in *volts.*

Second Method.

Make ρ of such a value that

$$(\rho + r_2)\left[g\left(1 + \frac{E_2}{E_1}\right) + \rho + r_2\right] = \frac{E_2^2}{E_1 c}$$

approximately.

If R is adjustable to $\frac{1}{10}$th or $\frac{1}{100}$th of an ohm, the right-hand side of the equation should be $\frac{E_2^2}{E_1\,10\,c}$ or $\frac{E_2^2}{E_1\,100\,c}$ respectively.

ρ_1 should be about equal to $\frac{\rho}{2}$.

In both methods E_1 should be the larger of the two batteries.

Possible Degree of Accuracy attainable.

First Method.

Where resistance of battery is very small,

$$\text{Percentage of accuracy} = \frac{100}{\rho} \times \frac{E_2}{E_1}.$$

Second Method.

$$\text{Percentage of accuracy} = \frac{200}{\rho - \rho_1} \times \frac{E_2}{E_1}.$$

Or, if ρ_1 is nearly equal to $\frac{\rho}{2}$,

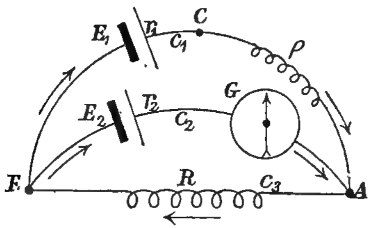
$$\text{Percentage of accuracy} = \frac{400}{\rho} \times \frac{E_2}{E_1}.$$

166. A great point in these methods of determining the comparative electromotive forces of batteries, lies in the fact that both batteries are working under exactly the same conditions; moreover, if the resistances R and ρ are high there is but little tendency for them to polarise. If one of the batteries be a constant one, such as a Daniell, then by varying the values of R and ρ we can test how the other battery behaves when worked through different resistances.

Poggendorff's Method.

167. In this method one battery is balanced against the other. The method is shown by Fig. 45. In this figure E_1 and E_2 are the electromotive forces to be compared. R and ρ are adjustable

Fig. 45.



resistances, r_1 and r_2 being the resistances of the batteries. G is the resistance of the galvanometer.

Before equilibrium is obtained we have

$$c_1 + c_2 - c_3 = 0 \qquad [1]$$

$$(r_2 + G)\, c_2 + R\, c_3 - E_2 = 0 \qquad [2]$$

$$(r_1 + \rho)\, c_1 + R\, c_3 - E_1 = 0. \qquad [3]$$

By substituting the value of c_1 obtained from equation [1], in equation [2], and then again the value of c_3 obtained from equation [2], in equation [3], we shall find that

$$c_2 = \frac{(r_1 + \rho)\, E_2 - R\,(E_1 + E_2)}{R\,(r_2 + G + r_1 + \rho) + (r_1 + \rho)\,(r_2 + G)}. \qquad [4]$$

If we put $c_2 = 0$, we get

$$(r_1 + \rho)\, E_2 - R\,(E_1 - E_2) = 0,$$

or

$$E_2\,(R + r_1 + \rho) = E_1\, R\,;$$

that is

$$E_1 : E_2 :: R + r_1 + \rho : R, \qquad [5]$$

or

$$\frac{E_1}{E_2} = 1 + \frac{r_1 + \rho}{R}. \qquad [6]$$

It will be observed that in order to get the ratio of E_1 to E_2 from this proportion, we must know the resistance r_1 of the

battery E_1. If, however, we decrease ρ to ρ_1 and again get equilibrium by readjusting R to R_1, we get a second proportion, viz.,

$$E_1 : E_2 :: R_1 + r_1 + \rho_1 : R_1, \qquad [7]$$

and by combining the two proportions, r_1 is eliminated in the manner shown in the last test (page 137) and we get

$$\frac{E_1}{E_2} = \frac{(R - R_1) + (\rho - \rho_1)}{R - R_1},$$

or

$$E_1 : E_2 :: (R - R_1) + (\rho - \rho_1) : (R - R_1), \qquad [A]$$

a proportion in which differences of resistance alone enter.

For example.

Two batteries whose electromotive forces E_1 and E_2 were to be compared, were joined up in circuit with a galvanometer and two resistances as shown in Fig. 45. The resistance ρ being 200 ohms, it was necessary in order to obtain equilibrium to adjust R to 500 ohms. ρ was then reduced to 100 ohms (ρ_1), and in order again to get equilibrium R had to be readjusted to 400 ohms (R_1), then

$$E_1 : E_2 :: (500 - 400) + (200 - 100) : (500 - 400);$$
$$\text{or as } 2 : 1.$$

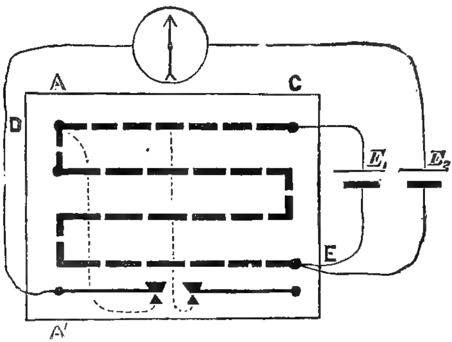
168. In making this test practically, the connections with the set of resistance coils shown by Fig. 6, page 13, would be as shown by Fig. 46. Having depressed the left-hand key, then, according to the example, we should take out the two 100 plugs between A and C, and proceed to adjust between D and E. This being done, we should insert one of the 100 plugs between A and C and readjust the resistance between D and E.

169. As only one of the batteries (the smaller) in this test has its electromotive force balanced, the other one should be a constant battery, whose electromotive force does not fall off on being worked continuously, such as a Daniell.

170. It is evident that the test can be made either by making ρ a fixed resistance and R an adjustable one, or by making R fixed and ρ adjustable. In order therefore to determine the *Best conditions for making the test*, one point for consideration will be—should R or ρ be the adjustable quantity?

Now by a similar reasoning to that given in (§ 160) we can see that in either case the value of the adjustable resistance should be *the highest one in which a change of* 1 *unit from its correct resistance produces a perceptible deflection of the galvanometer needle.*

FIG. 46.



If we refer to equation [6] (page 143) we can see that if $E_1 = 2\,E_2$ then $r_1 + \rho$ must be equal R, and that according as E_1 is greater or less than $2\,E_2$, so will $r_1 + \rho$ be greater or less than R. It is evident that the larger we make the adjustable resistance the greater will be the range of adjustment of which it is capable, therefore for this reason it follows that if E_1 is greater than $2\,E_2$ then $r_1 + \rho$ should be the resistance in which the adjustment is effected, whereas if E_1 is less than $2\,E_2$ then R should be the adjustable resistance.

Now if R be the adjustable resistance, then inasmuch as the value which it will have will depend upon the value given to ρ, therefore we must determine the highest value we can give to ρ.

Equation [4] (page 143) shows the current c_2 obtained through the galvanometer, when equilibrium is not produced. If in this equation we put $R - 1$ in the numerator instead of R, and then put

$$\frac{E_1}{E_2} = \frac{R + r_1 + \rho}{R}, \quad \text{or,} \quad R = (r_1 + \rho)\,\frac{E_2}{E_1 - E_2},$$

we shall get the current, c_2, corresponding to the change of 1 unit in the correct value of R. Thus

$$c_2 = \frac{(E_1 - E_2)^2}{(r_1 + \rho)\left[(r_1 + G)\,E_1 + (r_1 + \rho)\,E_2\right]}, \qquad [1]$$

or

$$(r_1 + \rho)\Big[(r_2 + G)\,E_1 + (r_1 + \rho)\,E_2\Big] = \frac{(E_1 - E_2)^2}{c_2}. \qquad [A]$$

And if in this equation we make c_2 the reciprocal of the figure of merit (page 47) of the galvanometer, then the value of ρ which satisfies the equation will be the highest value which it should have; as explained in the last test, ρ can be obtained by trial.

If ρ be the adjustable resistance, then what we have to determine is the value which R should have. To do this we must put $\rho + 1$ * instead of ρ in the numerator of equation [4] (page 143) and then put

$$\frac{E_1}{E_2} = \frac{R + r_1 + \rho}{R}, \quad \text{or,} \quad r_1 + \rho = \frac{R\,(E_1 - E_2)}{E_2},$$

we shall then get the current, c'_2, corresponding to the change of 1 unit in the current value of ρ. Thus

$$c'_2 = \frac{E_2^{\,2}}{R\,[(r_2 + G)\,E_1 + R\,(E_1 - E_2)]}.$$

or

$$R\,[(r_2 + G)\,E_1 + R\,(E_1 - E_2)] = \frac{E_2^{\,2}}{c'_2}, \qquad [B]$$

from which, as in the previous case, R can be obtained by trial.

We have next to determine the value which should be given to R, or to ρ_1. Let us in the first instance take R_1 to be the adjustable resistance, then what we have to do is to find the proper value to give to ρ_1. If, then, we suppose in the equation

$$\frac{E_1}{E_2} = \frac{(R - R_1) + (\rho - \rho_1)}{R - R_1}, \qquad [2]$$

or

$$\frac{E_1}{E_2} = 1 + \frac{\rho - \rho_1}{R - R_1}, \qquad [3]$$

that there is a small error λ in $\frac{E_1}{E_2}$ caused by a corresponding error $-\phi$ in R_1; then we have

$$\frac{E_1}{E_2} + \lambda = 1 + \frac{\rho - (\rho_1 - \phi)}{R - R_1}. \qquad [4]$$

* We put $\rho + 1$ in this case in preference to $\rho - 1$, simply in order to avoid giving c_2 a minus value. The general result obtained, however, would be similar whether the 1 be plus or minus.

By subtracting [3] from [4] we get

$$\lambda = \frac{\rho - (\rho_1 - \phi)}{\mathrm{R} - \mathrm{R}_1} - \frac{\rho - \rho_1}{\mathrm{R} - \mathrm{R}_1} = \frac{\phi}{\mathrm{R} - \mathrm{R}};$$

but from [3]

$$\mathrm{R} - \mathrm{R}_1 = \left(\frac{\mathrm{E}_2}{\mathrm{E}_1 - \mathrm{E}_2}\right)(\rho - \rho_1);$$

therefore

$$\lambda = \frac{\phi}{\left(\frac{\mathrm{E}_1}{\mathrm{E}_1 - \mathrm{E}_2}\right)(\rho - \rho_1)}.$$

This shows that with the definite error ϕ, λ is as small as possible when ρ_1 is as small as possible. λ would be very great if ρ_1 approaches in value to ρ, but it would be small when ρ_1 is about equal to $\frac{\rho}{2}$, and but little less if ρ_1 is made very small indeed. As our range of adjustment of R_1 is limited by making ρ_1 very small, it is advisable not to make it smaller than $\frac{\rho}{2}$.

A similar investigation would have proved that if ρ_1 were the adjustable resistance, then R_1 should be made small, though not smaller than $\frac{\mathrm{R}}{2}$.

171. From equation [1] (page 145) we can see that the test is impossible if E_1 and E_2 are equal, since $c_2 = 0$ with any value we can give to the resistances.* We can further see that the more the batteries differ in electromotive force the better; and also that it does not matter materially which is the stronger of the two.

172. As regards the *Possible degree of accuracy attainable*, this depends upon the degree of accuracy with which we can adjust both R and R_1 (or ρ and ρ_1, if R and R_1 are the fixed resistances), and as the errors in either of them may be + or −, the greatest possible error is that which will be produced by a + error in R and a − error in R_1 or *vice versâ.* Let these errors be both 1 unit, and let the corresponding error in $\frac{\mathrm{E}_1}{\mathrm{E}_2}$ be λ, then we have from equation [3] (page 146)

$$\frac{\mathrm{E}_1}{\mathrm{E}_2} + \lambda = 1 + \frac{\rho - \rho_1}{\mathrm{R} - 1 - (\mathrm{R}_1 + 1)} = 1 + \frac{\rho - \rho_1}{\mathrm{R} - \mathrm{R}_1 - 2}$$

* This is not the case in Lumsden's test.

and

$$\frac{E_1}{E_2} = 1 + \frac{\rho - \rho_1}{R - R_1}, \text{ or, } R - R_1 = \frac{E_2}{E_1 - E_2}(\rho - \rho_1);$$

therefore

$$\lambda = \frac{\rho - \rho_1}{R - R_1 - 2} - \frac{\rho - \rho_1}{R - R_1} = \frac{2(\rho - \rho_1)}{(R - R_1 - 2)(R - R_1)};$$

or since $R - R_1$ is very large,

$$\lambda = \frac{2(\rho - \rho_1)}{(R - R_1)^2} = \frac{2}{\rho - \rho_1} \times \left(\frac{E_1 - E_2}{E_2}\right)^2.$$

Since we require to know what *percentage* (λ') of error this represents, we have

$$\lambda = \frac{\lambda'}{100} \text{ of } \frac{E_1}{E_2},$$

or

$$\lambda' = 100\,\lambda\,\frac{E_2}{E_1} = \frac{200}{\rho - \rho_1} \times \frac{(E_1 - E_2)^2}{E_1\,E_2}.$$

In the case where ρ and ρ_1 are the adjustable resistances, we should get

$$\lambda = \frac{\rho - \rho_1 + 2}{R - R_1} - \frac{\rho - \rho_1}{R - R_1} = \frac{2}{R - R_1},$$

and calling, as before, λ' the *percentage* of error, we get

$$\lambda' = \frac{200}{R - R_1} \times \frac{E_2}{E_1}.$$

To sum up, then, we have

Best Conditions for making the Test.

173. First make a rough test to ascertain the approximate values of E_1, E_2, r_1, and r_2; then if E_1 is less than $2\,E_2$, make ρ a fixed resistance, and of such a value that

$$(r_1 + \rho)\left[(r_2 + G)\,E_1 + (r_1 + \rho)\,E_2\right] = \frac{(E_1 - E_2)^2}{c} \qquad [A]$$

approximately.

If R is adjustable to $\frac{1}{n}$th of an ohm, then the right-hand side of the last equation should be

$$\frac{(E_1 - E_2)^2}{c} \times \frac{1}{n},^*$$

c being the reciprocal of the figure of merit of the galvanometer, and E_1 and E_2 being both in *volts.*

ρ_1 should be about equal to $\frac{\rho}{2}$.

If E_1 is greater than 2 E_2, then make R a fixed resistance, and of such a value that

$$R\left[(r_2 + G)\,E_1 + R\,(E_1 - E_2)\right] = \frac{E_2^2}{c} \qquad [B]$$

approximately.

If ρ is adjustable to $\frac{1}{n}$th of an ohm, then the right-hand side of the last equation should be

$$\frac{E_2^2}{c} \times \frac{1}{n},$$

c being the figure of merit of the galvanometer, and E_1 and E_2 being both in *volts.*

R_1 should be about equal to $\frac{R}{2}$.

Possible Degree of Accuracy attainable.

When R and R_1 are the adjustable resistances, then

$$\text{Percentage of accuracy} = \frac{200}{\rho - \rho_1} \times \frac{(E_1 - E_2)^2}{E_1\,E_2};$$

or if ρ_1 nearly equals $\frac{\rho}{2}$

$$\text{Percentage of accuracy} = \frac{400}{\rho} \times \frac{(E_1 - E_2)^2}{E_1\,E_2};$$

When ρ and ρ_1 are the adjustable resistances, then

$$\text{Percentage of accuracy} = \frac{200}{R - R_1} \times \frac{E_2}{E_1};$$

* § 164, page 141.

or if R_1 nearly equals $\frac{R}{2}$

$$\text{Percentage of accuracy} = \frac{400}{R} \times \frac{E_2}{E_1}.$$

174. If the test is made by obtaining the result from formula [6] (page 143), the resistance r_1 of the battery being very small, then it is not difficult to see, from the investigation given in "Lumsden's test" (page 133) that when R is the adjustable resistance, then

$$\text{Percentage of accuracy} = \frac{100}{\rho} \times \frac{(E_1 - E_2)^2}{E_1 \, E_2}.$$

Also we should make ρ of such a value that

$$\rho \, (G \, E_1 + \rho \, E_2) = \frac{(E_1 - E_2)^2}{c}$$

approximately.

When ρ is the adjustable resistance, then

$$\text{Percentage of accuracy} = \frac{100}{R} \times \frac{E^2}{E_1}$$

Also we should make R of such a value that

$$R \left[G \, E_1 + R \, (E_1 - E_2) \right] = \frac{E_2{}^2}{c}$$

approximately.

Fahie's Method of Measuring Battery Resistance.

175. It may be pointed out* that the foregoing test also affords a means of ascertaining the *resistance*, r_1, of the battery E_1; thus from equations [5] and [7] (pages 143 and 144) we can see that

$$R + r_1 + \rho : R :: R_1 + r_1 + \rho_1 : R_1;$$

therefore

$$R_1 \, R + R_1 \, r_1 + R_1 \, \rho = R_1 \, R + R \, r_1 + R \, \rho_1;$$

therefore

$$r_1 \, (R - R_1) = R_1 \, \rho - R \, \rho_1,$$

or

$$r_1 = \frac{R_1 \, \rho - R \, \rho_1}{R - R_1};$$

* See Sabine's 'The Electric Telegraph,' p. 323.

thus if we take the example given on page 144, in which we have

$$
\begin{aligned}
R_1 &= 400 \\
R &= 500 \\
\rho &= 200 \\
\rho_1 &= 100
\end{aligned}
$$

we get

$$r_1 = \frac{(400 \times 200) - (500 \times 100)}{500 - 100} = 75 \text{ ohms.}$$

176. A resistance test made in this way, however, would not be an accurate one if the resistance r_1 of the battery were small in comparison with the resistance ρ_1 (which is in the same circuit with r_1), for in this case the high value of the latter would swamp, as it were, the low value of r_1. If, however, as suggested by Mr. Fahie,* we commence the test by having no resistance at first in circuit with the battery E_1, that is to say, if we have ρ_1 equal to 0, then we can obtain more satisfactory results; in this case we get

$$r_1 = \frac{R_1 \, \rho}{R - R_1}. \qquad [A]$$

177. With regard to the *Best conditions for making the test* according to formula [A], the resistance R_1 is the resistance required to produce balance in the first instance and it can have but one value; R, however, is dependent upon ρ, so that what is required is the value which should be given to the latter quantity. Now from formula [A] we can see that the larger we make ρ the larger will be the value of R, and the larger we make the latter the greater will be its range of adjustment, consequently, as in the electromotive force test, we should give it *the highest value in which a change of* 1 *unit from its correct resistance produces a perceptible deflection of the galvanometer needle;* this resistance we shall obtain by giving ρ such a value that

$$(r_1 + \rho)\left[(r_2 + G)\,E_1 + (r_1 + \rho)\,E_2\right] = \frac{(E_1 - E_2)^2}{c}$$

approximately,† c being the reciprocal of the figure of merit of the galvanometer.

As regards the *Possible degree of accuracy attainable,* this we shall obtain, as in previous cases, by supposing that there is an

* See 'Electrical Review,' vol. xii., p. 203. † Equation [A], p. 146.

error of $+1$ in R and an error of -1 in R_1, these errors causing a corresponding total error λ in r_1; thus

$$r_1 + \lambda = \frac{(R_1+1)\,\rho}{(R-1)-(R_1+1)} = \frac{(R_1+1)\,\rho}{R - R_1 - 2},$$

and since

$$r_1 = \frac{R_1\,\rho}{R - R_1}$$

we get

$$\lambda = \frac{(R_1+1)\,\rho}{R - R_1 - 2} - \frac{R_1\,\rho}{R - R_1} = \frac{\rho\,(R + R_1)}{(R - R_1 - 2)(R - R_1)},$$

or since $R - R_1$ is very large, we may say

$$\lambda = \frac{\rho\,(R + R_1)}{(R - R_1)^2}; \qquad [B]$$

but

$$\frac{E_1}{E_2} = \frac{R_1 + r_1}{R_1} = \frac{R + \rho + r_1}{R},$$

or

$$R_1 = \frac{E_2\,r_1}{E_1 - E_2}, \text{ and, } R = \frac{E_2\,(r_1 + \rho)}{E_1 - E_2},$$

therefore

$$R + R_1 = \frac{E_2}{E_1 - E_2}\,(2\,r_1 + \rho), \text{ and, } R - R_1 = \frac{E_2\,\rho}{E_1 - E_2}$$

and by substituting these values of $R + R_1$ and $R - R_1$ in equation [B] we get

$$\lambda = \frac{E_1 - E_2}{E_2}\left(\frac{2\,r_1}{\rho} + 1\right).$$

Or if we call λ' the *percentage* of error, then

$$\lambda = \frac{\lambda'}{100} \text{ of } r_1,$$

or

$$\lambda' = \frac{100\,\lambda}{r_1} = \frac{E_1 - E_2}{E_2}\left(\frac{2}{\rho} + \frac{1}{r_1}\right)100.$$

178. The relative electromotive forces of the batteries, it may be pointed out, are given by the proportion

$$E_1 : E_2 :: (R - R_1) + \rho : (R - R_1),$$

which is the same as proportion [A], page 144, except that ρ_1 is put equal to 0.

To sum up, then, we have

Best Conditions for making the Test.

179. Make ρ of such a value that

$$(r_1 + \rho)\,[(r_2 + G)\,E_1 + (r_1 + \rho)\,E_2] = \frac{(E_1 + E_2)^2}{c}$$

approximately, c being the reciprocal of the figure of merit of the galvanometer.

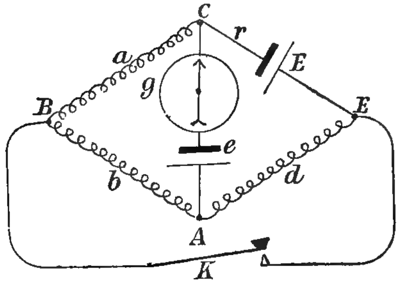
Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{E_1 - E_2}{E_2}\left(\frac{2}{\rho} + \frac{1}{r_1}\right)100.$$

FAHIE'S COMBINED METHOD OF COMPARING ELECTROMOTIVE FORCES AND MEASURING BATTERY RESISTANCE.

180. This is an extremely ingenious and elegant method, and although its application is rather limited it is well worth being noticed. The arrangement is a combination of Poggendorff's method of comparing electromotive forces (page 143) and Mance's method of measuring battery resistance (page 105).

FIG. 47.



Referring to Fig. 47, the following is the mode of making the test:—E is the stronger battery whose electromotive force is to be compared with the battery e, and whose internal resistance

is to be measured; d is a variable and $a + b$ a slide, resistance, B being the slider by the movement of which the ratio of a to b can be varied. The key K being open, the resistance d is adjusted until the needle of the galvanometer shows that no current is passing through the latter; when this is the case, then, as in Poggendorff's method (page 143), we have

$$E : e :: r + d + a + b : a + b. \qquad [1]$$

Balance being thus obtained, the key K is alternately depressed and raised and the slider B moved until the latter is brought to such a position that the movement of the key K ceases to affect the galvanometer needle, as in Mance's test (page 105). Now, inasmuch as the battery e merely acts as a counteracting force to the current which in Mance's test would cause a permanent deflection of the galvanometer needle, it must be evident that when the movement of the key K ceases to affect g, then we must have

$$r = \frac{a\,d}{b}, \qquad [2]$$

or

$$r + d = \frac{a\,d}{b} + d = \frac{d}{b}\,(a + b).$$

Substituting this value of $r + d$ in equation [1], we get

$$E : e :: \frac{d}{b}\,(a + b) + a + b : a + b,$$

or

$$E : e :: d + b : b. \qquad [3]$$

Equation [2], therefore, gives the resistance of the battery E, and equation [3] gives the relative electromotive forces of the two batteries.

For example.

The key K being raised, balance was obtained on the galvanometer g by adjusting d to 200 ohms. When the key K was alternately raised and depressed, the balance on g was disturbed until the slider B was moved to the position at which b was equal to 100 ohms; the total resistance of the slide resistance $a + b$ was 400 ohms, that is to say, a was equal to 300 ohms; then

$$r = \frac{300 \times 200}{100} = 600 \text{ ohms},$$

and

$$E : e :: 200 + 100 : 100,$$

or as

$$3 : 1.$$

181. The conditions for making this test so as to obtain accurate results must evidently be similar to those specified in the cases of Poggendorff's test and Mance's test made with a slide resistance. The nature of the method, however, is such that we cannot obtain the conditions which are best for the Poggendorff test without impairing the conditions necessary for making the Mance test accurately, so that practically we must arrange the resistances so as to suit the conditions necessary for making the latter satisfactorily; at the same time it may be pointed out that these conditions are such as to enable the Poggendorff test to be made with a considerable, though not with a *very* high, degree of accuracy. As in the case of Mance's test with a slide wire (page 108), the conditions required are that d shall be as large as possible, but not so large that the range of adjustment of the slider becomes excessively reduced. Now, practically, a slide resistance would not consist of more than about 100 coils, consequently if d were of such a value that the slider had to be set so that b was about 10 times as large as a (as would be the case when a slide wire is used), then the accuracy with which the latter could be adjusted would be extremely small, being only about 1 in 10, or 10 per cent. To make the test satisfactorily, therefore, it would be necessary to arrange so that the slider would have to come near the centre of its traverse, even though the sensitiveness of the whole arrangement became reduced in consequence. As long, however, as *sufficient* sensitiveness is obtained, that is to say, a sensitiveness such that a movement of the slider from its correct position to either of the contiguous coils produces a perceptible disturbance of the balance, then the nearer we can get the slider to the centre the better. It would not do, however, in any case to pass beyond the centre point, for in this case, although the error made in a by the slider being one coil out of adjustment is small, yet the error made in b becomes comparatively large. Now, in order that we may be able to get the slider near the centre of its traverse, it would be necessary that d should be approximately equal to r, but since, in order to obtain balance in the first instance, we must have

$$E : e :: a + b + d + r : a + b,$$

or

$$\frac{E}{e} = 1 + \frac{d + r}{a + b},$$

d could not be made equal to r unless

$$\frac{E}{e} = 1 + \frac{2r}{a + b}, \text{ or, } e = \frac{E(a + b)}{a + b + 2r}.$$

Now if E and e were both fixed quantities and were not of such relative values that the above equation held good, then it would be impossible to obtain the conditions necessary for making the test favourably; the method of testing we are considering, however, would usually be employed for the purpose of measuring the electromotive force of a battery in terms of the electromotive force of one or more standard cells whose number could be varied to suit any particular requirement; in such a case it would usually be possible to give to e the value which would enable the above equation to be satisfied. Thus, for example, suppose the resistance, r, of the battery E were estimated to be about 100 ohms, and suppose the slide resistance $a + b$ consisted of 100 coils of 10 ohms each, that is, 1000 ohms in all, then we must have

$$e = \frac{E\,1000}{1000 + (2 \times 100)} = \frac{E\,10}{12};$$

that is to say, the electromotive forces of the batteries E and e should be in the proportion of 10 to 12. Now, it is evident that if E were a battery of one or two cells only then it would practically be impossible to give to e the required value, but if E consisted of a considerable number of elements, 20 or 30 for example, then there would be no difficulty in adjusting e. From these considerations it must be evident that Fahie's method, although extremely ingenious and elegant, and in some special cases very convenient, is very limited in its application.

182. With respect to the *Possible degree of accuracy attainable*, this as regards the *resistance* test is directly dependent upon the accuracy with which we can adjust the ratio of a to b, thus if $a + b$ consisted of 100 coils, then if the ratio of E to e were such that the slider when adjusted stood near the centre position of its traverse, the error caused by the slider being 1 coil out of position would be 1 in 50 in a, and 1 in 50 in b, consequently the total error would be 1 in 25, or 4 per cent. With n coils, in

fact, the *Possible degree of accuracy attainable* would be one 1 in $\frac{n}{4}$, or, $\frac{100 \times 4}{n}$ per cent.

To determine the degree of accuracy attainable in the electromotive force test, we must suppose that d is 1 unit, and b 1 coil, out of adjustment. If we call λ the error caused in $\frac{E}{e}$, then we must have

$$\frac{E}{e} + \lambda = 1 + \frac{d+1}{b - \frac{a+b}{n}}, \quad \text{or,} \quad \lambda = 1 + \frac{d+1}{b - \frac{a+b}{n}} - \frac{E}{e};$$

and since

$$\frac{E}{e} = 1 + \frac{d}{b}$$

we get

$$\lambda = 1 + \frac{d+1}{b - \frac{a+b}{n}} - 1 - \frac{d}{b} = \frac{b\,(n+d) + a\,d}{b\,[b\,(n-1) - a]}.$$

If λ' be the *percentage* of error, then we have

$$\lambda = \frac{\lambda'}{100} \text{ of } \frac{E}{e}, \quad \text{or,} \quad \lambda' = 100\,\lambda \frac{b}{b+r};$$

therefore

$$\lambda' = \frac{100\,[b\,(n+d) + a\,d]}{(b+r)\,[b\,(n-1) - a]}.$$

If the test is made under the best conditions, that is, if we have $a = b$, and $d = r$, approximately, then we get

$$\lambda = \frac{100\,[b\,(n+r) + b\,r]}{(b+r)\,[b\,(n-1) - b]} = \frac{100\,(n+2\,r)}{(b+r)\,(n-2)},$$

or since n is large, we may say

$$\lambda' = \frac{100\,(n+2\,r)}{n\,(b+r)}.$$

For example.

In determining the relative electromotive forces, E and e, of two batteries by Fahie's method, the resistance, r, of E being

approximately 100 ohms, a slide resistance having 100 coils (n) of 10 ohms each was employed. What was the *greatest possible degree* of accuracy attainable?

$$\lambda' = \frac{100\,[100 + (2 \times 100)]}{100\,(500 + 100)} = \tfrac{1}{2} \text{ per cent.}$$

To sum up, then, we have

Best Conditions for making the Test.

183. Make

$$e = \frac{E\,(a + b)}{a + b + 2\,r}$$

approximately, r being the approximate resistance of the battery E.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100\,[b\,(n + d) + a\,d]}{(b + r)\,[b\,(n - 1) - a]}.$$

If $a = b$, and $d = r$, and n is large, then

$$\text{Percentage of accuracy} = \frac{100\,(n + 2\,r)}{n\,(b + r)},$$

n in both cases being the number of coils of which the slide resistance is composed.

184. It may be as well to point out that Fahie's test cannot be made (except under very exceptional circumstances, rarely met with in practice) with a slide *wire;* for, as a rule, the latter has such an extremely low resistance that it would be impossible to obtain equilibrium in the first instance; the proportion

$$E : e :: r + d + a + b : a + b,$$

which is necessary for equilibrium, could not, in fact, be satisfied unless the resistance of the battery E and the resistance d were both extremely small; in which case, moreover, the latter would have to be adjustable to a very small fraction of an ohm.

Clark's Method.

185. This is a valuable modification of Poggendorff's method, and is shown in theory by Fig. 48. $a\,b$, which takes the place of R in Poggendorff's method (page 143), is a slide resistance;

E_3 is a third battery which is connected to a slider through a galvanometer G_3.

Now if we suppose equilibrium to be obtained in both galvanometers, we must have from [5], page 143,

$$E_1 : E_2 :: r_1 + \rho + a + b : a + b,$$

and also

$$E_1 : E_3 :: r + \rho + a + b : a;$$

from which we get

$$E_2 : E_3 :: a + b : a.$$

If then we take $a + b$ to represent the electromotive force of the standard battery E_2, a will represent the electromotive force of the battery E_3.

FIG. 48.

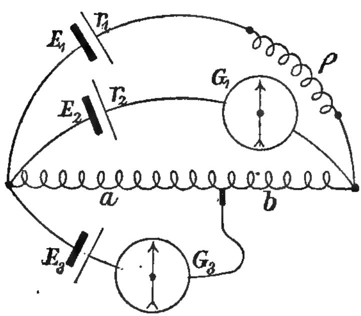
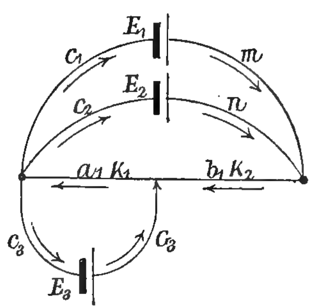


FIG. 49.



In making this test practically, the battery E_3, which would be the trial battery, being disconnected from the slide resistance, balance would be obtained with the standard battery E_2 by adjusting ρ until no deflection is observed on the galvanometer G_1. E_3 would then be connected up and the slider moved until no deflection is observed on the second galvanometer G_3.

The great advantage of Clark's method is that both the standard and the trial battery are compared under the same conditions, that is, when no current is flowing in either of them; this is a great point, as errors due to polarisation are avoided.

186. It must be evident that if equilibrium is not produced with the trial cell, then the balance in the standard cell circuit will also be disturbed; it would therefore seem to be possible to

dispense with the galvanometer G_3, but inasmuch as the current which would flow through the galvanometer G_1 would only be a fraction of that flowing out of the battery E_3, we should not be able to make a measurement with nearly such a degree of accuracy as we could if we employed the galvanometer G_3, which would be acted upon by the full force of the current.

187. To determine the best arrangement of resistances, &c., for making the test, let us suppose that there is a small error, λ, in E_3, caused by a corresponding small error in a, and let us find what effect this error has upon the current which would flow through the galvanometer G_3. Supposing then that a_1 is the new value of a which causes this error, then, keeping in mind that $a + b$ being a slide resistance is not altered by changing a, we have

$$E_3 + \lambda = \frac{E_2 a_1}{a + b}, \qquad [1]$$

or

$$a_1 = \frac{(E_3 + \lambda)(a + b)}{E_2}. \qquad [2]$$

We next have to determine what the current flowing through the galvanometer, when equilibrium is disturbed, is equal to.

Referring to Fig. 49, in which m, n, a, b, and g, represent the resistances, and c_1, c_2, c_3, κ_1, and κ_2 the current strengths in the various circuits, we have by Kirchoff's laws (page 134)

$$c_1 + c_2 + c_3 - \kappa_1 = 0$$
$$\kappa_1 - c_3 - \kappa_2 = 0$$
$$\kappa_2 - c_1 - c_2 = 0$$
$$c_1 m + \kappa_1 a_1 + \kappa_2 b_1 - E_1 = 0$$
$$c_2 n + \kappa_1 a_1 + \kappa_2 b_1 - E_2 = 0$$
$$c_3 G_3 + \kappa_1 a_1 - E_3 = 0.$$

We know also that

$$E_1 : E_2 : : m + a_1 + b_1 : a_1 + b_1,$$

and

$$a_1 + b_1 = a + b.$$

By finding then the value of c_1 from the first equation and substituting its value throughout the others, and then again

the value of c_2 from any other equation, and again substituting throughout and so on, and also substituting the value of E_1 obtained from the proportion, and again the value of $a_1 + b_1$, we shall find that

$$c_3 = \frac{E_3 - E_2 \dfrac{a_1}{a+b}}{G_3 + \dfrac{a_1\left(b_1 + \dfrac{m\,n}{m+n}\right)}{a+b+\dfrac{m\,n}{m+n}}} = \frac{E_3 - E_2 \dfrac{a_1}{a+b}}{K}. \qquad [3]$$

If in this equation we substitute the value of a_1 given by equation [2], we shall get

$$c_3 = \frac{\lambda}{G_3 + \dfrac{a_1\left(b_1 + \dfrac{m\,n}{m+n}\right)}{a+b+\dfrac{m\,n}{m+n}}};$$

or as a_1 and b_1 are very nearly equal to a and b, we may say

$$c_3 = \frac{\lambda}{G_2 + \dfrac{a\left(b + \dfrac{m\,n}{m+n}\right)}{a+b+\dfrac{m\,n}{m+n}}}. \qquad [4]$$

On examining this equation we see that to make c_3 as *large* as possible we must make $\dfrac{a\left(b + \dfrac{m\,n}{m+n}\right)}{a+b+\dfrac{m\,n}{m+n}}$ as *small* as possible, but we also see that it is no use making it much smaller than G_3, as c_3 is but very little increased by so doing.

Now the quantity $\dfrac{a\left(b + \dfrac{m\,n}{m+n}\right)}{a+b+\dfrac{m\,n}{m+n}}$ is the resistance a combined in multiple arc with the resistance b plus m and n combined in multiple arc, consequently this quantity can never be greater

than a. As long therefore as a is smaller than G_3, the highest values that can be given to the other resistances cannot make c_3 less than $\frac{\lambda}{G_3 + a}$, whilst, on the other hand, however low we make these resistances, we can never make c_3 greater than $\frac{\lambda}{G_3}$. The value therefore we give to a practically determines the sensitiveness of the system. But as a is only a portion of the slide resistance $a + b$, and as it may include the whole of the latter, as for instance when the slider is moved quite to the end of $a + b$, the sensitiveness is practically dependent upon the value given to $a + b$. This must then be made as much lower than G_3 as may be desirable.

It would not do, however, to have its resistance excessively low, for the following reason :—

In order to get equilibrium on the galvanometer G_2, it is necessary that the relation

$$E_1 : E_2 :: r_1 + \rho + a + b : a + b,$$

or

$$a + b = \frac{r_1 + \rho}{\frac{E_1}{E_2} - 1},$$

should hold good. This cannot be the case, however, if $\frac{r_1 + \rho}{\frac{E_1}{E_2} - 1}$ is greater than $a + b$; that is to say, if $a + b$ is very small $\frac{r_1 + \rho}{\frac{E_1}{E^2} - 1}$ must be very small also; but to make the latter small we must make E_1 large and $r_1 + \rho$ small, but since r_1, the resistance of E_1, will increase by increasing E_1 it may be impossible to do this. Practically we may say the resistance of $a + b$ should be a fractional value of G_3.

188. Let us now determine the *Possible degree of accuracy attainable* by the method. In equation [1] (page 160) we have supposed that an error λ has been caused in E_3 by a being out of adjustment; that is to say, from the slider being moved a little too far, so that a becomes a_1. If we call ϕ the distance the slider has been moved beyond its correct position, then we have

$$E_3 + \lambda = \frac{E_2(a + \phi)}{a + b} = \frac{E_2\, a}{a + b} + \frac{E_2\, \phi}{a + b},$$

but

$$E_3 = \frac{E_2\, a}{a + b},$$

therefore

$$\lambda = \phi \frac{E_2}{a + b},$$

that is to say, the distance the slider is out of position represents directly the error λ in E_3. The degree of accuracy therefore with which we adjust the position of the slider will be the degree of accuracy with which we can measure E_3.

We have pointed out that if $a + b$ is small, then

$$\frac{a\left(b + \frac{m\,n}{m + n}\right)}{a + b + \frac{m\,n}{m + n}}$$

will be smaller still; if therefore G_3 is large compared with $a + b$, equation [4] (page 161) becomes

$$c_3 = \frac{\lambda}{G_3}.$$

If in this equation we put the value of λ, given above, we have

$$c_3 = \frac{\phi\, E_2}{G_3\,(a + b)},$$

or

$$\frac{\phi}{a + b} = \frac{c_3\, G_3}{E_2}.$$

This equation enables us to determine what movement of the slider produces a perceptible deflection on the galvanometer. With a Thomson galvanometer of 5000 ohms resistance and figure of merit = 1,000,000 (page 47) we should have, supposing E_2 to be 1 volt,

$$\frac{\phi}{a + b} = \frac{5000}{1{,}000{,}000{,}000} = \frac{1}{200{,}000},$$

or a movement of the slider equal to $\frac{1}{200{,}000}$th of the length of $a + b$ would produce a perceptible deflection; that is to say, we could determine the accuracy of an electromotive force E_3 of about 1 volt to an accuracy of $\frac{1}{200{,}000}$th.

To obtain this accuracy, however, it would be necessary to have the wire $a + b$ graduated into 200,000 parts, each of which would be very small, unless indeed the wire were very long. If a lesser number of graduations were employed, we could practically subdivide each of them by noting what the galvanometer deflections were when the slider stood, first at one division mark, and then at the contiguous mark.

Suppose the slider stood at a distance a from the end of the slide wire, and a deflection due to a current c_1 was produced to one side of zero, and suppose that when the slider was moved 1 division forward, that is to $a + 1$, the deflection was on the other side of zero, or was produced by a current $- c_2$. Then we have from equation [3] (page 161), since a and $a + 1$ are very nearly equal,

$$c_1 = \frac{E_3 - E_2 \frac{a}{a+b}}{K}$$

and

$$- c_2 = \frac{E_3 - E_2 \frac{a+1}{a+b}}{K} = \frac{E_3 - E_2 \frac{a}{a+b} - \frac{E_2}{a+b}}{K};$$

therefore

$$c_1 E_3 - c_1 E_2 \frac{a}{a+b} - c_1 \frac{E_2}{a+b} = - c_2 E_3 + c_2 E_2 \frac{a}{a+b},$$

or

$$E_3 (c_1 + c_2) = E_2 \frac{a}{a+b} (c_1 + c_2) \frac{E_2}{a+b} c_1;$$

therefore

$$E_2 : E_3 :: a + b : a + \frac{c_1}{c_1 + c_2}.$$

The subdivision of the division beyond a is therefore given by the fraction $\frac{c_1}{c_1 + c_2}$. We have seen that we could get a deflection of 1 division on the galvanometer if the slider were moved a distance of $\frac{1}{200,000}$th beyond the distance required to give equilibrium. If the wire $a + b$ were divided into 20,000 parts, then a movement of the slider through 1 part or division would give 10 divisions of deflection on the galvanometer, each division representing a tenth of one of the wire graduations. If in making a measurement we got a deflection of 7 divisions (c_1) to the left when the slider stood at a distance

a from the end of the wire, and a deflection of 3 divisions (c_2) to the right when the slider was moved 1 wire graduation beyond a, then the position of the slider for exact equilibrium would be

$$a + \frac{7}{7+3} = a + \cdot 7.$$

The galvanometer can thus be made to act as a *vernier*, and the greater the deflection produced by a movement of the slider through one division of the graduated wire, the greater will be the accuracy with which a test can be made.

The general results that we arrive at from the foregoing investigations are as follows:—

Best Conditions for making the Test.

189. Let the slide wire $a + b$ be a fractional value of the resistance of the galvanometer G_2, but not so low that it is less than $\dfrac{r_1 + \rho}{\dfrac{E_1}{E_2} - 1}$.

The values given to the other resistances and electromotive forces do not affect the sensitiveness of the arrangement.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{c_3\, G_3\, 100}{E_3}.$$

190. Mr. Latimer Clark employs a platinum-iridium wire of 40 ohms resistance, wound spirally on an ebonite cylinder, for the slide resistance. The edge of the cylinder being divided into 1000 equal parts, and there being twenty turns to the cylinder, the whole wire is divided into 20,000 equal parts. By employing with this instrument (which combined with the batteries and resistances is called a "Potentiometer") a galvanometer with a high figure of merit, and a standard battery E_2 of one Daniell cell, a 1 division movement of the slider, after equilibrium has been produced, will produce a deflection of 50 divisions. It is possible, therefore, to measure an electromotive force of one Daniell cell to an accuracy of

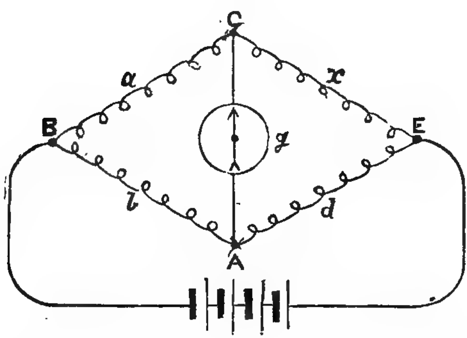
$$\frac{1}{20{,}000 \times 50} = \frac{1}{1{,}000{,}000}\text{th.}$$

CHAPTER VIII.

THE WHEATSTONE BRIDGE.

191. The theoretical arrangement of the Wheatstone Bridge, or Balance, is shown by Fig. 50. It consists of four resistances *a*, *b*, *d*, and *x*, arranged in the form of a parallelogram, a battery occupying the place of one, and a galvanometer the place of the other, diameter. When the four resistances are so adjusted

FIG. 50.



that equilibrium is produced, that is to say, when no current passes through the galvanometer, then these resistances bear a certain relation to one another. This relation may be thus determined:—

When equilibrium is produced, then since there is no tendency for a current to flow between the points A and C, the galvanometer may be removed without altering the strengths of current in the other parts of the bridge; and, further, we may join the points A and C without affecting the strengths. Let us first suppose the points A and C to be separated; then the joint resistance given by the four resistances between the points B and E will be

$$\frac{(a + x)(b + d)}{a + x + b + d}.$$

If, now, we join A and C, the resistance may be written

$$\frac{ab}{a+b}+\frac{dx}{d+x},$$

which must be equal to the former expression, that is,

$$\frac{(a+x)(b+d)}{a+x+b+d}=\frac{ab}{a+b}+\frac{dx}{d+x}.$$

By multiplying up and simplifying we get

$$a^2d^2+b^2x^2-2abdx=0;$$

therefore

$$(ad-bx)^2=0,$$

from which

$$\frac{a}{b}=\frac{x}{d}.$$

If, now, three of the quantities in this equation are known, the fourth can be determined; thus:—

$$x=\frac{a\,d}{b}.$$

In the most general form of bridge, two of the resistances are fixed, and a third is adjustable, the fourth being the resistance whose value is to be determined.

As a rule, we should make a and b the fixed resistances, x the resistance whose value it is required to find, and d the adjustable resistance.

In the simplest method of measuring we should make a and b of equal value, in which case

$$x=d;$$

that is to say, the resistance which is between A and E when equilibrium is produced, gives the value of the unknown resistance.

It is absolutely necessary that there be some resistance in a and b, for otherwise the galvanometer is short-circuited, and equilibrium, as far as the galvanometer is concerned, will be always produced, no matter what resistances we have in the other two branches.

192. Besides using equal resistances in a and b, we can make one of the two to be 10 or 100 times as great as the other, or, in

fact, any multiple of it we like, but multiples of 10 are those most commonly used. If, when we are measuring a resistance x, we make b 10 times as large as a, then every unit of resistance in d represents $\frac{1}{10}$th of a unit in x, for in this case

$$x = \frac{d}{10}.$$

We can, therefore, by this device determine the value of a resistance to an accuracy of $\frac{1}{10}$th of a unit, although d is adjustable only to units. In like manner, if we make b 100 times as large as a, then every unit of resistance in d represents $\frac{1}{100}$th of a unit in x; for in this case

$$x = \frac{d}{100},$$

and we can thus determine the value of a resistance to an accuracy of $\frac{1}{100}$th of a unit. In the first instance, however, the value of d when adjusted would be 10 times that of x; we could not, therefore, in that case measure a resistance whose value was greater than $\frac{1}{10}$th of the total resistance we could insert in d; and in the second instance d would be 100 times as great as x; we could not, therefore, in that case measure a resistance greater than $\frac{1}{100}$th of the total resistance in d. In fact, the larger we make d the closer will be the degree of accuracy with which a measurement can be made, but, at the same time, the smaller will be the resistance which can be measured, unless extra resistance coils are added in between A and E.

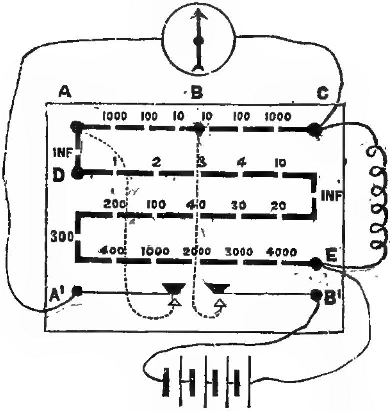
There is, however, a limit to the degree of accuracy with which a resistance can be thus measured, which is dependent upon the sensitiveness of the galvanometer; of this we shall speak hereafter.

If, now, we wish to measure a resistance which is greater than the total resistance we can insert in d, we must make a larger than b. If a be made 10 times as great as b, we can then measure any resistance which is not greater than 10 times the resistance we can insert in d; but as in this case 1 unit in d represents 10 units in x, we can only be certain of the value of x within 10 units. In like manner, if we make a 100 times as great as b, we can measure any resistance which is not greater than 100 times d, but we can only determine its value within 100 units.

193. The practical method of joining up one form of the bridge (Figs. 6 and 7, page 13) is shown by Fig. 51. When

the connections are made, and the proper plugs removed from A B (b) and B C (a), the *right*-hand key must be pressed down to put on the battery current. Plugs are now removed from E A (d) until we have inserted a resistance, as near as we can guess, equal to the resistance we are going to measure. The

Fig. 51.



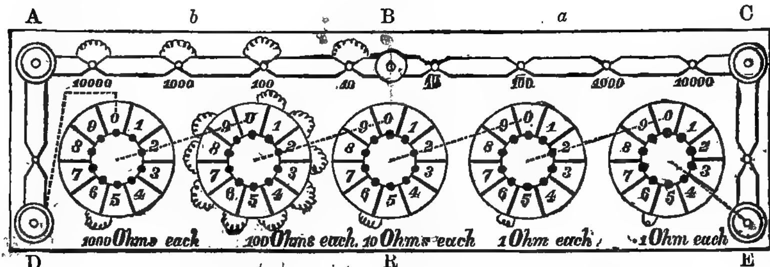
left-hand (galvanometer) key is next pressed down, and plugs removed or shifted from E A (d) until no movement of the galvanometer needle is produced upon raising and depressing the key. The connections in the case of the set of coils shown by Figs. 4 and 5 (page 12) would be similar to the foregoing, but separate keys, in circuit with the battery and galvanometer respectively, would have to be employed.

194. If the galvanometer used is a very sensitive one, with a fine filar suspension, the key must not, at first, be pressed firmly down, but only snapped down sharply; for otherwise, if equilibrium is not very nearly produced when it is depressed, there is a danger of breaking the fibre of the galvanometer needle by the violent deflection. When, however, after repeated trials, we have very nearly obtained equilibrium, then the key may be firmly depressed, and the final adjustment of plugs made.

195. Fig. 52 shows a plan of the internal connections of the set of resistance coils which were shown in general view by Fig. 8, page 14. The method of joining up these coils to form a bridge would be as follows:—The resistance to be measured is connected between C and E, the "Infinity" plug between the two being removed; the galvanometer is joined

between A and C; the battery is connected between B and E. The "Infinity" plug between A and D is inserted firmly in its place. Besides the connections referred to, it is necessary to have a key in circuit with the galvanometer, and another in circuit with the battery.

Fig. 52.



b : a : R : x

In this form of bridge, when balance is obtained, we have

$$\text{Resistance to be measured} = R\,\frac{a}{b}.$$

An advantage of the foregoing set of coils lies in the fact that there are only five plugs to be shifted, for the insertion of these plugs brings the resistances into circuit, instead of short circuiting them, as in the ordinary coils. The reading, also, of the total value of the resistance in circuit is a very easy matter, as must be obvious.

Inasmuch as the withdrawal of a plug causes a disconnection, i. e. makes $R = \infty$, great care must be taken that the galvanometer key is raised previous to shifting a plug, otherwise a violent deflection of the galvanometer needle will be produced; this fact renders the use of the "Dial" pattern objectionable in certain tests, especially in "fault" testing.

Conditions for Accurate Measurements.

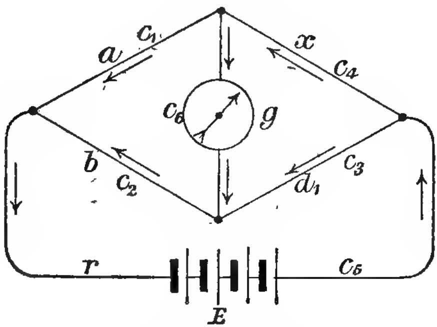
196. Besides the method of joining up, as shown by Fig. 51, we may also join up by placing the battery between A′ and C, and the galvanometer between B′ and E; this, under certain conditions, renders the action of the galvanometer more sensitive than by the common arrangement. What these conditions are, and what should be the general arrangement of the resistances in the bridge in order that a test may be made under the best possible conditions for ensuring accuracy, we will now proceed to consider.

To investigate these questions it is first of all necessary to

find what relation the current which flows through the galvanometer when equilibrium is not produced, bears to the different resistances which make up the bridge.

In Fig. 53 let a, b, d_1, x, r, and g be the resistances of the differents parts of the bridge, also let c_1, c_2, c_3, c_4, c_5, and c_6 be the current-strengths in the same, and let E be the electromotive force of the battery.

FIG. 53.



Applying Kirchoff's laws (page 134), we get the following six equations as representing the connection between the resistances, current strengths, and the electromotive force:—

$$c_5 - c_1 - c_2 = 0. \qquad [1]$$

$$c_4 - c_6 - c_1 = 0. \qquad [2]$$

$$c_3 + c_6 - c_2 = 0. \qquad [3]$$

$$c_5 r + c_3 d_1 + c_2 b - \mathrm{E} = 0. \qquad [4]$$

$$c_1 a - c_2 b - c_4 g = 0. \qquad [5]$$

$$c_3 d_1 - c_4 x - c_6 g = 0. \qquad [6]$$

From these equations we have to find the value of c_6, the current flowing through the galvanometer.

By finding the value of c_1 from equation [1] and substituting its value in equations [2] and [5] we get rid of c_1; and in like manner, by finding the value of c_2 from equation [3] and substituting throughout we get rid of c_2. By adopting the same process with respect to c_3 and c_4 we shall finally get equations [5] and [6] to become

$$c_5 a - c_6 a - c_6 b - c_6 g - (a + b)\,\frac{\mathrm{E}\, c_6 b - c_5 r}{b + d_1} = 0. \qquad [5]$$

$$c_5 x + c_6 g - (d_1 + x)\,\frac{\mathrm{E} - c_6 b - c_5 r}{b + d_1} = 0. \qquad [6]$$

From these two equations we get

$$\frac{c_5\{r(d_1+x)+x(b+d_1)\}}{c_5\{r(a+b)+a(b+d_1)\}}=\frac{-c_6(bg+d_1g+bx+bd_1)+\text{E}(d_1+x)}{c_6(ad_1+bd_1+bg+d_1g)+\text{E}(a+b)}. \quad [7]$$

from which

$$c_6=\frac{\text{E}(ad_1-bx)}{g\{(a+x)(b+d_1)+r(a+b+d_1+x)\}+r(d_1+x)(a+b)+bd_1(a+x)+ax(b+d_1)}=\frac{\text{A}}{\text{B}_1}. \quad [8]$$

This equation gives the strength of the current which would flow through the galvanometer if the resistances were arranged as shown by Fig. 53.

197. Suppose now the battery occupied the place taken by the galvanometer and *vice versâ*, or in other words, suppose the galvanometer connected the junctions of a with b, and d_1 with x, and the battery connected the junctions of a with x, and b with d_1, then the currrent (c_7) flowing through the galvanometer would be

$$c_7=\frac{\text{E}(ad_1-bx)}{g\{(d_1+x)(a+b)+r(a+b+d_1+x)\}+r(b+d_1)(a+x)+ab(d_1+x)+d_1x(a+b)}=\frac{\text{A}}{\text{B}_2}. \quad [9]$$

If we subtract equation [9] from equation [8] we get

$$c_6-c_7=\frac{\text{A}}{\text{B}_1}-\frac{\text{A}}{\text{B}_2}=\frac{\text{A}}{\text{B}_1\,\text{B}_2}(\text{B}_2-\text{B}_1),$$

and if in (B_2-B_1) we substitute the values of B_1 and B_2 given in equations [8] and [9], respectively, and then multiply up, cancel, &c., we finally get

$$c_6-c_7=\frac{\text{A}}{\text{B}_1\,\text{B}_2}(g-r)(a-d)(x-b).$$

In this equation, if g is larger than r, and both a and x are respectively larger or smaller than d_1 and b; or if r is greater than g and at the same time both a and b are greater than d_1 and x, then c_6-c_7 will be a positive quantity, that is, c_6 will be greater than c_7.

But c_6 is the current obtained by the arrangement of the bridge indicated by Fig. 53; and on examination it will be found that when the resistances have the relative magnitudes indicated, the greater of the two resistances g and r connects the junction of the two greater with the junction of the two lesser resistances; consequently, as this arrangement gives the greatest current through the galvanometer when equilibrium is not produced, it must be the best one to employ.

In practice it is almost always the case that the galvanometer has a higher resistance than the testing battery.

198. We have next to consider what should be the relative values of a, b, d, and x, in order that the bridge test may be made under the best possible conditions.

There are several different considerations involved in these questions, but we will investigate the problem from a general point of view first.

Equation [8] shows the relation between the current and the resistances. In this equation, as equilibrium is very nearly produced, we may, except where differences are concerned, put

$$a\,d_1 = a\,d = b\,x, \quad \text{or,} \quad b = \frac{a\,d}{x},$$

d being the adjusted resistance when equilibrium is exactly produced.

We then get

$$c_6 = \frac{\mathrm{E}\,x\,(a\,d_1 - b\,x)}{\{g\,(a+x) + a\,(d+x)\}\,\{r\,(d+x) + d\,(a+x)\}}. \quad [1]$$

Let us now suppose that the error in d_1, which causes the current c_6, produces an error λ in x, or that

$$x + \lambda = \frac{a\,d_1}{b}, \quad \text{or,} \quad a\,d_1 = b\,x + b\,\lambda,$$

and as

$$b = \frac{a\,d}{x},$$

therefore

$$c_6 = \frac{\mathrm{E}\,a\,d\,\lambda}{\{g\,(a+x) + a\,(d+x)\}\,\{r\,(d+x) + d\,(a+x)\}}$$

$$= \frac{\mathrm{E}\,\lambda}{\left\{d + g + x + \frac{g\,x}{a}\right\}\left\{r + a + x + \frac{r\,x}{d}\right\}} \quad [2]$$

or

$$\lambda = \frac{c_6}{\mathrm{E}}\left\{d + g + x + \frac{g\,x}{a}\right\}\left\{r + a + x + \frac{r\,x}{d}\right\} \quad [3]$$

Now our object is to make λ as small as possible, and this we shall do by making the error in d as small as possible. But the accuracy with which we can adjust d is limited by the degree of closeness with which the movement of the galvanometer needle from zero can be observed. In other words, if c_6 is the smallest current which will produce a perceptible deflection on the galvanometer, that is to say, if c_6 is the reciprocal of its "figure

of merit" (page 47), then the value of λ which corresponds to c_6 will be the amount of the error which we are likely to make in x.

If we write equation [3] in the form

$$\lambda = \frac{c_6\, r\, x \left\{d + \left(g + x + \frac{g\,x}{a}\right)\right\} \left\{\frac{1}{d} + \frac{1}{\left(\frac{r\,x}{r + x + a}\right)}\right\}}{\mathrm{E}} \qquad [4]$$

we can see that λ is smallest when the numerator of the fraction is smallest, and we must determine the values of d and a, which make this numerator as small as possible.

In order to do this let us simplify the above equation by putting

$$\left(g + x + \frac{g\,x}{a}\right) = \mathrm{X}, \text{ and, } \left(\frac{r\,x}{r + x + a}\right) = \mathrm{Y};$$

we then get

$$\lambda = \frac{c_6\, r\, x \left\{d + \mathrm{X}\right\} \left\{\frac{1}{d} + \frac{1}{\mathrm{Y}}\right\}}{\mathrm{E}},$$

or

$$\lambda = \frac{c_6\, r\, x \left\{\frac{\mathrm{Y} + \mathrm{X} + \left(d + \frac{\mathrm{XY}}{d}\right)}{\mathrm{Y}}\right\}}{\mathrm{E}}. \qquad [A]$$

From this equation we can see that to make λ as small as possible we must make

$$d + \frac{\mathrm{X\,Y}}{d}$$

as small as possible.

Now

$$d + \frac{\mathrm{X\,Y}}{d} = 2\sqrt{\mathrm{X\,Y}} + \left(\sqrt{d} - \frac{\sqrt{\mathrm{X\,Y}}}{\sqrt{d}}\right)^2,$$

and in order to make the right-hand side of the equation as small as possible, we must make

$$\sqrt{d} - \frac{\sqrt{\mathrm{X\,Y}}}{\sqrt{d}}$$

as small as possible; that is to say, we must have

$$\sqrt{d} - \frac{\sqrt{\mathrm{X\,Y}}}{\sqrt{d}} = 0, \quad \text{or,} \quad \sqrt{d} = \frac{\sqrt{\mathrm{X\,Y}}}{\sqrt{d}},$$

from which we get

$$d = \sqrt{\mathrm{X\,Y}};$$

that is to say, we must make d equal to the *geometric mean* of the quantities $\left(g + x + \frac{g\,x}{a}\right)$ and $\left(\frac{r\,x}{r + x + a}\right)$.

Now, although the value "$d = \sqrt{\mathrm{X\,Y}}$" is one which gives a minimum value to λ, yet it is not the value which makes λ an *absolute* minimum, for X and Y both contain the variable quantity a. In order, therefore, to make λ an absolute minimum, we must determine what value a should have.

If in equation [A] we put $d = \sqrt{\mathrm{X\,Y}}$, we get

$$\lambda = \frac{c_6\,r\,x\left\{\dfrac{\mathrm{Y} + \mathrm{X} + \left(\sqrt{\mathrm{X\,Y}} + \dfrac{\mathrm{X\,Y}}{\sqrt{\mathrm{X\,Y}}}\right)}{\mathrm{Y}}\right\}}{\mathrm{E}}$$

$$= \frac{c_6\,r\,x\left\{\dfrac{\mathrm{Y} + \mathrm{X} + 2\sqrt{\mathrm{X\,Y}}}{\mathrm{Y}}\right\}}{\mathrm{E}} = \frac{c_6\,r\,x\left\{1 + \sqrt{\dfrac{\mathrm{X}}{\mathrm{Y}}}\right\}^2}{\mathrm{E}}. \qquad \text{[B]}$$

In order to make λ an absolute minimum, we can see that we must make $\frac{\mathrm{X}}{\mathrm{Y}}$ a minimum. Now

$$\frac{\mathrm{X}}{\mathrm{Y}} = \frac{\left(g + x + \dfrac{g\,x}{a}\right)}{\left(\dfrac{r\,x}{r + x + a}\right)} = \left(g + x + \frac{g\,x}{a}\right)\left(\frac{r + x + a}{r\,x}\right)$$

$$= \frac{g}{r}\left\{a + (r + x)\right\}\left\{\frac{1}{a} + \frac{1}{\left(\dfrac{g\,x}{g + x}\right)}\right\},$$

consequently we can see from the reasoning in the previous investigation, that to make $\frac{\mathrm{X}}{\mathrm{Y}}$ a minimum we must make

$$a = \sqrt{(r + x)\frac{g\,x}{g + x}}. \qquad \text{[C]}$$

Having now obtained the required value of a in terms of the known quantities, r, g, and x, we can also determine the value of d in terms of r, g, and x; for we have

$$d = \sqrt{\mathrm{X\,Y}} = \sqrt{\left(g + x + \frac{g\,x}{a}\right)\left(\frac{r\,x}{r + x + a}\right)}$$

$$= \sqrt{\left\{g + x + \frac{g\,x}{\sqrt{\frac{(r+x)\,g\,x}{g+x}}}\right\}\left\{\frac{r\,x}{r + x + \sqrt{\frac{(r+x)\,g\,x}{g+x}}}\right\}}$$

$$= \sqrt{\left\{\frac{(g+x)\sqrt{r+x} + \sqrt{(g+x)g\,x}}{\sqrt{r+x}}\right\}\left\{\frac{r\,x\sqrt{g+x}}{(r+x)\sqrt{g+x} + \sqrt{(r+x)gx}}\right\}}$$

$$= \sqrt{\frac{g+x}{r+x}\left\{\sqrt{g+x}\sqrt{r+x} + \sqrt{g\,x}\right\}\left\{\frac{r\,x}{\sqrt{r+x}\sqrt{g+x} + \sqrt{g\,x}}\right\}}$$

or

$$d = \sqrt{(g + x)\frac{r\,x}{r + x}} \qquad \text{[D]}$$

199. Although equations [C] and [D] show the values of a and d which are necessary for making the error λ an *absolute* minimum, yet practically we may make both a and d to vary considerably from these exact values without increasing λ to any great extent.

As regards d it is preferable to make it as high as possible, so that its range of adjustment may be as high as possible. Referring to equation [4] (page 174), we have proved that for an absolute minimum we must make d equal to the geometric mean of the quantities $\left(g + x + \frac{g\,x}{a}\right)$ and $\left(\frac{r\,x}{r + x + a}\right)$, consequently we can see that in this case d must be less than $\left(g + x + \frac{g\,x}{a}\right)$. If we suppose the value of d for a minimum to be very small compared with $\left(g + x + \frac{g\,x}{a}\right)$, then we can see that even if we increase d up to an equality with $\left(g + x + \frac{g\,x}{a}\right)$, we cannot increase λ beyond twice its minimum value, especially if we consider that by increasing d we diminish the value of

$\left\{\frac{1}{d}+\frac{1}{\left(\frac{r\,x}{r+x+a}\right)}\right\}$. If we only increase d up to $g+x$, then λ will of course be increased still less. Should the value of d for a minimum happen to be only a little less than $\left(g+x+\frac{g\,x}{a}\right)$, then of course the increase of d referred to will have but little effect on λ. In any case, however, by keeping d below $g+x$, the increase in λ must be less, and may be considerably less, than $2\,\lambda$. The importance of this fact may be seen if we suppose g, x, and r to have the following values :—

$$g = 4899, \quad x = 1, \quad r = 100,$$

then for a minimum we must make

$$d = \sqrt{(4899+1)\frac{100 \times 1}{100+1}} = 70\,;$$

but we have proved that if we may make "$d = (4899 + 1) = 5000$," then by so doing we cannot possibly increase λ to more than $2\,\lambda$, and actually the increase must be to less than $2\,\lambda$.

We next have to consider to what extent we may vary a. To do this let us write equation [3] in the form :—

$$\lambda = \frac{c_6\,g\,x\left\{a+\left(r+x+\frac{r\,x}{d}\right)\right\}\left\{\frac{1}{a}+\frac{1}{\left(\frac{g\,x}{g+x+d}\right)}\right\}}{\mathrm{E}}\,.$$

From this equation we can see that if d has the value necessary to make λ a minimum, then as long as we do not make a less than $\frac{g\,x}{g+x}$ we cannot possibly increase λ to more than $2\,\lambda$. But then the question arises—Suppose we have already increased λ by making d as great as $g+x$, under these conditions what will be the effect of also decreasing a to $\frac{g\,x}{g+x}$?

If we refer to the last equation, we can see from the investigation made in the case of equation [4] (page 174), that the value of a which makes λ a minimum must be

$$a = \sqrt{\left(r+x\frac{r\,x}{d}\right)\left(\frac{g\,x}{g+x+d}\right)},$$

and this value is one which makes λ *a* minimum whatever be the value of *d*, though to make λ an *absolute* minimum we must also have

$$d = \sqrt{(g + x)\frac{r\,x}{r + x}}.$$

Now, if we increase *d*, we can see that to make λ *a* minimum we shall have to decrease the value of *a*, for by increasing *d* we decrease both $\left(r + x + \frac{r\,x}{d}\right)$ and $\left(\frac{g\,x}{g + x + d}\right)$; consequently a decrease in *a* after *d* has been increased will tend to decrease again the increased value of λ. We cannot, however, bring back λ to its original absolute minimum, although we may bring it near to it; for after a certain point the decrease in the value of *a* causes λ to increase again; as long, however, as we avoid making *a* less than $\frac{g\,x}{g + x}$ this increase cannot be great.

As the value which *d* has must depend upon the value given to *b*, therefore after we have determined what values to give to *a* and *d*, we must ascertain the value of *b* from the equation "$b = \frac{a\,d}{x}$."

For example.

It being required to measure exactly a resistance *x* whose value was found by a rough test to be about 500 ohms, a ten-cell Daniell battery (E = 10·7) whose resistance was 200 ohms (r) was used for the purpose, and also a galvanometer whose resistance was 5000 ohms (g) and figure of merit 1,000,000,000 $\left(\frac{1}{c_6}\right)$. What resistances should be given to the arms *a* and *b* of the bridge in order that the test may be made under the most favourable conditions, also what percentage of accuracy would be obtainable under these conditions?

$$x = 500$$
$$g = 5000$$
$$r = 200;$$

therefore

$$a = \sqrt{(200 + 500)\frac{5000 \times 500}{5000 + 500}} = 560 \text{ ohms},$$

$$d = \sqrt{(5000 + 500)\frac{200 \times 500}{200 + 500}} = 890 \text{ ohms};$$

also we must have

$$b = \frac{560 \times 890}{500} = 1000 \text{ ohms.}$$

In practice we could make d as high as 5500 ohms $(g + x)$, and a as low as 450 ohms $\left(\frac{g\,x}{g + x}\right)$, without seriously increasing λ.

Supposing, however, we actually gave a and d their best values, then by equation [3] (page 173) we should have

$$\frac{\left\{890+5000+500+\frac{5000\times 500}{560}\right\}\left\{200+560+500+\frac{500\times 200}{890}\right\}}{1{,}000{,}000{,}000 \times 10 \cdot 7} = \cdot 0014\,;$$

that is to say, we may be $\cdot 0014$ units out when we measure x exactly; this is equivalent to an error of $\frac{\cdot 0014 \times 100}{500} = \cdot 0003$ per cent. approximately.

In order to make the test as accurately as this, it would be necessary that d be adjustable to a small fraction of a unit; if we call ϕ the value of the latter, then we should have

$$x + \lambda = \frac{a\,(d + \phi)}{b} = \frac{a\,d}{b} + \frac{a\,\phi}{b}$$

and

$$x = \frac{a\,d}{b}\,;$$

therefore

$$\lambda = \frac{a\,\phi}{b}, \quad \text{or,} \quad \phi = \frac{b\,\lambda}{a}.$$

We therefore have

$$\phi = \frac{1000 \times \cdot 0014}{500} = \cdot 003,$$

showing that d ought to be adjustable to $\cdot 003$ of an ohm or less. If we make it adjustable to $\cdot 001$ or $\frac{1}{1000}$th of an ohm therefore, we shall be able to make the test properly.

200. The facts we have arrived at by the foregoing investigation are these, that with $a = 560$ ohms and $b = 1000$ ohms, then when equilibrium is exactly produced, an alteration in the value of d equal to $\cdot 003$ of an ohm (which quantity would mean an error, λ, of $\cdot 0014$ units, or $\cdot 0003$ per cent. approximately, in x)

would produce a perceptible deflection (1 division) on the galvanometer.

We have, then,

Best Conditions for making the Test.

201. First make a rough test to ascertain approximately the value of x.

Make d not greater than $g + x$, or less than $\sqrt{(g+x)\frac{r\,x}{r+x}}$, and preferably make it as near to the latter quantity as possible, provided the range of adjustment of d is not reduced to too great an extent by so doing.

Make a not less than $\frac{g\,x}{g+x}$ and not greater than $\sqrt{(r+x)\frac{g\,x}{g+x}}$, and preferably make it as near to the latter quantity as possible in the case where d is made nearly equal to $\sqrt{(g+x)\frac{r\,x}{r+x}}$; but if d is made more nearly equal to $g + x$, then a should preferably be made more nearly equal to $\frac{g\,x}{g+x}$.

It is clearly advantageous that E should be as large and r as small as possible.

Possible Degree of Accuracy attainable.

Percentage of accuracy $= \frac{\lambda\,100}{x}$, where

$$\lambda = \frac{C_6}{E}\left\{d + g + x + \frac{g\,x}{a}\right\}\left\{r + a + x + \frac{r\,x}{d}\right\},$$

c_6 being the reciprocal of the figure of merit of the galvanometer.

In order to obtain this percentage of accuracy, d must be adjustable to not less than $\frac{d\,\lambda}{x}$ units, or $\frac{1}{\left(\frac{x}{d\,\lambda}\right)}$th of a unit.

202. In the foregoing investigation we have considered the exact conditions required for a maximum degree of accuracy, and we have seen that in order to attain this it is necessary that d be adjustable to a fraction of a unit. At the commencement of the chapter (§ 192, page 167), however, we saw that if d is only

adjustable to units, then in order to obtain the greatest possible accuracy we should make d as much larger than x as possible, as by so doing we get a great range of adjustment. But, as we also stated, there is a limit to thus increasing d, for unless we are able to adjust d accurately, we can gain nothing by having the range of adjustment so large. Now to adjust d we note the deflection of the galvanometer needle, and when this becomes 0 we know that d is adjusted exactly right; but if an alteration of several units produces no perceptible effect on the deflection we may just as well have d of a smaller value. Thus, supposing we have b 10 times as great as a, that is d 10 times x; then if an alteration of 10 units in d only just affects the galvanometer needle, it is evident that we cannot adjust d to a closer accuracy than 10 units, and consequently we cannot obtain the value of x to a closer accuracy than 1 unit. If we have b equal to a, that is, d equal to x, then if we can adjust d within 1 unit, we shall in this case obtain the value of x to an accuracy of 1 unit, that is, with just as much accuracy as we could in the first case, when d was 10 times x. It is even possible that we could obtain the value of x more accurately in the latter case, for it may be that an alteration of 1 unit in d when b equals a may produce a much greater movement of the galvanometer needle than does the alteration of 10 ohms when b is 10 times a. Whether this is so or not is a point we have to determine.

We have also to find what should be the absolute values of a and b.

We have seen that in order to obtain accuracy it is necessary to make d as high as possible, but the highest useful value we could give to d would be *that which produces the smallest perceptible deflection when it is* 1 *unit out of adjustment.*

Now if λ be the error in x caused by d being 1 unit out of adjustment, we must have

$$x + \lambda = \frac{a\,(d+1)}{b} = \frac{a\,d}{b} + \frac{a}{b}; \qquad [A]$$

and since

$$x = \frac{a\,d}{b}, \text{ or, } \frac{a}{b} = \frac{x}{d},$$

therefore

$$\frac{a\,d}{b} + \lambda = \frac{a\,d}{b} + \frac{x}{d},$$

or

$$\lambda = \frac{x}{d}.$$

We have, then, from equation [2] (page 173)

$$c_6 = \frac{E\,x}{d\left\{d + g + x + \frac{g\,x}{a}\right\}\left\{r + a + x + \frac{r\,x}{d}\right\}}. \qquad [B]$$

From this equation we have to determine the highest value we can give to d; this will be limited by the "figure of merit" of the galvanometer, and also by the value of a. Let us write the above equation in the form

$$d\left\{a + r + x + \frac{r\,x}{d}\right\}\left\{\frac{1}{a} + \frac{1}{\frac{g\,x}{g + x + d}}\right\} = \frac{E}{g\,c_6}.$$

Now since $\frac{E}{g\,c_6}$ is a fixed quantity, therefore in order that d may have as large a value as possible we must give a such a value that

$$\left\{a + r + x + \frac{r\,x}{d}\right\}\left\{\frac{1}{a} + \frac{1}{\frac{g\,x}{g + x + d}}\right\}$$

is as small as possible. From the investigation given in (§ 198, page 173) we can see that if we make a as low as possible, but not lower than, say, $\frac{g\,x}{g + x}$, then

$$\left\{a + r + x + \frac{r\,x}{d}\right\}\left\{\frac{1}{a} + \frac{1}{\frac{g\,x}{g + x + d}}\right\}$$

will be very close to its minimum value, no matter how high d may be.

For the purpose of determining the actual numerical value which d can have, let us write equation [B] in the form

$$\left\{d + g + x + \frac{g\,x}{a}\right\}\left\{d + \frac{r\,x}{r + x + a}\right\} = \frac{E\,x}{c_6(r + x + a)};$$

this equation, being an ordinary quadratic * (see note on next page), would enable the value of d to be obtained in terms of the other quantities in the usual manner, but inasmuch as we only require to determine the value of d within, say, 10 per cent., it is a much simpler and shorter operation to adopt the "trial" method; that is to say, to give d different values until we arrive at one which approximately satisfies the equation.

For example.

Suppose, as in the last example,

$$E = 10,$$
$$x = 500 \text{ (from a rough test)},$$
$$g = 5000,$$
$$r = 200,$$
$$c_6 = \frac{1}{1{,}000{,}000{,}000},$$

then make $a = 500$.

We then get

$$\left\{ d + 5000 + 500 + \frac{5000 \times 500}{500} \right\} \left\{ d + \frac{200 \times 500}{200 + 500 + 500} \right\} = \frac{10 \times 500 \times 1{,}000{,}000{,}000}{200 + 500 + 500},$$

or,

$$\{d + 10{,}500\} \{d + 83 \cdot 3\} = 4{,}170{,}000{,}000.$$

If we make $d = 60{,}000$ we shall very nearly satisfy the equation, for

$$\{60{,}000 + 10{,}500\} \{60{,}000 + 83 \cdot 3\} = 4{,}236{,}000{,}000.$$

As the value which d will have will depend upon the value given to b, the latter must be amde equal to

$$b = \frac{500 \times 60{,}000}{500} = 60{,}000.$$

---

* The solution of the quadratic equation is as follows:—
Let

$$g + x + \frac{g\,x}{a} = \text{A},$$
$$\frac{r\,x}{r + x + a} = \text{B},$$
$$\frac{\text{E}\,x}{c_6 (r + x + a)} = \text{K},$$

then we get

$$\{ d + \text{A} \} \{ d + \text{B} \} = \text{K},$$

therefore

$$d^2 + d(\text{A} + \text{B}) = \text{K} - \text{A}\,\text{B},$$

or

$$d^2 + d(\text{A} + \text{B}) + \left(\frac{\text{A} + \text{B}}{2}\right)^2 = \text{K} - \text{AB} + \frac{\text{A}^2 + 2\text{AB} + \text{B}^2}{4} = \frac{4\text{K} + (\text{A} - \text{B})^2}{4},$$

therefore

$$d = \frac{\sqrt{4\text{K} + (\text{A} - \text{B})^2}}{2} - \frac{\text{A} + \text{B}}{2} = \frac{\sqrt{4\text{K} + (\text{A} - \text{B})^2} - (\text{A} + \text{B})}{2}.$$

As regards the *Possible degree of accuracy* with which the test can be made, we have seen on page 181 that

$$\lambda = \frac{x}{d}.$$

We therefore have

$$\lambda = \frac{500}{60{,}000} = 0083,$$

which equals

$$\frac{\cdot 0083 \times 100}{500} = \cdot 0017 \text{ per cent.};$$

this compares unfavourably with the result obtained when the test was made with d of a low value and adjustable to $\frac{1}{100}$th of a unit, the percentage of accuracy in the latter case being ·0003 per cent. To summarise the results of the investigation, we have

Best Conditions for making the Test.

203. Make a as low as possible, but not lower than $\frac{g\,x}{g + x}$.

Make d as high as possible, but not so high that

$$\left\{d + g + x + \frac{g\,x}{a}\right\}\left\{d + \frac{r\,x}{r + x + a}\right\}$$

is greater than

$$\frac{\mathrm{E}\,x}{c_6\,(r + x + a)},$$

c_6 being the reciprocal of the figure of merit of the galvanometer.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{\frac{x}{d} \times 100}{x} = \frac{100}{d}.$$

If we make d adjustable to any particular fraction of a unit, we can tell the degree of accuracy with which x could be measured, for if in equation [A] (page 181) we put $\frac{1}{n}$ instead of 1, we get

$$\lambda = \frac{x}{n\,d};$$

and equation [B] (page 182) becomes

$$\left\{d + g + x + \frac{g\,x}{a}\right\}\left\{d + \frac{r\,x}{r + x + a}\right\} = \frac{1}{n} \cdot \frac{\mathrm{E}\,x}{c_6\,(r + x + a)}. \quad \text{[B]}$$

If in this last equation we give to $\frac{1}{n}$ the fractional value to which d is adjustable, we determine the degree of accuracy with which we can make the test.

For example.

Suppose d was adjustable to $\frac{1}{10}$th of a unit $\left(\frac{1}{n}\right)$, then we have (giving to x, a, g, and r the values used in the previous examples)

$$\{d + 10{,}500\}\ \{d + 83 \cdot 3\} = 417{,}000{,}000.$$

If we make $d = 16{,}000$, we shall very nearly satisfy the equation, and the percentage of accuracy, λ', with which x would be measured would be

$$\lambda' = \frac{\frac{x}{n\,d} \times 100}{x} = \frac{100}{n\,d} = \frac{100}{10 \times 16{,}000} = \cdot 00062 \text{ per cent.}$$

204. At the commencement of the chapter (§ 192, page 167), we saw that by making b 10 or 100 times as great as a, and consequently d 10 or 100 times as great as x, we were enabled to measure x to an accuracy of $\frac{1}{10}$th or $\frac{1}{100}$th of a unit, although d was adjustable to units only. Every unit in d, in fact, represented $\frac{1}{10}$th or $\frac{1}{100}$th of a unit in x. But to measure to an accuracy of $\frac{1}{100}$th of a unit, with the forms of bridge shown in Chapter II., page 10, the resistances in a and b have to be 10 and 1000 respectively, we have no other choice of resistances.

In the investigation we have made, we have seen that a should be not less than $\frac{g\,x}{g + x}$, but in the bridge as usually arranged, if we wished to have a and b in the proportion of 1 to 100, so that we could measure to the accuracy of $\frac{1}{100}$th of a unit, we might find that we should have to very considerably transgress the rule of not making a smaller than $\frac{g\,x}{g + x}$, unless, indeed, x were a low resistance; for inasmuch as we could adjust the resistances in the bridge so as to theoretically measure a

resistance of 100 ohms to an accuracy of $\frac{1}{100}$th of a unit, if the resistance were as high, or nearly as high, as 100, it might be 10 times, or nearly 10 times, as high as we could make a. Under these conditions, then, the bridge is not in a favourable condition for ensuring an accurate test.

We say it is not in a favourable condition for ensuring accuracy, but it does not follow therefore that we *cannot* measure a resistance of 100 ohms accurately to an accuracy of $\frac{1}{100}$th of a unit with such an arrangement. A galvanometer if it has a high figure of merit may, although the conditions are unfavourable, still give a sufficient deflection to enable us to exactly adjust.

What, then, it may be asked, is the practical value of the results we have theoretically arrived at? The value is this: if we find we have *not* got sufficient sensitiveness to obtain a good test, then we can see what may be the cause of it, and therefore how we can remedy it. The results further show that the values given to a and b in the bridges as ordinarily arranged are such that only certain resistances can be measured under the best conditions for ensuring accuracy.

205. It should not be overlooked that the conditions for obtaining a good test are, to a very great extent, dependent upon the resistance of the galvanometer used, since the value which a must have is dependent upon both g and x. But it must not therefore be imagined that we can make these conditions anything we please by employing a galvanometer of a low resistance, for such galvanometers have a low figure of merit, and consequently what is gained in one direction by having g low, is more than counterbalanced by having the figure of merit low. It must be evident, then, that the whole question of the accuracy with which a bridge test can be made is dependent, in the first instance, upon both the resistance and figure of merit of the galvanometer, and, as we shall see in certain cases, it is absolutely necessary that the resistance be very low, although the figure of merit has consequently to be low also.

Measurement of a Resistance when exact Equilibrium cannot be obtained.

206. It very often happens, especially when measuring small resistances, that exact equilibrium cannot be obtained in the bridge; thus one unit too much in d may give a deflection to one side of zero, and one unit too little, a deflection to the other side of zero, and as no nearer adjustment can be made, the exact value of x is not directly determinable. If, however, the values

of the deflections be noted, the true value of x can be obtained very closely.

On page 172 we have an equation [8] which gives the value of the current (c_6) passing through the galvanometer when equilibrium is not produced.

Let, then, c' be the current which produces, say, a left-hand deflection of the galvanometer needle, and let this current be caused by d being too small; also let c'' be the current which produces a right-hand deflection, and let this current be caused by d being too large. Then if d' and d'' be the smaller and larger resistances respectively, we have two equations, viz.,

$$c' = \frac{ad' - bx}{B'}, \quad \text{and,} \quad -c'' = \frac{ad'' - bx}{B''}$$

where B' and B'' are quantities corresponding to B_1 in equation [8].

Now, since d' and d'' are very nearly equal, B' and B'' may be taken as being equal without sensibly altering the relative values of c' and c''; therefore, we may say

$$-\frac{c'}{c''} = \frac{ad' - bx}{ad'' - bx},$$

that is,

$$c'ad'' - c'bx = c'bx - c''ad',$$

or

$$x = \frac{a\,(c'\,d'' + c''d')}{b\,(c' + c'')}.$$

But as d'' would be only 1 unit larger than d', that is, as

$$d'' = d' + 1,$$

therefore

$$x = \frac{a\,(c'd' + c' + c''d')}{b\,(c' + c'')} = \frac{a\,(d'\,(c' + c'') + c')}{b\,(c' + c'')} = \frac{a}{b}\left(d' + \frac{c'}{c' + c''}\right).$$

For example.

a and b being 10 and 100 ohms respectively, when d' was adjusted to 156 ohms a deflection of 15 divisions (c') was obtained to one side of zero, and when d' was increased to 157 ohms, a deflection of 20 divisions (c'') to the other side of zero, was observed. What was the exact value of x?

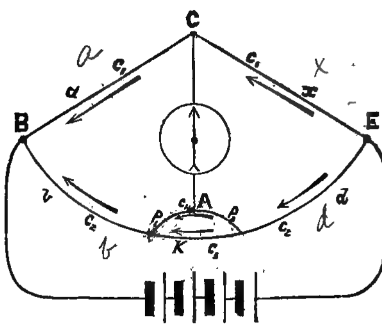
$$x = \frac{10}{1000}\left(156 + \frac{15}{15 + 20}\right) = 1{\cdot}5643 \text{ ohms}.$$

Slide Resistance Coils Bridge.

207. Instead of fixing a and b and varying d, we may make a a fixed resistance, and $b + d$ a slide resistance, and vary the ratio of b to d. Either a slide wire or a set of slide resistance coils, such as that indicated by Fig. 9 (page 14) may be used. The former would be employed if $b + d$ is required to be a low resistance, the latter if a high resistance is necessary.

A set of coils allows of but few different ratios being given to b and d, unless indeed the number of coils is very large, which would be both a cumbersome and an expensive arrangement. Mr. Varley, by means of a movable derived circuit, reaching across two of the coils, has devised a means of subdividing each of the latter. This arrangement is shown by means of Figs. 54 and 55. Referring to Fig. 54, let us suppose that

Fig. 54.



equilibrium is produced so that no current circulates through the galvanometer. This being the case, the points C and A may be joined without altering the current strengths in the various circuits. Let us suppose this junction to be effected; then, by applying Kirchoff's laws (page 134), we have the following relations existing between the current strengths and the resistances in the system:—

$$c_2 - c_3 - c_4 = 0.$$

$$c_1\, a_1 - c_2\, b - c_4\, \rho_1 = 0.$$

$$c_1\, x - c_2\, d - c_4\, \rho_2 = 0.$$

$$c_4\, \rho_1 + c_4\, \rho_2 - c_3\, \kappa = 0.$$

By substitution we get

$$c_1 a = c_4 \left(\frac{\rho_1 + \rho_2}{\kappa} b + b + \rho_1 \right),$$

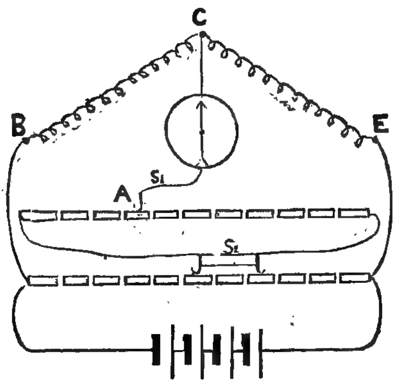
$$c_1 x = c_4 \left(\frac{\rho_1 + \rho_2}{\kappa} d + d + \rho_2 \right).$$

Now if in these equations we make $\kappa = \rho_1 + \rho_2$, and divide the one equation by the other, we get

$$\frac{a}{x} = \frac{b + \frac{\rho_1}{2}}{d + \frac{\rho_2}{2}}.$$

This equation shows that if the slide resistance $\rho_1 + \rho_2$ be made equal to the portion κ of the slide resistance $b + \kappa + d$ which it encloses, then the values of the resistances between the

FIG. 55.



points B A and E A will be to one another, as the resistance b, plus half the resistance ρ_1, is to the resistance d, plus half the resistance ρ_2.

If, therefore, we have $b + \kappa + d$ formed of 101 coils of, say, 1000 ohms each, and $\rho_1 + \rho_2$ of 100 coils of 20 ohms each, that is, 2000 ohms ($\rho_1 + \rho_2$) in all, and further, if the slider S_2 (Fig. 55) bridges across two of the 1000-ohm coils so as to enclose a resistance of 2000 ohms (κ), then a movement of slider S_1 from one contact to the next represents an alteration of 10 ohms in the

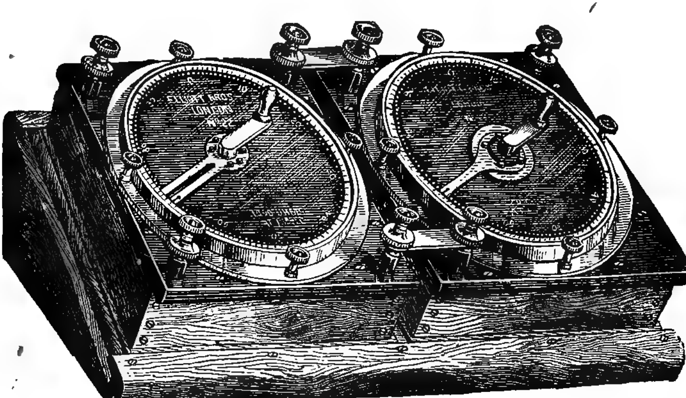
ratio of B A to E A, whilst a similar movement of the slider S_2 represents an alteration of 1000 ohms. We can thus, by means of the 201 coils, 101 of 1000 ohms each and 100 of 20 ohms each, obtain 10,000 ratios of B A and E A, each differing from the next by 10 ohms.

We could, if required, have a second slider like S_2 to move along $\rho_1 + \rho_2$, and connected to a third set of coils along which the slider S_1 would move; by this means the differences of 10 ohms could be subdivided into differences of $\frac{1}{10}$th of an ohm. In fact, we could have any number of sets of coils with sliders, each carrying out the subdivision to any required degree.

When we come to make very small subdivisions, such, for instance, as subdividing $\frac{1}{10}$th of an ohm into 100 parts of $\frac{1}{1000}$th of an ohm each, it would be inconvenient to employ a set of small resistances, as they are difficult to adjust exactly; slide wires may therefore be employed with advantage for the purpose.

208. Fig. 56 shows a convenient arrangement of the Slide Resistance Coils Bridge; the coils in this case are arranged in

Fig. 56.

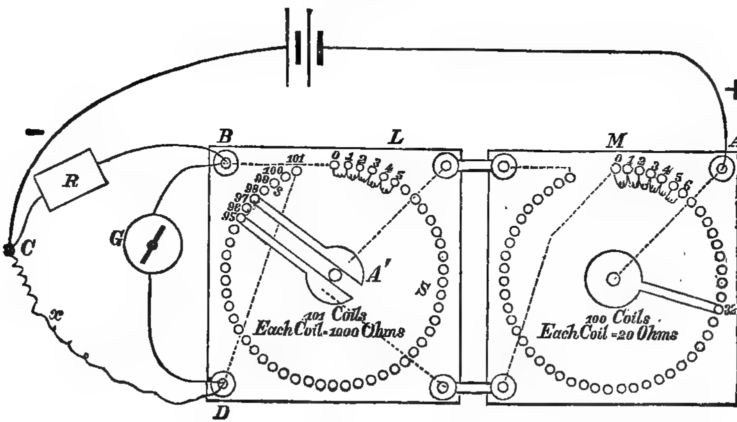


a circle instead of in a straight line as represented by the theoretical diagram Fig. 55. The left-hand dial contains the contacts and double slider for the 1000-ohm coils, and the right-hand dial the contacts and single slider for the 20-ohm coils.

Fig. 57 shows a theoretical arrangement of the foregoing Slide Resistance Coils Bridge; the connections in this diagram differ

from those shown in Fig. 55 in so far that the relative positions of the battery and galvanometer are reversed, but this reversal is not essential for the principle, as either arrangement can be employed.

FIG. 57.



SLIDE WIRE OR METRE BRIDGE.

209. The simple slide wire bridge is a very useful arrangement, as a very close adjustment can be made by means of it, and great accuracy of measurement thereby be obtained. It is especially useful for measuring small resistances accurately.

The form in which this description of bridge is usually constructed is shown by Fig. 58.

The slide wire, which is 1 metre long and about 1·5 mm. in diameter, is stretched upon an oblong board (forming the base of the instrument) parallel to a metre scale divided throughout its whole length into millimetres, and so placed that its two ends are as nearly as possible opposite to the divisions 0 and 1000 respectively of the scale.

The ends of the wire are soldered to a broad, thick copper band, which passes round each end of the graduated scale, and runs parallel to it on the side opposite to the wire.

This band is interrupted by four gaps, at m_1, a, x, and m_2. On each side of these gaps, and also at B, C, and E, are terminals.

In the ordinary use of the apparatus, the wires from the battery are attached to the terminals B and E, and the galvano-

meter is connected between C and the slider A; by pressing down a knob this latter is put in contact with the wire.

The conductor whose resistance has to be measured, and a standard resistance, are placed in the gaps at x and a respectively.

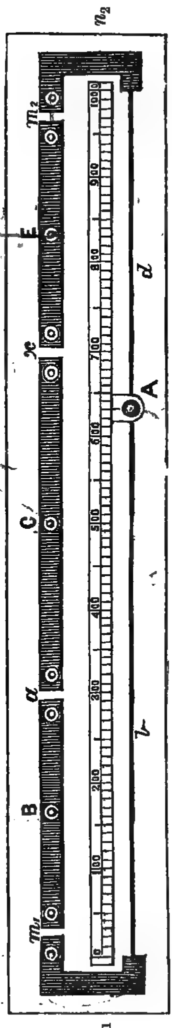


FIG. 58.

The two gaps at m_1 and m_2 can either be bridged across by thick copper straps, or resistances of known values can be inserted in them; it is easy to see that these are simply ungraduated prolongations of the slide wire.

210. If we have no resistance in these gaps, then when we have equilibrium,

$$\frac{x}{d} = \frac{d}{b} \text{ a}, \quad \text{or,} \quad x = a\frac{d}{b};$$

as $\frac{d}{b}$ is merely a ratio, we do not require to know the absolute values of d and b, but only their relative values, that is to say, we only require to know the *lengths* of the portions on either side of the slider A, and not the *resistances* of those portions.

As the length k of the slide wire is constant, that is, since

$$b + d = k, \quad \text{or,} \quad d = k - b,$$

therefore

$$x = a\,\frac{k - b}{b} = a\left(\frac{k}{b} - 1\right);$$

but k = 1000 millimetres, and b is usually called the *scale reading*, therefore we have

$$x = \left(\frac{1000}{\text{scale reading}} - 1\right). \qquad [A]$$

For example.

The standard resistance a being 1 ohm, equilibrium was obtained when the scale reading was 510; what was the value of the unknown resistance x?

$$x = 1\left(\frac{1000}{510} - 1\right) = \cdot 961 \text{ ohms.}$$

211. It has been pointed out by Mr. Martin F. Roberts that equation [A] is the same as

$$x = a\left(\left\{\begin{matrix}\text{reciprocal of}\\ \text{scale reading}\end{matrix}\right\} \times 1000 - 1\right),$$

and that consequently, by the use of a table of reciprocals, calculations can be considerably simplified in working out the value of x.

212. Equation [A] is only true if the resistances between the ends of the slide wire and the terminals B and E are zero. But, although it may not appear so, it is by no means easy to make these resistances inappreciable; even the careful soldering of the ends of the wire to the copper straps introduces a resistance which is sufficient to affect very accurate tests. Referring to Fig. 53, in which n_1 and n_2 are these resistances, we know that strictly speaking

$$\frac{x}{a} = \frac{d + n_2}{b + n_1};$$

or that

$$x = a\left(\frac{1000 + n_1 + n_2}{\text{scale reading} + n_1} - 1\right).$$

To make a strictly accurate test, then, we must know the values of n_1 and n_2 in terms of the equivalent length of the slide wire. These may be obtained in the following manner:—

Having bridged across the gaps at m_1 and m_2 with thick copper straps, taking care that the surfaces in contact are scraped bright, insert known resistances at a and x, a being rather larger than x; then, having obtained equilibrium, we have

$$a\,(d + n_2) = x\,(b + n_1);$$

now reverse a and x, and again obtain equilibrium. Let the new scale readings be b_1 and d_1; we then have

$$x\,(d_1 + n_2) = a\,(b_1 + n_1).$$

By multiplying up and arranging the quantities, we have

$$a\,n_2 = x\,b + x\,n_1 - a\,d$$

and

$$x\,n_2 = a\,b_1 + a\,n_1 - x\,d_1;$$

therefore

$$\frac{a}{x} = \frac{x\,b + x\,n_1 - a\,d}{a\,b_1 + a\,n_1 - x\,d_1};$$

that is,

$$a^2 n_1 - x^2 n_1 = x^2 b - a x d - a^2 b_1 + a x d_1,$$

therefore

$$n_1 = \frac{a x (d_1 - d) + x^2 b - a^2 b_1}{a^2 - x^2}.$$

In a similar manner we should find

$$n_2 = \frac{a x (b - b_1) + x^2 d_1 - a^2 d}{a^2 - x^2}.$$

Or since

$$b + d = b_1 + d_1 = 1000,$$

that is,

$$d = 1000 - b, \text{ and } d_1 = 1000 - b_1,$$

we have

$$n_1 = \frac{a x (b - b_1) + x^2 b - a^2 b_1}{a^2 - x^2} = \frac{b x - b_1 a}{a - x}$$

and

$$n_2 = \frac{(1000 - b_1) x - (1000 - b) a}{a - x}.$$

For example.

In order to determine n_1 and n_2, resistances were inserted at a and x equal to 3 and 2 ohms respectively. Balance was obtained when the scale reading b was 603. On reversing a and x, balance was obtained when the scale reading b_1 was 399. What were the values of n_1 and n_2?

$$n_1 = \frac{(603 \times 2) - (399 \times 3)}{3 - 2} = 8 \text{ mm.}$$

$$n_2 = \frac{(1000 - 399) 2 - (1000 - 603) 3}{3 - 2} = 11 \text{ mm.}$$

The value of x, then, would be given by the equation

$$x = a\left(\frac{1000 + 9 + 11}{\text{scale reading} + 9} - 1\right) = a\left(\frac{1020}{\text{scale reading} + 9} - 1\right).$$

213. Although perfectly satisfactory results may be obtained with the metre bridge when the latter is properly made, and when the measurements are carefully carried out, yet considerable trouble is often occasioned to inexperienced persons by results being obtained which are obviously erroneous. One most frequent cause of error is that occasioned by imperfect con-

tacts; great care should therefore be taken that the important connections, viz. those at the gaps, should be well made; this should be ensured by having the various surfaces in contact made clean and bright by scraping. Good contacts are best assured by having mercury cups at the gaps instead of screw terminals; care should be taken that the mercury in these cups is in good metallic contact with them, that is to say, it should *wet* the metallic surfaces. The mercury should also, of course, be in similar good contact with the ends of the wires or rods (the latter are usually attached to the standard resistances), which may be dipped into the cups.

The amalgamation of the metallic surfaces is best effected by scouring the latter with emery paper, and then moistening them with a solution of nitrate of mercury.

Even if good contacts be assured, correct results cannot be obtained if the standard resistances are incorrect, or if the slide wire is not uniform in its resistance throughout its length. A metre bridge to be really useful, therefore, requires to be carefully made.

214. The accuracy with which a test can be made, as in the ordinary form of bridge, depends upon the values given to the various resistances, and amongst these upon the value given to k.

In order to be able to vary the value of this quantity, the gaps at m_1 and m_2 are provided.

As the resistances placed in these gaps are simply prolongations of the slide wire, it is necessary that their values should be known in terms of equivalent lengths of the slide wire; that is, we must know how many millimetres of the wire they are equal to. This is best done in the following manner:—

Close the gaps at m_1 and m_2 with the thick copper straps, and place resistances of known values at a and x. Adjust the slides so that equilibrium is produced, then

$$x = a\left(\frac{1000 + n_1 + n_2}{b + n_1} - 1\right),$$

or

$$x(b + n_1) = a(1000 + n_2 - b).$$

Now insert one of the resistances, whose equivalent length m_1 in millimetres is required, at the left-hand gap, and again obtain equilibrium; calling the new scale reading b_1 we then have

$$x(b_1 + n_1 + m_1) = a(1000 + n_2 - b_1).$$

By subtracting the one equation from the other ge get

$$x(b - b_1) - x m_1 = a(b_1 - b),$$

that is,

$$(b - b_1)(a + x) = m_1 x,$$

or

$$m_1 = (b - b_1)\frac{a + x}{x}.$$

For example.

It being required to know how many millimetres of the slide wire a resistance m_1 was equal to, the scale reading b, with the two gaps closed, was 500 mm., and the scale reading b_1, with m_1 inserted, was 480 mm., the resistances at a and x being 6 and 4 ohms respectively. What was the value of m_1?

$$m_1 = (500 - 480)\frac{6 + 4}{4} = 50 \text{ mm.}$$

If we have a and x equal, we get the simplification

$$m = (b - b_1)\,2.$$

There are other methods of determining the value of m_1, but the one given, besides being extremely simple, is very accurate, as it is independent of the quantities n_1 and n_2.

The values of the resistances to be placed at m_1 and m_2 being thus determined, the value of x is given by the equation

$$x = a\left(\frac{1000 + n_1 + n_2 + m_1 + m_2}{\text{scale reading} + n_1 + m_1} - 1\right).$$

215. Let us now consider the *Best arrangement of resistances*, &c., for making a test with the metre bridge, under favourable conditions.

Now a mistake of a millimetre in the position of the slider will make a much greater error in the result of x worked out from the formula, when the slider is near the ends of the wire than when it is near the middle. Thus, for example, suppose x was 1 ohm and a was also 1 ohm, then we should have the slider standing exactly at 500 if it were properly adjusted. Suppose, however, it was 1 millimetre out, then the apparent value of x would be

$$x = 1\left(\frac{1000}{501} - 1\right) = \cdot 996,$$

that is, we make x, $1 - \cdot 996$, or $\cdot 004$ ohms too small.

Next suppose $a = 9$ ohms, then for equilibrium the scale reading would be 900, and if we make a mistake of 1 millimetre we should have

$$x = 9\left(\frac{1000}{901} - 1\right) = \cdot 990;$$

that is, we make x, $1 - \cdot 990$, or $\cdot 010$ ohms too small.

Lastly let us suppose $a = \frac{1}{9}$ ohm, then the scale reading for exact equilibrium would be 100, and supposing there to be an error of 1 millimetre, we have

$$x = \frac{1}{9}\left(\frac{1000}{101} - 1\right) = \cdot 989;$$

that is, we make x, $1 - \cdot 989$, or $\cdot 011$ ohms too small.

To summarise the results, then, we see that with

a larger than x,	error	was	$\cdot 010$, or 1	per cent.
,, equal to	,,	,,	,,	$\cdot 004$, or $\frac{2}{5}$	,,
,, smaller than,,	,,	,,	$\cdot 011$, or 1	,,

The error, in fact, was smallest when the slider was at the middle of the wire. We must, however, determine whether the middle is really the point at which the error is least.

Calling k' the resistance of the slide wire and its prolongations m_1, m_2, and b' the scale reading plus the prolongation m_1, let there be an error λ in x caused by an error $-\delta$ in b', then

$$x + \lambda = a\left(\frac{k'}{b' - \delta} - 1\right) \quad \text{or} \quad \lambda = a\left(\frac{k'}{b' - \delta} - 1\right) - x.$$

But

$$x = a\left(\frac{k'}{b'} - 1\right), \quad \text{or,} \quad a = \frac{x}{\dfrac{k'}{b'} - 1};$$

therefore

$$\lambda = x\left[\frac{\dfrac{k'}{b' - \delta} - 1}{\dfrac{k'}{b'} - 1} - 1\right] = x\,\frac{k'\,\delta}{(b' - \delta)\,(k' - b')};$$

or since δ is a very small quantity, we may say,

$$\lambda = x\,\frac{k'\,\delta}{b'\,(k' - b')}. \qquad \text{[A]}$$

Now we have to make λ as *small* as possible; this we shall do, since x and k' are constant quantities, by making $b'\,(k' - b')$ as *large* as possible.

But

$$b'(k'-b') = \frac{k'^2}{4} - \left(\frac{k'}{2} - b'\right)^2,$$

and to make this expression as large as possible, we must make $\frac{k'}{2} - b'$ as small as possible; that is, since b' must be positive, we must make it equal to 0, or

$$\frac{k'}{2} - b' = 0; \text{ that is, } b' = \frac{k'}{2};$$

which proves the truth of the supposition.

To obtain the slider as near to the middle of the wire as possible when equilibrium is produced, we must make a as nearly as possible equal to x.

If in equation [A] we put

$$\lambda = \frac{\lambda'}{100} \text{ of } x, \quad \text{and,} \quad b' = \frac{k'}{2},$$

we get

$$\lambda' = \frac{400\,\delta}{k'}$$

so that if when the slider is near to the centre of k' we can adjust the slider to an accuracy of 1 division (δ), then if k' consisted of 1000 parts (as would be the case if there were no prolongations m_1, m_2), we could measure the value of x to an accuracy of

$$\frac{400 \times 1}{1000} = \cdot 4 \text{ per cent.}$$

216. In order to make a measurement in this manner, as we have seen, it is necessary for a to be approximately equal to x. Now in many cases there would be no difficulty in arranging that such should be the case. Thus, for example, suppose it was required to measure the conductivity of a sample of wire, then in this case we should take a sufficient length of the wire to give a resistance *approximately* equal to a, and then having measured the exact length taken, we should ascertain its *exact* resistance by adjusting the slider until equilibrium was obtained.

217. If we wish the measurement to be made to a higher percentage of accuracy than can be made with the slide wire k alone, then we must add equal resistances, m_1 and m_2, at each end of the wire so as to increase the value of k.

Since
$$\lambda' = \frac{400\,\delta}{k'}\,,$$
therefore
$$k' = \frac{400\,\delta}{\lambda'}\,;$$
so that if we wish to measure x to an accuracy, say, of $\cdot 1$ per cent., then we must make k' equal to
$$\frac{400 \times 1}{\cdot 1} = 4000\,;$$
that is to say, we must add resistances m_1 and m_2 at each end of k, each equivalent to 1500 millimetres of the wire k. It must be recollected, however, that there will be no advantage in thus increasing the length of k, unless the figure of merit of the galvanometer employed is sufficiently high to enable a movement of the slider to a distance of 1 division from its correct position, to produce a perceptible movement of the needle.

If the resistance to be measured is not one which admits of adjustment, then in order to obtain a satisfactory measurement we must add a resistance on to one or other of the ends of k, according as x is larger or smaller than a; or we may add resistances to both ends, their values being unequal.

If in equation [A] (page 197) we put
$$x = a\left(\frac{k'}{b'} - 1\right),\quad \text{or,}\quad k' = b'\,\frac{a+x}{a}\,, \qquad [1]$$
then we get
$$\lambda = \frac{(a+x)\,\delta}{b'}\,;$$
or if we put
$$\lambda = \frac{\lambda'}{100} \text{ of } x,$$
we have
$$\lambda' = \frac{100\,(a+x)\,\delta}{b'\,x},\quad \text{or,}\quad b' = \frac{100\left(\frac{a}{x}+1\right)\delta}{\lambda'}.$$

From this equation we can see that no matter what are the relative values of a and x still b' can always have a value which will enable x to be obtained to any percentage of accuracy λ'; that is, of course, provided the figure of merit of the galvanometer be sufficiently high for the purpose.

For example.

It is required to measure the exact value of a resistance x, whose approximate value is five times that of the resistance a; what must be the value of b' in order that the measurement may be made to an accuracy of ·5 per cent.? The adjustment of the slider can be determined to an accuracy of 1 division.

$$b' = \frac{100\left(\frac{1}{5} + 1\right)1}{\cdot 5} = 240.$$

From equation [1] (page 199) we get

$$k' = 240\left(1 + \frac{5}{1}\right) = 1440,$$

consequently since k consists of 1000 divisions we must add a prolongation m_2 equal to not less than 440 divisions on to k.

We may of course make the prolongation larger than 440; in fact, in practice we should have to do so unless we had a resistance available of the exact required value; but it must not be too large, otherwise the position of balance for the slider would be at some point on m_2 instead of on the wire k. In fact, m_2 must not be greater than $\frac{k\,x}{a}$.

If it should happen that in order to obtain a particular percentage of accuracy it is necessary that b' should exceed k, then in this case it would be necessary to have a prolongation m_1 in addition to the prolongation m_2; the latter quantity in this case must not exceed $(k + m_1)\frac{x}{a}$.

In the last example we have supposed x to be *less* than a. If, however, x is *greater* than a, then b' will probably have to be greater than k, in which case of course we should have to add the prolongation m_1 in the place of the prolongation m_2, the value of m_1 being such that it does not exceed $k\frac{a}{x}$, unless we also add a prolongation m_2 in addition to m_1, in which case m_1 must not exceed $(k + m_2)\frac{a}{x}$.

We have seen that by means of m_1 and m_2—the values of which can be determined in the manner shown in (§ 214, page 195)—we can theoretically arrange that the value of x can be assured to any required degree of accuracy, no matter what the relative values of x and a may be. This, however, can only be the case

provided the figure of merit of the galvanometer is such as to enable the slider to be adjusted to an accuracy of 1 division. The figure of merit of the galvanometer, therefore, as in other tests, is the limit to the "Possible degree of accuracy attainable." This limit can be determined from equation [2] (page 173) in the following manner:—

Let λ be the error in x, caused by b' being $\frac{1}{n}$th of a unit out of adjustment, then we have

$$x + \lambda = a\frac{d' + \frac{1}{n}}{b' - \frac{1}{n}}, \text{ or, } \lambda = a\frac{d' + \frac{1}{n}}{b' - \frac{1}{n}} - x = \frac{a\,d' + \frac{a}{n} - b'x + \frac{x}{n}}{b' - \frac{1}{n}};$$

and since $a\,d' = b'\,x$, and $\frac{1}{n}$ is a very small quantity, we get

$$\lambda = \frac{1}{n}\cdot\frac{a + x}{b'};$$

We have then from equation [2] (page 173) by putting $d' = \dfrac{b\,x}{a}$,

$$c_6 = \frac{\mathrm{E}\frac{1}{n}\cdot\frac{a + x}{b'}}{\left\{\frac{x}{a}(b' + g) + x + g\right\}\left\{r + a + x + \frac{r\,a}{b'}\right\}}.$$

In order, therefore, that b' may be able to have the value necessary to ensure x being measured to the required degree of accuracy, the value of c_6 must not be less than that given by the above equation.

As the values of g, d, x, and r are most easily obtained in ohms, the value of b' corresponding to the number of divisions of which it would consist must be in ohms also; $\frac{1}{n}$, likewise, will have to be the resistance, in the fraction of an ohm, corresponding to 1 division (or fraction of a division, if the slider can be adjusted to a closer accuracy than 1 division) of the wire k.

For example.

In the last example it was required to be known whether a galvanometer whose resistance was 1 ohm (g), and the reciprocal of whose figure of merit was $\cdot 0002$ (c_6) would be suitable for the purpose of making the measurement in question. The resistance of the slide wire, which was divided into 1000 divisions (k), was 5 ohms; the resistance a was 1 ohm, and the resistance x, 5 ohms approximately. The actual value of the prolongation added to k was such as to make k' equal to 1560. The resistance

of the battery was 5 ohms (r), and its electromotive force 2 volts (E) approximately.

Since $k = 1000$, therefore $\frac{1}{n} = \frac{\cdot 5}{1000} = \cdot 0005$.

Also (from equation [1], page 199) we have

$$b' = \frac{a\,k'}{a + x} = \frac{1 \times 1560}{1 + 5} = 260 \text{ divisions} = \frac{\cdot 5 \times 260}{1000} = \cdot 13 \text{ ohms};$$

therefore

$$c_6 = \frac{2 \times \cdot 0005 \times \frac{1 + 5}{\cdot 13}}{\left\{\frac{5}{1}(\cdot 13 + 1) + 5 + 1\right\}\left\{5 + 1 + 5 + \frac{5 \times 1}{\cdot 13}\right\}} =$$

$$\frac{\cdot 046}{(11 \cdot 65)\,(49 \cdot 46)} = \cdot 0008,$$

which is greater than $\cdot 0002$ the reciprocal of the figure of merit of the galvanometer in question, consequently the latter instrument is well suited for the purpose for which it is required.

218. The resistance of the galvanometer employed in making a bridge test is an important point, especially as regards the measurement of small resistances.

In the case of the ordinary bridge test, we can adjust within 1 unit, and in the case of the slide wire bridge, we can adjust within 1 millimetre of the wire; if then the galvanometers employed in these cases are such that when we are 1 unit or 1 millimetre from exact equilibrium we obtain perceptible deflections of the needles, then we have what we require, whatever the resistances of the galvanometers may be.

In the ordinary form of bridge, where the adjustable resistances are not capable of being adjusted to a greater accuracy than 1 unit, a Thomson's galvanometer, such as that described in Chapter III. (page 31), and which has a resistance of about 5000 ohms, gives, under all circumstances, a very large deflection when the adjustment is only 1 unit from equilibrium. In the case of the slide wire bridge, however, where to be 1 millimetre from exact equilibrium means to be only $\frac{1}{10{,}000}$th of an ohm, or even less, out, a galvanometer of such a high resistance as 5000 ohms would not be found to give a perceptible deflection.

The reason of this is, that such a galvanometer is practically short circuited by the very low resistance it has between its terminals.

The question of galvanometer resistance is considered at length in Chapter XXV., and it is there shown that it is best that the instrument should have a resistance not more than about 10 times, or less than about $\frac{1}{10}$th, $\frac{a(d+x)}{a+x}$. Of course in practice we cannot adjust the resistance to meet every particular case, but the limits given are sufficiently wide to enable an instrument to be made which would prove satisfactory for most purposes for which the metre bridge is adapted; moreover, if a particular galvanometer does not prove to be suitable for a particular purpose, we can ascertain, by the help of the above rule, whether the cause is due to its resistance being too high or too low.

It should be clearly understood that when we speak of the resistance of the galvanometer we mean the resistance of the instrument itself, and not the resistance in its circuit; thus, if according to calculation it were proved that the galvanometer resistance should be 1 ohm, then it would not be carrying out the rule if we took an instrument having a resistance of, say $\frac{1}{4}$ of an ohm, and added a resistance of $\frac{3}{4}$ of an ohm in its circuit, for this $\frac{3}{4}$ of an ohm would be an addition to the *external* current, and not an addition to the galvanometer itself.

Under no conditions should the battery be joined between A and C, and the galvanometer between B and E, for in such a case the battery current in passing from the slider to the wire would be liable to injure the surface of the latter.

To sum up, then, we have

Conditions necessary for making the Test to any required Degree of Accuracy.

219. The number of divisions of which b' must consist in order that x may be measured to an accuracy of λ' per cent. must be not *less* than

$$\frac{100\left(\frac{a}{x}+1\right)\delta}{\lambda'},$$

δ being the number of divisions, or the fraction of a division, to which it is possible to adjust the slider.

If prolongations are necessary, then m_1 must not exceed $\left(k+m_2\right)\frac{a}{x}$, and m_2 must not exceed $\left(k+m_1\right)\frac{x}{a}$.

The reciprocal of the figure of merit of the galvanometer must be not less than

$$\frac{E\,\delta.\dfrac{a+x}{b'}}{\left\{\dfrac{x}{a}\left(b'+g\right)+x+g\right\}\left\{r+a+x+\dfrac{r\,a}{b'}\right\}}$$

where E is in volts and *all* the other quantities (including b' and d') are in ohms.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100}{b'}\left(\frac{a}{x}+1\right)\delta.$$

Measurement of the Conductivity Resistance of a Telegraph Line.

Direct Method.

220. When, by means of the bridge, Fig. 51 (p. 169), we are measuring the *conductivity* resistance of a wire whose farther end is not at hand, we should join one end to terminal C, put the farther end to earth, put terminal E to earth, and then measure in the usual way.

Loop Method.

221. It is always as well, however, when possible, to measure without using an earth, by looping two wires together at their farther ends, the nearer ends being joined to terminals E and C respectively; this gives the joint conductivity resistance of the two. Errors consequent from earth currents, or a defective earth, &c., are thereby avoided. We cannot, however, by this means, obtain the conductivity resistance of each wire separately. If, however, we have three wires at hand, we can by three measurements obtain the conductivity resistance of each wire, without using an earth. This is effected as follows:—

Let the three wires be numbered respectively 1, 2, and 3. First loop wires 1 and 2, at their further ends, and let their resistance be R_1. Next loop wires 1 and 3, and let their resistance be R_2. Lastly, loop 2 and 3, and let their resistance be R_3. Supposing the respective resistances of 1, 2, and 3 to be r_1, r_2, and r_3, we get

$$r_1 + r_2 = R_1$$
$$r_1 + r_3 = R_2$$
$$r_2 + r_3 = R_3.$$

Now, since each of the wires is looped first with one and then with the other of the other two, it is evident that the sum of the three measurements will be the sum of the individual resistances of the three wires taken twice over, and consequently $\frac{R_1 + R_2 + R_3}{2}$ must be the sum of the resistances of the three wires. If, then, we subtract R_1 from this result, the remainder must be the resistance of r_3. Similarly, if we subtract R_2 from the same, the remainder will give us r_2; and lastly, by subtracting R_3, we get the value of r_1.

For example.

The conductivity resistance of each of three wires, Nos, 1, 2, and 3 was required. Nos. 1 and 2 being looped, the resistance (R_1) was found to be 300 ohms. Nos. 1 and 3 looped gave a resistance (R_2) of 400 ohms. Lastly, Nos. 2 and 3 looped gave a resistance (R_3) of 500 ohms; then

Added resistance of the three wires will be

$$\frac{300 + 400 + 500}{2} = 600 \text{ ohms.}$$

Therefore,

Resistance	(r_1)	of No. 1	wire	$= 600 - 500 = 100$	ohms.
„	(r_2)	„ 2	„	$= 600 - 400 = 200$	„
„	(r_3)	„ 3	„	$= 600 - 300 = 300$	„

By this device, then, we are enabled to eliminate all sources of error without making a greater number of measurements than would be required if we measured each wire separately, by using an earth.

Measurement of the Resistance of an Earth.

222. By means of a method very similar to the foregoing we can, if we have two wires at our disposal, measure the resistance of the *earths* at the end of the lines. The following is the way in which this can be done:—

Let the two wires be numbered respectively 1 and 2. First loop the two wires at their further ends, and let the measured resistance of the loop be R_1. Next have No. 1 wire put to the earth at its further end, and measure the resistance, which will be that of the wire and earth combined; let this total resistance be R_4. Lastly, have wire No. 2 put to the earth at the distant station, and measure the total resistance, which we will call R_5;

then by adding R_1, R_4 and R_5 together, and dividing the result by 2, we get the sum of the resistances of the two wires and the earth; by subtracting from this result the resistances of the two looped wires the remainder will be the resistance of the earth.

223. By means of a test made in this manner we can determine not only the resistance of an earth, but also the individual resistance of two wires; for if we subtract R_4 from $\frac{R_1 + R_4 + R_5}{2}$, the result will be the resistance of wire No. 2, and if we subtract R_5 instead of R_4, then the result will be the resistance of wire No. 1. Such a test, however, although it eliminates errors due to defective earths, does not eliminate errors due to earth currents. But inasmuch as it is a test which is applicable when only *two* wires can be had, it is useful, since the earth current errors can be eliminated by a method which we shall investigate.

Measurement of the Insulation Resistance of a Telegraph Line.

224. In measuring the insulation resistance of a wire, the connections would be the same as for conductivity resistance, except that the farther end of the wire, instead of being put to earth, would be insulated.

225. It sometimes happens that we require to find the insulation resistance of two sections of one wire, but we can only test from one end.

Now, if we join several wires together, one in front of the other, it is evident that the total *insulation* resistance of the combination will diminish according to the number of the wires and according to the insulation resistance of each of them. The law for the total resistance, in fact, will be the same as that for the joint *conductor* resistance of a number of wires joined up in multiple arc (page 52). That is to say, *the total insulation resistance of any number of wires joined together will be equal to the reciprocal of the sum of the reciprocals of their respective insulation resistances.* As a matter of fact, it is immaterial whether the wires be joined together one in front of the other or all be bunched together; the law of the joint insulation resistance is the same in both cases.*

* This is not the case if the insulation resistances are very low, as the resistance of the conductor then comes into question and modifies the result.

Suppose, then, A C to be the wire which is required to be tested for insulation resistance from A in two sections, A B and B C. Let a be the insulation resistance of the section A B, and b the insulation resistance of the section B C; and suppose x to be the insulation resistance of the whole wire from A to C, then we have

$$x = \frac{a\,b}{a + b},$$

from which

$$b = \frac{a\,x}{a - x}.$$

All we have to do, therefore, supposing we are testing from A, is first to get the end C insulated and to measure the insulation resistance; this gives us x. Next get the wire separated at B, and the end of the section A B insulated. Again measure the insulation resistance; this gives us a. Then from the two results b can be calculated.

For example.

The *insulation* resistance (x) of the whole wire, from A to C, was found to be 6000 ohms, and that from A to B (a), 24,000 ohms. What was the insulation resistance (b) of the section B?

$$b = \frac{24{,}000 \times 6000}{24{,}000 - 6000} = 8000 \text{ ohms.}$$

226. To obtain the *conductivity* resistance of one section of a wire when the resistance of the other section, and also of the whole wire, is known, we have only to subtract the resistance of the one section from the resistance of the whole section. The truth of this is obvious.

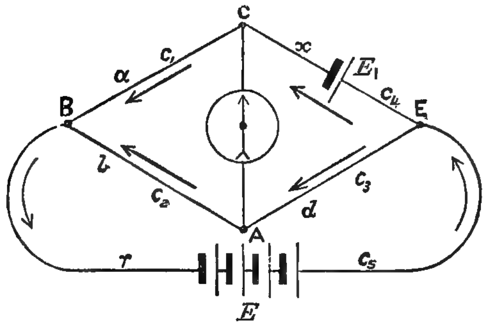
Measurement of the Conductivity Resistance of Wires traversed by Earth Currents.

227. When the conductivity resistance of a line of telegraph is measured by having the farther end of the line put to earth, the presence of earth currents, that is to say, the currrents set up by electrical disturbances over the surface of the earth, and also currents due to the polarisation of the earth plates, renders the formula $x = d\,\frac{a}{b}$, when equilibrium is produced, incorrect. To obtain the true value of the resistance of the wire, therefore, a different formula is necessary.

Equilibrium Method.

228. In Fig. 59 let E be the electromotive force of the testing battery, E_1 the electromotive force of the earth current, whose value will be $+$ or $-$ according to its direction, and let a, b, d, x, and r be the resistances of the various parts of the bridge; then

FIG. 59.



c_1, c_2, c_3, c_4, and c_5 being the current strengths in the different branches, we have by Kirchoff's laws (page 134), when equilibrium is produced, the following equations connecting the resistances, current strengths, and electromotive forces:—

$$c_1 - c_4 = 0$$
$$c_2 - c_3 = 0$$
$$c_5 - c_3 - c_4 = 0$$
$$c_1\,a - c_2\,b = 0$$
$$c_4\,x - c_3\,d = \pm\,E_1$$
$$c_5\,r + c_3\,d + c_2\,b = E.$$

By elimination we obtain two values of c_4, one in terms of the battery E_1, and the other in terms of E, thus

$$c_4 = \pm\frac{E_1}{x - \dfrac{a\,d}{b}}$$

and

$$c_4 = \frac{E}{\dfrac{a\,(d + r) + b\,(a + r)}{b}}.$$

229. If we equate the two values of c_4 we can get the relation between the two electromotive forces E and E_1, and thus obtain a method of determining the relative electromotive forces of the batteries, for we have

$$\frac{\pm E_1}{E} = \frac{b\,x - a\,d}{a\,(d + r) + b\,(a + r)}.$$

230. From the latter equation we find

$$x = \frac{a\,d}{b} \pm \frac{E_1}{E}\left[\frac{a\,(d + r) + b\,(a + r)}{b}\right].$$

To make this equation useful it is necessary that E_1 and E be known. If, however, we reverse the testing battery and again obtain equilibrium by readjusting d to d_1, we get a second equation, viz.,

$$\frac{\pm E_1}{-E} = \frac{b\,x - a\,d_1}{a\,(d_1 + r) + b\,(a + r)};$$

we therefore have

$$\frac{b\,x - a\,d}{a\,(d + r) + b\,(a + r)} + \frac{b\,x - a\,d_1}{a\,(d_1 + r) + b\,(a + r)} = 0.$$

By multiplying up we get

$$\begin{aligned} &b\,x\,[a\,(d_1 + r) + b\,(a + r)] - a\,d\,[a\,(d_1 + r) + b\,(a + r)] \\ &+ b\,x\,[a\,(d + r) + b\,(a + r)] - a\,d_1\,[a\,(d + r) + b\,(a + r)] \\ &= 0; \end{aligned}$$

that is

$$x = \frac{a}{b} \cdot \frac{d\,[a\,(d_1 + r) + b\,(a + r)] + d_1\,[a\,(d + r) + b\,(a + r)]}{a\,(d_1 + r) + b\,(a + r) + a\,(d + r) + b\,(a + r)}$$

$$= \frac{a}{b}\left[\frac{2\,(d + k)\,(d_1 + k)}{(d + k) + (d_1 + k)} - k\right] \qquad [A]$$

where

$$k = \left[r\left(1 + \frac{b}{a}\right) + b\right].$$

For example.

In making a conductivity test of a wire in which an earth current existed, with the zinc pole of the battery to line equilibrium was obtained when d_1 was 8000 ohms. On reversing the testing current, equilibrium was obtained when d was

6000 ohms. The resistances a and b being 100 and 1000 ohms respectively, and the resistance, r, of the battery 200 ohms, what was the resistance, x, of the line?

$$k = \left[200\left(1 + \frac{1000}{100}\right) + 1000\right] = 3200,$$

therefore

$$x = \frac{100}{1000}\left[\frac{2\,(6000+3200)\,(8000+3200)}{(6000+3200)+(8000+3200)} - 3200\right] = 690 \cdot 2 \text{ ohms.}$$

It may be pointed out that the quantity $\frac{2\,(d+k)\,(d_1+k)}{(d+k)+(d_1+k)}$ in equation [A] is the *harmonic* mean of the quantities $(d+k)$ and (d_1+k).

Various abbreviations of formula [A] have been suggested, but none of them are satisfactory except under certain conditions, and inasmuch as the formula is only required occasionally, the advantage of a simplification which at the best is only an approximation is a doubtful one.

Equal Deflection or "False Zero" Method.

231. Referring to page 172, if we suppose that there is the electromotive force E_1 in the branch x (Fig. 59), then equation [7] becomes

$$\frac{c_5\,[r\,(d_1 + x) + x\,(b + d_1)]}{[c_5\,(r\,(a + b) + a\,(b + d_1]} \qquad [A]$$

$$= \frac{-\,c_6\,(b\,g + d_1\,g + b\,x + b\,d_1) + E\,(d_1 + x) - E_1\,(b + d_1)}{c_6\,(a\,d_1 + b\,d_1 + b\,g + d_1\,g) + E\,(a + b)};$$

or say

$$K = \frac{-\,c_6\,k' + E\,(d_1 + x) - E_1\,(b + d_1)}{c_5\,k'' + E\,(a + b)},$$

that is

$$c_6 = \frac{E\,(d_1 + x) - E_1\,(b + d_1) - E\,(a + b)\,K}{k''\,K + k'}.$$

Now, supposing the electromotive force E is removed without altering r, and suppose at the same time that c_6 and the other quantities remain unaltered, then we have

$$K = \frac{-\,c_6\,k' - E_1\,(b + d_1)}{c_6\,k''},$$

that is

$$c_6 = \frac{-\,E_1\,(b + d_1)}{k''\,K + k'},$$

therefore

$$E(d_1 + x) - E_1(b + d_1) - E(a + b)K = - E_1(b + d_1);$$

therefore

$$d_1 + x = (a + b)K.$$

Or giving the value [A] of K, we have

$$d_1 + x = (a + b)\frac{r(d_1 + x) + x(b + d_1)}{r(a + b) + a(b + d_1)};$$

therefore

$$\begin{aligned} &r(a + b)(d_1 + x) + a(b + d_1)(d_1 + x) \\ = &r(a + b)(d_1 + x) + x(b + d_1)(a + b); \end{aligned}$$

therefore

$$a(d_1 + x) = x(a + b),$$

or

$$x = \frac{a\,d_1}{b}.$$

If therefore we have a key so arranged that on depressing it a resistance equal to that of the battery is inserted in the place of the latter, then on adjusting the resistances so that no alteration in the deflection of the galvanometer needle is produced by the movement of the key, we get the value of x at once from the above equation.

232. When the battery connections for measuring conductivity are made, as shown by Fig. 51 (page 169), then in order to put the zinc current to line, we should put the cable or line to C and the earth to E. To put the copper to line we can either reverse the battery or put the cable to E and the earth to C, whichever is most convenient to the experimenter.

Measurement of the Conductivity Resistance of a Submarine Cable.

233. When we are measuring the conductivity of a submarine cable, which requires to be carefully done, the best method to adopt is the following :—

Put on the battery current for half a minute by pressing down the right-hand key; at the expiration of that time, proceed to adjust the plugs, pressing down the left-hand key as required until equilibrium is produced; continue to adjust, if the needle does not remain at zero, and at the expiration of half a minute note the resistance. Now reverse the battery connections, put on the current for half a minute; again measure,

again reverse and measure, and so on until about a dozen measurements with either current have been taken. It will usually be found that about half the measurements made with the negative current are the same, and also half the measurements made with the positive current; these results may be taken as the correct measurements for d and d_1.

234. In order to reverse the current through the cable, we can either reverse the battery, or the line and earth, connections (§ 232). There is an advantage in doing the latter, as by this means the galvanometer deflection due to, say, too much resistance being inserted between D and E (Fig. 51, page 169), is always on the same side of zero, although the direction of the current through the cable is reversed. Thus it is easy to see at a glance in every case, and without chance of a mistake, whether balance is out in consequence of too much or too little resistance being inserted.

235. The presence of earth currents can be detected when the line, galvanometer, and earth are joined to the resistance box, by pressing down the left-hand key alone. This will cause the galvanometer needle to be deflected if there are any currents present. A line is seldom, if ever, quite neutral in this respect.

236. It is almost immaterial what battery power is used in measuring conductivity; sufficient, however, should be used to obtain a good deflection on the galvanometer needle when equilibrium is not exactly produced. About 10 or 20 cells is a convenient number to employ. There is no danger of heating the resistance coils with such a power if the battery be a Daniell charged with plain water, or even a Leclanché, as their internal resistances are considerable. It would not be advisable, however, to use a Grove or a Bunsen battery, or a Daniell charged with acidulated water, as their heating power is great in consequence of their small internal resistances.

Elimination of the Resistance of Leading Wires.

237. In order to determine the exact resistance of the conductor of a cable, or coil of cable core, for example, it is of course necessary that the resistance of the wires leading from the testing-room to the tank in which the cable or core is placed, should be deducted from the total measured resistance. This involves a calculation which, although slight, still might be avoided with advantage, especially if a large number of measurements have to be made. At Messrs. Siemens' works, at Charlton, a very simple device is adopted which enables the resistance of the

leading wire to be eliminated, thus rendering any deduction unnecessary. For this purpose a small supplementary slide wire resistance (§ 18, page 15) is connected in the arm A E of the bridge (Fig. 50, page 166); the leading wires (when connected to the bridge) being looped together at their further ends, and all the plugs being inserted in A E, the slide resistance is adjusted till balance is obtained on the galvanometer. The leads are now connected to the cable or core to be tested, and then balance is again obtained on the galvanometer by removing plugs from A E in the usual manner. This being done, the resistance unplugged in A E (allowing for the ratio of the arms A B, B C, of the bridge, if the two are unequal) obviously gives the exact value of the resistance required, since the resistance of the leads is balanced by the slide resistance.

Measurement of Battery Resistance.

238. The resistance of a battery which consists of a large number of cells may in many cases be measured with a considerable degree of accuracy by means of the Wheatstone bridge, in the following manner:—

Divide the battery into two equal parts, and connect the two halves together so that their electromotive forces oppose one another; under these conditions the battery may be treated as an ordinary resistance, and measured as such.

CHAPTER IX.

LOCALISATION OF FAULTS.

239. The theoretical methods of testing for the localities of faults are comparatively simple, but their practical application presents some difficulties.

Localisation of a full Earth Fault.

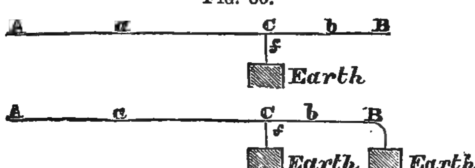
240. The simplest kind of fault to localise is a complete fracture where the fault offers no resistance, and the conductivity resistance at once gives its position. Thus, a line which was 100 miles long, and in its complete condition had a resistance of 1350 ohms, that is to say, a resistance of $\frac{1350}{100} = 13 \cdot 5$ ohms per mile, gave a resistance of 270 ohms when broken. Then distance of fault from testing station was

$$\frac{270}{13 \cdot 5} = 20 \text{ miles.}$$

Localisation of a partial Earth Fault.

241. When the fault has a resistance, the localisation becomes somewhat difficult. The following are the theoretical methods generally adopted (Fig. 60).

Fig. 60.



First Method.

242. Let A B be a line which has a fault f at C, A being the testing station. A first gets B to insulate his end of the

line. He then measures the resistance, which we will call l, then

$$a + f = l;$$

therefore

$$f = l - a. \qquad [1]$$

Next, B puts his end to earth, and A again measures. Let the new resistance be l_1, then

$$a + \frac{b\,f}{b + f} = l_1. \qquad [2]$$

Calling L the resistance of the line, we have also

$$a + b = L;$$

therefore

$$b = L - a. \qquad [3]$$

From these three equations we have to determine a. Substituting in [2] the values of f and b obtained from [1] and [3], we get

$$a + \frac{(l - a)\,(L - a)}{L + l - 2\,a} = l_1,$$

therefore

$$a^2 - 2\,a\,l_1 = L\,l - L\,l_1 - l\,l_1;$$

from which, since a must be less than l_1, and the root consequently negative,

$$a = l_1 - \sqrt{(l - l_1)\,(L - l_1)}.$$

For example.

A faulty cable, whose total conductivity resistance when perfect was 450 ohms (L), gave a resistance of 350 ohms (l) when the farther end was insulated, and 270 ohms (l_1) when the end was put to earth. What was the resistance of the conductor up to the fault?

$$\text{Resistance} = 270 - \sqrt{(350 - 270)\,(450 - 270)} = 150 \text{ ohms.}$$

If the length of the cable were 50 miles, then conductivity per mile equals $\frac{450}{50} = 9$ ohms, and distance of fault from testing station consequently equals $\frac{150}{9} = 16\frac{2}{3}$ miles.

Second Method.

243. Two measurements are made, one by station A, and the other by station B, A and B insulating their end in turn. Thus resistance measured from A when B insulates, as before, is

$$a + f = l. \qquad [1]$$

Resistance measured from B when A insulates

$$b + f = l_2, \quad [2]$$

also

$$a + b = \mathrm{L}. \quad [3]$$

Subtracting [2] from [1]

$$a - b = l - l_2,$$

and adding [3]

$$2a = \mathrm{L} + l - l_2;$$

therefore

$$a = \frac{\mathrm{L} + l - l_2}{2}.$$

For example.

A faulty cable, whose total conductivity resistance when perfect was 450 ohms (L), when measured from A with the end at B insulated, gave a resistance of 350 ohms (l); and when measured from B with the end A insulated, a resistance of 500 ohms (l_2). What was the resistance of the conductor from A to the fault?

$$\text{Resistance} = \frac{450 + 350 - 500}{2} = 150 \text{ ohms.}$$

Practical Execution of Tests.

244. So far the testing is simple; the practical application, however, presents some difficulty. This is owing to the variation of the resistance of the fault when the testing current is put to the cable, in consequence of this current acting on the copper conductor, and through the agency of the sea water covering it with a salt, which besides increasing the resistance of the fault, also sets up a current opposing the testing current. To make a proper test, then, it is necessary so to manipulate the testing apparatus and battery as to get rid of the polarisation and resistance set up by the salt formed on the fault, and to measure the resistance at the moment this is done. The following is known as:—

Lumsden's Method.

245. The further end of the cable being insulated, the conductor is cleaned at the fault by applying a zinc current from 100 cells for ten or twelve hours, the current being occasionally

reversed for a few minutes. A rough resistance test is then made with a copper current.

A positive current is now applied to the cable for about one minute, using two or three cells for every 100 units of resistance which have to be measured. This coats the conductor with chloride of copper.

The cable is now again connected to the resistance coils, and the battery and galvanometer connections made as shown by Fig. 51 (page 169), the zinc pole being to terminal B′ and the copper to terminal E. The cable must be joined to C, and earth to E.

Both keys being depressed, the galvanometer needle is carefully watched and plugs inserted and shifted unit by unit, so as to keep the needle at zero; for the action of the negative current is to clean off the chloride of copper, and thereby to reduce the resistance of the fault. At a certain point this decomposition becomes complete, and the needle of the galvanometer flies open with a jerk, showing that a disengagement of hydrogen has taken place at the fault which enormously increases its resistance. The resistance in the resistance coils at that moment is the required resistance.

The fault being once cleaned by the application of the 100 cells for ten or twelve hours, it is unnecessary on repeating the measurement, which should always be done, to apply the battery for so long a time; ten or twenty minutes, or even less, will generally suffice.

When the measurement is made with the farther end of the cable to earth, the same process of preparation can be employed.

The rate at which the decomposition of the salts at the fault takes place, depends to a very great extent upon the strength of the current flowing out at the fault; now, if the latter be very near the end at which the test is being made, the resistance between the testing battery and the fault will be so small that the changes at the latter will take place with great rapidity, and it would be a matter of great difficulty to adjust the resistance in the bridge quickly enough to follow up the change of resistance at the fault as it takes place. To avoid this difficulty the best plan is to insert a resistance between the bridge and the end of the cable; this will retard the changes by reducing the strength of the current flowing in the circuit. The value of this resistance will depend entirely upon circumstances, and will be a matter of judgment with the person making the test, but in any case it should not be out of proportion to the actual conductor resistance of the cable.

The amount of battery power used is also a matter dependent

upon circumstances, but the higher the power it is found possible to use the less will the effect of earth currents influence the accuracy of the test.

The resistance employed in the arms A B, B C of the bridge (Fig. 51, page 169), will, to some extent, modify the rate at which the changes at the fault take place, and here again discretion must be used, as no definite rule can well be laid down.

It might be imagined that a "slide resistance" (page 14) would be very advantageous for making a test of this kind, but practical experience shows that the plug resistances are preferable in many cases.

The galvanometer with which this and the following test must be made, must be an ordinary astatic one (page 16) with filar suspended or pivoted needles. A Thomson's reflecting galvanometer is quite useless for the purpose.

Before making the test, A must of course arrange with B, or *vice versâ*, at what time and for how long he is to insulate, put to earth, &c., his end of the cable.

Fahie's Method.

246. Mr. J. J. Fahie, in a paper read before the Society of Telegraph Engineers,* has given the results of some very careful experiments and tests which he has made, bearing upon the subject of testing for faults. His method contains many valuable points of novelty, and is, in the author's words as nearly as possible, as follows:—

The cable-current is eliminated by sending into the line the current of the opposite sign to that coming from it, and arranging the strength and duration of this current to suit the strength of the one from the cable. Thus, if the latter be strong and negative, put (say) sixty cells positive to line for a couple of minutes, and then note the condition of the cable-current; if it be still negative, but weaker, put the battery on again for a short time, and continue to do so until the galvanometer needle indicates a weak positive current from the fault. If the latter be now left to itself and the cable put to earth through a galvanometer the needle will steadily, and as a rule leisurely, fall to zero and pass over to the other side, indicating a negative current again from the fault. While the needle is on zero the line is free and in a fit state for the subsequent test.

If the cable-current be positive, put sixty cells negative on until the fault is depolarised; the effect in this case is more brief

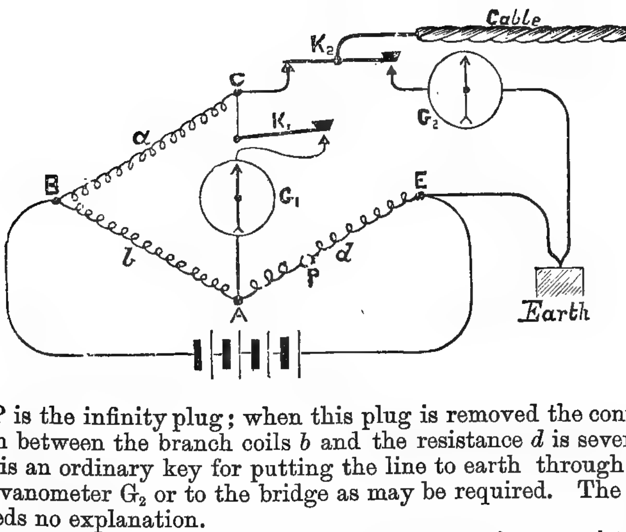
* 'Journal of the Proceedings of the Society of Telegraph Engineers,' No. IX.

than in the other, the needle falling quickly to zero and crossing to its original position.

Having once eliminated the current from the fault (and the operation very rarely exceeds ten minutes in the most obstinate cases) the cable can always be kept free by momentary applications of the necessary battery pole. Thus, if the needle begin to move off zero in the direction indicating a negative current from the fault, a positive current applied for a moment will bring it back, and *vice versâ*. In practice it is best to repolarise the fault slightly in the opposite direction, as a little time is thereby gained to arrange the bridge for a test.

Having shown how to prepare the cable, the test will now be described. The bridge is arranged as shown by Fig. 61.

Fig. 61.



P is the infinity plug; when this plug is removed the connection between the branch coils *b* and the resistance *d* is severed; K_2 is an ordinary key for putting the line to earth through the galvanometer G_2 or to the bridge as may be required. The rest needs no explanation.

First ascertain by an ordinary test the approximate resistance of the faulty cable and leave it unplugged in *d*. Next allow the line to rest for a few minutes in order that it may recover itself from the effects of the current employed in this preliminary test, and then depress K_2, and observe the cable-current on the galvanometer G_2; let it be positive, open the key K_1, remove the plug P, and send a negative current from the testing battery of (say) sixty cells into the cable *viâ* the branch-coils *a*, which should be plugged-in to avoid heating. When the cable-current

has been repolarised—a fact which may be ascertained by putting the cable to earth at intervals through G_2—arrange the bridge, close the key K_1, and, keeping the cable to G_2, watch till the needle comes to zero; at that moment let K_2 fly back, and send a negative current through the bridge system, observing the instantaneous effect on the galvanometer G_1. If d be too great the needle will be deflected in a direction (say to the right) indicative of this, but immediately after it will rush across zero and up the other side of the galvanometer (to the left), showing that the cable-current has again set in. If d be too small the needle will pass to the left, at first slowly, but immediately after with a bound. d is now adjusted, resistance is inserted or removed as required, and the eliminating process begun again. As d more nearly resembles the resistance of the cable, the first and instantaneous deflections after battery-contact become smaller; and, when d and the cable resistance are equal, the needle trembles over the zero-point for a moment, and then rushes over to the left under the influence of the cable-current.

Should the current given off by the fault be negative, having arranged the bridge as before, repolarise the fault with a positive battery current, and, waiting till G_2 shows the cable free, proceed to test as before, but using a positive current instead of a negative. Should d be too great the needle of G_1 will be deflected in this case, first to the left and then to the right. Should it be too small the needle will move to the right, at first slowly, but immediately after with a rush. The galvanometer G_1 must always be ready, and not short circuited, else the first and instantaneous deflections after battery-contact will not be perceived.

In practice it is found that when the cable-current is positive it is easily eliminated by a negative current, but that when it is negative the operation with a positive current is more difficult. Indeed, it is better not to employ a positive testing current at all, except for a moment when it is required to eliminate a weak negative cable-current. A positive current applied for a few seconds in this manner has only time to depolarise a fault, but when contiuued longer it seems to actually coat the exposed wire with badly conducting substances, by which the total resistance is increased.

It will be noticed that when the fault is depolarised by a positive current of any duration it does not recover itself for a long time. If a galvanometer be joined in circuit, its needle will remain at or near zero for a considerable time, occasionally oscillating feebly. The depolarisation by a negative current, on the other hand, lasts only a few moments.

The whole of the foregoing observations do not appear to be

applicable to every fault. Thus, when the fault has considerable resistance in itself, or when more faults than one exist, it is not always possible to eliminate the cable-current. Again, when the fault possesses resistance, the direction and strength of the cable-current, when the distant end is alternately insulated and put to earth, do not always coincide. For example, a fault occurred on a six-mile piece of shore-end cable, which reduced the insulation resistance to about 2000 units absolute. Now, when the further end of this piece was to earth, a strong negative current was often obtained, but when it was insulated the cable-current was slight and positive. Again, when the fault is further off than about 150 miles, and the intervening cable perfect, the charge-current interferes with the test.

247. The principal obstacle found in testing for faults is the presence of earth currents. If it were not for these there would really be but comparatively little difficulty in making satisfactory tests. But even earth currents would not create any serious difficulties provided they kept constant in strength and direction for any length of time; this, however, is unfortunately seldom the case, and it is often only by patient watching that a few seconds can be obtained when the cable is in a quiescent condition, and a test of correct value made.

The earth current difficulty is especially met with in long cables, and it is not uncommon for days to pass without a satistory test being taken.

248. Practice is required before these tests can be satisfactorily made. An artificial line, however, can easily be made with resistance coils to represent the resistance of the line up to the fault, and a short piece of cable core which has been pierced with a needle for the fault itself. This piece of core is immersed in a vessel of sea-water, using a piece of galvanised iron plate or wire for an earth. By this means a very fair idea of some of the difficulties encountered in testing for faults in cables may be obtained, and good practice made.

Kempe's Loss of Current Test.

249. In this test, which is shown by Fig. 62, a battery E is permanently connected, through a galvanometer G_1, to one end A of the cable, the further end B being connected to earth through a second galvanometer G.

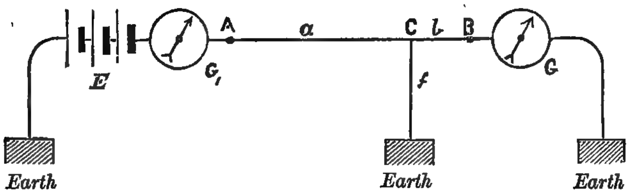
Let C_s be the current sent through the galvanometer G_1, and let C_r be the current received on the galvanometer G, then

$$C_r = C_s \frac{f}{f+b+G}, \quad \text{or,} \quad \frac{f}{f+b+G} = \frac{C_r}{C_s}.$$

Let the resistance beyond A be l_3, then

$$l_3 = a + \frac{f(b+G)}{f+b+G} = a + \frac{C_r}{C_s}(b+G);$$

FIG. 62.



also, as in the previous tests, let

$$a + b = L, \quad \text{or,} \quad b = L - a,$$

then by substitution we get

$$l_3 = a + \frac{C_r}{C_s}(L - a + G);$$

therefore

$$C_s\, l_3 = C_s\, a + C_r\,(L + G) - C_r\, a,$$

that is

$$a\,(C_s - C_r) = C_s\, l_3 - C_r\,(L + G),$$

or

$$a = \frac{C_s\, l_3 - C_r\,(L+G)}{C_s - C_r}.$$

L in this equation is known, it being the conductor resistance of the cable when sound. l_3 is easily determined, when the observations with the cable are completed, by joining up the galvanometer G_1 and battery E in circuit with a set of resistance coils, and then adjusting the latter until the deflection on the galvanometer G_1 is observed to be the same as it was when the cable was in circuit; the resistance in the resistance coils then gives the value of l_3.*

In order to determine C_s and C_r we must compare the deflections they produce on the respective galvanometers with the deflections obtained on the same galvanometers from a standard current, such, for instance, as that given by a standard Daniell cell (1·079 volts) (page 118), working through 1079 ohms, that is a current of 1 milliampère.

* See § 3, page 1.

Supposing both stations are furnished with standard cells, then each station having noted the deflection obtained when in circuit with the cable, disconnects his galvanometer from the, latter and puts it in circuit with a standard cell and a certain definite resistance, say, 1079 ohms, including the resistance of the galvanometer. The deflection is again noted; then this deflection, divided into the deflection obtained when the cable was in circuit, gives the value of C_s or C_r, as the case may be.

For example.

In testing a cable by the foregoing test, the connections being made as in Fig. 62, station A obtained a deflection on his galvanometer equivalent to 2800 divisions; station B obtained a deflection equivalent to 1520 divisions.

The deflection obtained by A on his galvanometer with a standard cell through 1079 ohms was 100 divisions, and the deflection obtained by station B with a similar battery working through 1079 ohms was 95 divisions; then

$$C_s = \frac{2800}{100} = 28; \quad C_r = \frac{1520}{95} = 16.$$

The value of l_3 was found to be 280 ohms, and the values of L and G were known to be 345 ohms and 5 ohms respectively. What was the value of a?

$$a = \frac{(28 \times 280) - [16 \times (345 + 5)]}{28 - 16} = 186 \cdot 7 \text{ ohms.}$$

If the cable had a conductivity resistance of 10 ohms per mile, then the distance of the fault from A would be

$$\frac{186 \cdot 7}{10} = 18 \cdot 67 \text{ miles.}$$

A great advantage which this test possesses lies in the fact that all the necessary observations with the cable can be made simultaneously, station A arranging with station B that at a definite time the observations are to be made on the galvanometers; there is thus no chance of error from the fault changing its resistance between two independent observations, as might occur in the other tests.

It has been assumed that this test has been made with Thomson galvanometers, and it is advisable if possible to employ them; the directing magnets in the instruments would, however, have to be placed very low down and very low shunts

employed, otherwise the deflections obtained would be beyond the range of the scale.

It will sometimes be found that the cable is traversed by an earth current. The effects of this may be eliminated by means of a compensating battery of one or two *large*-sized *Daniell* cells, inserted between the end of the cable and the galvanometer. The number of these cells used should be slightly in excess of that required to counteract the earth current, exact balance being obtained by means of a shunt inserted between the terminals of the battery. To effect this adjustment, previous to putting on the battery E, we should connect the galvanometer to earth, and then adjust the compensating battery shunt until no deflection is obtained. This being done, the battery E is connected up and the test made as if no earth current existed.

It will seldom be found that a larger compensating battery than one or two cells is required to produce a balance, and if these be of a large size their internal resistance may practically be ignored.

It is advisable to make the current from the testing battery flow in the same direction as the current which tends to flow from the compensating battery; thus, if the latter requires to be inserted, so that the zinc pole is connected to one terminal of G_1 and the copper pole to the end A of the cable, then the copper pole of the testing battery should be connected to the second terminal of G_1 and the zinc pole to earth.

Best Conditions for making the Test.

The resistances of the battery E and galvanometers G and G_1 should be as low as possible.

THE LOOP TEST.

250. When a faulty cable is lying in the tanks at a factory so that both ends of it are at hand, or when a submerged cable can be looped at the end farthest from the testing station with either a second wire, if it contains more than one wire, or with a second cable which may be lying parallel with it, as is often the case, then the simplest and most accurate test for localising the position of the fault is the loop test.

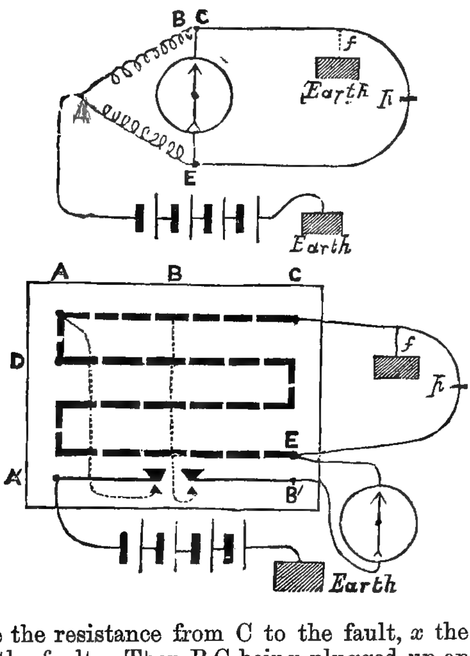
This test is independent (within certain limits) of the resistance of the fault, thus doing away with the necessity of cleaning and depolarising as would be necessary in the ordinary tests.

There are two ways of making this test with the form of apparatus hitherto described.

MURRAY'S METHOD.

251. Fig. 63 shows the theoretical and practical arrangements. p is the point where the two wires or cables are looped together at the farther station, f being the fault.

FIG. 63.



Let y be the resistance from C to the fault, x the resistance from E to the fault. Then B C being plugged up and A B and E A adjusted until equilibrium is produced,

$$\mathrm{A\,B} \times x = \mathrm{E\,A} \times y.$$

Let L be the total conductivity resistance of the whole loop, then

$$x + y = \mathrm{L},$$

therefore

$$x = \mathrm{L} - y.$$

Substituting this value of x in the above equation, we get

$$A\,B\,(L - y) = E\,A \times y,$$

from which

$$y = L\left(\frac{A\,B}{A\,B + E\,A}\right).$$

To obtain L, we should simply join up for the ordinary conductivity test, as shown by Fig. 51 (page 169). The fault in this case has no effect upon the test, provided it is not caused by the complete fracture of the cable; in the latter case the broken ends become covered with salts, which would make the resistance appear higher than it really is. When, however, the fault is due to a simple imperfection in the insulating sheathing, the ordinary conductivity test gives the correct result.

252. It is advisable to keep a record of the conductivity resistance, so that it can be ascertained without the necessity of making a measurement.

253. In the practical execution of this loop test, the connections being made as shown by the figure, all the plugs between B and C must be inserted; this is necessary, because the galvanometer connection is made on to the terminal B′, which is the same as B, instead of on to C. The test could be made by placing the galvanometer on to C, but in that case we should lose the advantage of the key, which it is always best to use.

The plugs being inserted between B and C, and the other plugs being in their places, we should remove, say, the 1000 plug from between A and B, and having pressed down the left-hand key, to put the battery current on, which should be a *zinc* (or negative) one as shown, we should adjust the plugs between D and E, pressing down the right-hand key as required until equilibrium is produced. The different resistances being inserted in the formula, y is found, which being divided by the conductivity resistance per mile of the cable, gives the position of the fault.

For example.

A cable 50 miles long, whose total conductivity resistance was 450 ohms, that is, 9 ohms per mile, was looped with a second cable, which had the same length and conductivity resistance as the first cable—the resistance of the loop being $450 \times 2 = 900$ ohms. The adjusted resistance in E A to obtain equilibrium was 4000 ohms, A B being 1000 ohms, then

$$y = 900\left(\frac{1000}{1000 + 4000}\right) = 180 \text{ ohms.}$$

Dividing this by the conductivity per mile, which is 9 ohms, we get distance of fault from testing station $= \frac{180}{9} = 20$ miles.

In making a test of this kind it is advisable to use as high resistances as possible in A B and E A, because the greater these resistances are the greater will be the range of adjustment.

254. We know that the best galvanometer to employ would be one whose resistance does not exceed 10 times the joint resistance of the resistances on either side of it.* In practice, the resistances A B and E A would always be greater than the resistance of the looped cables, and the joint resistance of the two resistances would consequently never be more than one-half the resistance of the looped cables; if, therefore, we do not use a galvanometer with a resistance more than, say, five times the resistance of the looped cables, we may be sure that the conditions are very favourable for making an accurate test.

The value which E A should have depends upon the value given to A B, and since the range of adjustment is large in proportion as E A is large, therefore for this reason it is advantageous to make A B as large as possible; but it is not advisable to make it higher than is requisite to obtain what may be considered to be a sufficient range of adjustment, for by making A B and E A large the current which passes out of the battery becomes diminished, and consequently the effect on the galvanometer will also be diminished. This can of course be compensated for by adding on extra batteries, but as the number of the latter may have to be inconveniently large, it is as well to avoid doing so, otherwise there is no limit to the values which may be given to A B and E A.

It is possible to avoid making A B and E A high by making the latter resistance adjustable to a fraction of a unit.

If the fault has a very high resistance the employment of high battery power is inevitable, as this high resistance is directly in circuit with the battery. In such a case, however, we may make A B and E A as high as we like, for, inasmuch as the current flowing out of the battery depends upon the *total* resistance in its circuit, the result of making A B and E A high is to add but very little to the *total* resistance, unless indeed they are very excessive, which in practice can hardly be the case.

To sum up, then, we have

Best Conditions for making Murray's Loop Test.

255. Make A B as high as is necessary to obtain the required range of adjustment in E A; if A B and E A would in this case

* Chapter XXV.

require to be excessive compared with the resistance of the loop, E A must be adjustable to a fraction of a unit.

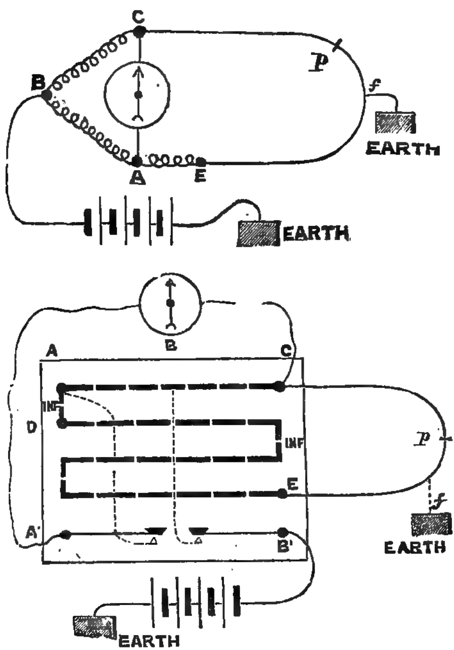
Employ a galvanometer whose resistance is not more than about five times the resistance of the looped cables.

Employ sufficient battery power to obtain a perceptible deflection of the galvanometer needle when E A is 1 unit, or a fraction of a unit, out of exact adjustment.

Varley's Method.

256. This is shown theoretically and practically by Fig. 64.

Fig. 64.



In this test, B C and A B are fixed resistances, and E A is adjusted until equilibrium is produced. Then, y and x being the resistances of the fault from E and C respectively,

$$B\,C\,(E\,A + y) = A\;B\,x,$$

and

$$x = \mathrm{L} - y;$$

therefore

$$\mathrm{B\,C}\,(\mathrm{E\,A} + y) = \mathrm{A\,B}\,(\mathrm{L} - y),$$

from which

$$y = \frac{(\mathrm{A\,B} \times \mathrm{L}) - (\mathrm{B\,C} \times \mathrm{E\,A})}{\mathrm{A\,B} + \mathrm{B\,C}}.$$

If A B = B C, then

$$y = \frac{\mathrm{L} - \mathrm{E\,A}}{2}.$$

For example.

The two cables being of the same length and conductivity as in the last example, and A B being equal to B C, equilibrium was obtained by making E A = 600; then

$$y = \frac{900 - 600}{2} = 150 \text{ ohms}.$$

257. It is necessary that the faulty one of the two looped cables be attached to E, or else it would be impossible to obtain equilibrium. If we were testing a looped cable, and after having joined it up we found that we could not obtain equilibrium, we may be sure that the fault lies between C and p. The cable must then be reversed, and a fresh test made.

258. The conditions for making this test with accuracy are not quite so simple as they were in Murray's test. In this case they are almost precisely similar to what they are in an ordinary bridge test, for the resistance $\mathrm{E\,A} + y$ takes the place of the resistance d in the latter test, and if we determine the best conditions for finding x we practically determine the best conditions for finding y, as the test is made in the same manner for determining either quantity.

It is, however, always best to have the relative positions of the battery and galvanometer as indicated in the figure. For if the galvanometer took the place of the battery, and *vice versâ*, it would be affected by any earth or polarisation currents which might enter at the fault, and this would render adjustment difficult. We have, then,

Best Conditions for making Varley's Loop Test.

259. Make B C as low as possible, but not lower than $\dfrac{g\,x}{g + x}$. Make A B of such a high value that E A when 1 unit out from

exact adjustment produces a perceptible movement of the galvanometer needle.

A rough test would first have to be made to ascertain approximately the values of x and y, and then if necessary the resistances must be readjusted so that the above conditions are satisfied, and then exact adjustment of E A be made.

Best General Conditions for making the Loop Test.

260. Although the loop test avoids errors due to *earth* currents it does not avoid errors due to *cable* currents, that is to say, currents set up by chemical action at the fault itself; this action causes a current to flow in *opposite* directions through the branches of the cable on either side of the fault, in other words, it causes a current to *circulate* in the loop. This current, although comparatively weak, yet is sufficient to cause errors which it is advisable to avoid if possible. Mr. A. Jamieson states that by balancing to a "false zero" (page 210) the above cause of error may be eliminated and a very considerable increase in the accuracy of the test be obtained.

Correction for the Loop Test.

261. It sometimes happens that the resistance of the fault in a cable approaches the normal insulation resistance of the latter; then the position of the fault indicated by the loop test will not be its true position. The reason of this is, that the current flowing in a faulty cable has two paths open to it: one through the fault and the other through the whole of the insulated sheathing. The cable, in fact, possesses two faults: the actual fault, and the fault due to the conducting power of the insulating sheathing. This second or *resultant* fault, as it is called, in a homogeneous cable is equivalent to a fault in the centre of the cable of a resistance equal to the insulation resistance of the cable itself when in good condition. If the cable is not homogeneous throughout, this resultant fault will lie away from the centre. Its position can be found, however, by the ordinary loop test when the cable is sound.

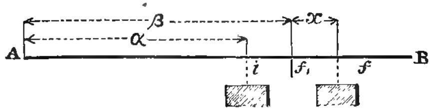
We have then to determine the true position of the fault when the position and resistance of the resultant fault, the insulation resistance of the cable when imperfect, and the position of the fault indicated by the ordinary loop test, are known. The following shows how this may be done approximately:—

In Fig. 65 let A B be the cable joined up for the loop test,

f being the actual fault, i the resultant fault, and f_1 the apparent position of the fault given by the loop test.

Let P equal the resistance of i, that is, the insulation resistance of the cable when perfect; also let I equal the insulation

FIG. 65.



resistance when the cable has a fault, which resistance is due to the joint resistances, of the fault (which we will call c) and the insulation P; then

$$\mathrm{I} = \frac{\mathrm{P}\,c}{\mathrm{P}+c};\ \text{whence } c = \frac{\mathrm{P\,I}}{\mathrm{P}-\mathrm{I}}.$$

Now it is evident that the position of f_1 with respect to i and f will depend upon the relative values of P and c: thus if P and c were equal, then f_1 would lie midway between i and f; if P were greater than c, then f_1 would be nearer f; or again, if P were less than c, then f_1 would be nearer i. This being the case, we have the proportion

$$\mathrm{P} : \begin{pmatrix}\text{distance between} \\ f_1 \text{ and } i\end{pmatrix} :: c : \begin{pmatrix}\text{distance between} \\ f_1 \text{ and } f\end{pmatrix}.$$

Let distance A $f_1 = \beta$ and A $i = a$, therefore distance $i\,f_1 = \beta - a$; also let distance $f_1\,f = x$, then

$$\mathrm{P}\,x = c\,(\beta - a),$$

or

$$\mathrm{P}\,x = \frac{\mathrm{P\,I}}{\mathrm{P}-\mathrm{I}}\,(\beta - a);$$

therefore

$$x = \frac{\mathrm{I}}{\mathrm{P}-\mathrm{I}}\,(\beta - a),$$

which gives us the position of the true fault beyond the apparent one.

Or the distance of the fault from A will be

$$\beta + \frac{\mathrm{I}}{\mathrm{P}-\mathrm{I}}\,(\beta - a) = \frac{\beta\,\mathrm{P} - a\,\mathrm{I}}{\mathrm{P}-\mathrm{I}}.$$

For example.

In a looped cable, whose total length was 100 miles, and total conductivity resistance 900 ohms, the ordinary loop test showed the apparent position of a fault which existed in it to be 700 ohms from A, that is,

$$\beta = 700.$$

The position of the resultant fault given by the loop test when the cable was new was found to be 500 ohms from A, that is,

$$\alpha = 500.$$

The insulation resistance of the cable when new was 3,000,000 ohms, and when faulty 600,000 ohms, that is,

$$P = 3,000,000.$$
$$I = \quad 600,000.$$

Where was the true position of the fault?

Distance of fault from A

$$= \frac{(700 \times 3,000,000) - (500 \times 600,000)}{3,000,000 - 600,000} = 750 \text{ ohms};$$

that is to say, distance of fault beyond distance given by loop test was

$$750 - \beta = 50 \text{ ohms}.$$

Or, supposing the cable to have a resistance of 9 ohms per mile, the true distance of the fault beyond the apparent distance was $\frac{50}{9}$, or $5\frac{5}{9}$, miles.

If the cable be homogeneous throughout, the resultant fault will appear in the middle of it. In this case α will equal $\frac{L}{2}$, where L is the total length of the loop.

If in the equation,

$$\text{Distance of fault from A} = \frac{\beta P - \alpha I}{P - I},$$

we put $P = I$, or, what is the same thing, $\alpha = \beta$, then

$$\text{Distance of fault from A} = \beta,$$

as in the ordinary loop test.

262. In order to make this test satisfactorily, it is necessary to know what are the insulation resistances of the cable when

good and also when faulty, at the moment when equilibrium is obtained. Now, as will be shown in Chapter XV., the insulation resistance (P) of a sound cable alters in proportion to the time a current is kept on it; but the rate at which this alteration takes place is definite, and can be obtained by reference to previous tests of the cable made when the latter was sound. The insulation resistance (I) of the cable when faulty cannot, however, be determined by any reference to previous tests; some plan of enabling it to be measured accurately is therefore necessary.

A method suggested by Mr. S. A. Phillips enables this to be done in a very satisfactory manner. The whole of the testing apparatus is carefully insulated by being placed on a sheet of ebonite, or on insulated supports; the experimenter also stands on an insulated stand or a sheet of ebonite. The battery for making the loop test, instead of being connected directly on to the terminal of the resistance coils, is connected thereto through the medium of a second galvanometer. By noting the deflection on the latter at the moment equilibrium is obtained on the first galvanometer, and comparing it afterwards with the deflection obtained through a known resistance, we obtain the value of I plus the combined resistance of the resistances in the bridge, which quantity will, however, be insignificant compared with I, and need not be taken into account.

A note should be made of the time at which the battery is connected to the instruments, and then, when the plugs are adjusted, equilibrium obtained, and the deflection on the second galvanometer observed, the time must again be noted, so that the period during which the battery current has acted may be known and the value of P correctly obtained.

The method of determining the value of P will be considered hereafter.

Individual Resistance of Two Wires by the Loop Test.

263. Mr. S. A. Phillips has pointed out that the loop test may be made very useful for determining the individual resistance of two wires, the leads in a cable factory, for instance, whose ends cannot be got at to connect to the testing apparatus.

To do this, the further ends of the leads would be joined together, and the junction put to earth. It is evident, then, that the loop test applied to the wires would give the resistance of either of them to their junction.

CHAPTER X.

KEYS, SWITCHES, CONDENSERS, AND BATTERIES.

SHORT CIRCUIT KEYS.

264. Although the short-circuit plug-hole is convenient to avoid accidental currents being sent through the galvanometer when the various resistance coils, batteries, &c., are being joined up for making a measurement, yet a key which in its normal condition short circuits the galvanometer, is extremely convenient and useful.

Such a key is represented by Fig. 66. In its normal condition the spring rests against a platinum contact, and, when pressed down, against an ebonite one.

FIG. 66.

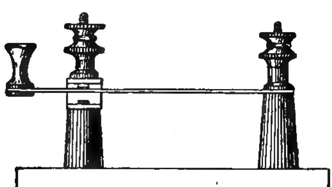


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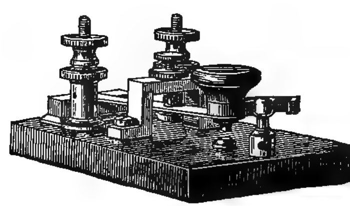
FIG. 67.



The two terminals of the shunt are connected to the terminal of the key, which in this and most keys are double, so as to enable the wires leading to the resistance coils, batteries, &c., to be conveniently connected to them.

If it is required to keep the key pressed down for a lengthened period, a small piece of sheet ebonite or gutta-percha can be slipped in between the contacts, so as to prevent their making connection when the finger is taken off the key. Some keys of this kind are provided with a catch (Fig. 67), which keeps the spring down when it is depressed.

The advantage of the short-circuit key over the short-circuit plug may not seem obvious, but actual practice will soon show its value.

Reversing Keys.

265. Besides the short-circuit key, a *Reversing Key* is usually inserted in the galvanometer circuit, so that the deflections of the needle may always be obtained on the same side of the scale. A form of reversing key very commonly used is shown

Fig. 68.

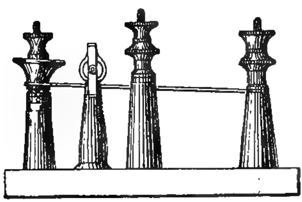
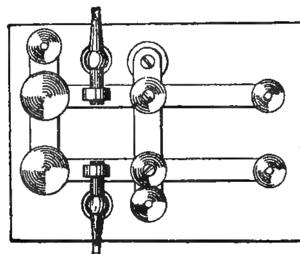


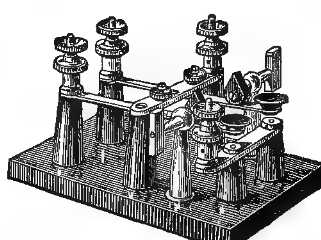
Fig. 69.



in elevation and plan by Figs. 68 and 69, and in general view by Fig. 70.

The galvanometer terminals would be connected to the two end terminals of the reversing key, or, if the short-circuit key is inserted, to the terminals of the latter. By pressing down one or other of the springs, the current will pass through the galvanometer in one direction or the other. The two handles on either side of the two springs are for the purpose of clamping either of them down when required.

Fig. 70.



Particular care should be taken, when procuring the key, to see that the terminals, &c., are not fixed on the top of the ebonite pillars by means of bolts running right through them, as in such a case the advantage of the pillars is entirely lost, and the terminals might just as well be screwed direct into the base board.

Care should also be taken that the contacts of the keys are clean, as when there are several contacts considerable resistance might be introduced into the circuit from their being dirty.

Reversing Switches.

266. In addition to the reversing key for the galvanometer, a *Reversing Switch* for the testing battery is very useful: it need not, however, be such an elaborate one as that used for the galvanometer.

Figs. 71 and 72 represent such a switch. It consists of four brass segments screwed firmly down to an ebonite base. Each segment is provided with a screw, to which to attach the testing wires.

Fig. 71.

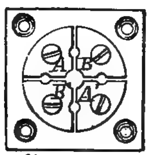


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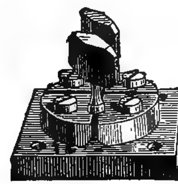
Fig. 72.



In some cases each segment is supported on an ebonite pillar, which improves its insulation very much, and, indeed, would be absolutely necessary for some tests we shall describe.

The poles of the battery would be attached to two opposite terminal screws, say A and A′, and the leading wires to the two other screws, B and B′.

To make the current flow in one direction, we should place the plugs between the segments A and B, and A′ and B′, and to make it flow in the other direction, between the segments A and B′, and A′ and B. If one or both the plugs are removed the battery current will be cut off altogether. It is always best, in order to do this, to remove both the plugs in preference to one only, for if the battery is not well insulated a portion of the current may still be able to flow out of the battery and disturb the accuracy of a test.

Two other pieces of apparatus are necessary to form a very complete set, viz. a "Condenser" and a "Discharge key."

Condensers.

267. A *Condenser* is merely a Leyden jar exposing a large surface within a small space; those constructed for testing purposes are made of sheets of tin-foil placed in layers between thin sheets of mica coated with shellac. The alternate layers of tin-foil are connected together, so that sets are formed corresponding to the outside and inside coatings of the Leyden jar.

A very convenient form of condenser, manufactured by Messrs. Warden, is shown in plan and elevation by Figs. 73 and 74.

The layers of tin-foil and mica are placed in a round brass box with an ebonite top, on which are fixed the connecting terminals. These terminals are placed on brass blocks, the ends

FIG. 73.

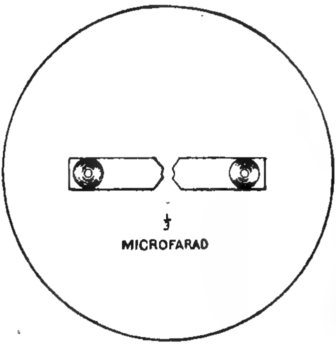
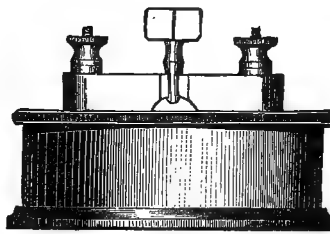
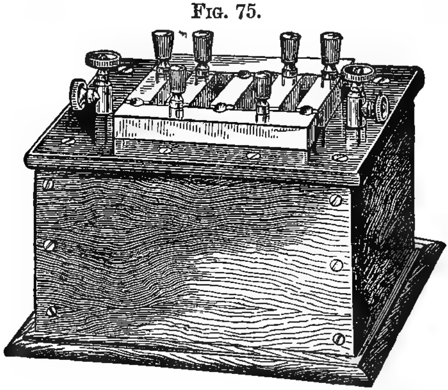


FIG. 74.



of which are in close proximity to one another, so that a plug can be inserted between them for the purpose of enabling the apparatus to be short circuited. This should always be done when the condenser is not in use, so that any residual charge which may remain in it after use may be entirely dissipated.

FIG. 75.



The "electrostatic capacity" of these condensers is usually $\frac{1}{3}$ microfarad, the "farad" being the unit of electrostatic capacity. They are also made, however, so that several capacities can be

obtained, by inserting plugs in different holes. Those having five different capacities (Fig. 75), viz. ·05, ·05, ·2, ·2, and ·5 microfarads, enable any value from ·05 to 1 to be obtained by inserting one or more plugs. It is very often extremely useful to be able to vary the capacity, so that it is better to have the last form rather than the first, although it may be a little more expensive.

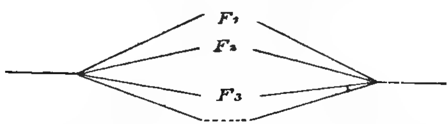
A good condenser should not lose, through leakage, more than 1 per cent. of its charge in one minute.

268. Condensers, like batteries, can be combined for "quantity" or in "series," and advantage may often be taken of this power of combination to obtain a large number of capacities from a small number of condensers.

When condensers are connected together for "quantity" the capacity of the combination will be equal to the sum of the respective capacities of the several condensers. Thus, if we call F_1, F_2, F_3, &c., these capacities, then the capacity of the combination will be

$$F_1 + F_2 + F_3 + \ldots .$$

This may be expressed symbolically thus:—



When the combination is made in "series" (corresponding to the "cascade" arrangement of Leyden jars) the joint capacity of the series follows the law of the joint resistance of parallel circuits,* thus

$$\frac{1}{\frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3} + \ldots}.$$

This may be symbolically expressed thus:—

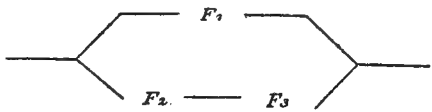
$$\text{———} \; F_1 - F_2 - F_3 \; \text{———} \; \ldots . \; \text{———}$$

By following out these laws, if we had two condensers, F_1 and F_2, we could obtain four different capacities, viz. F_1, F_2, $F_1 + F_2$, and $\frac{F_1 F_2}{F_1 + F_2}$.

* See Appendix.

With three condensers we could obtain fourteen different capacities, viz. F_1, F_2, F_3, $F_1 + F_2$, $F_1 + F_3$, $F_2 + F_3$, $F_1 + F_2 + F_3$, $\frac{F_1 F_2}{F_1 + F_2}$, $\frac{F_1 F_3}{F_1 + F_3}$, $\frac{F_2 F_3}{F_2 + F_3}$, $F_1 + \frac{F_2 F_3}{F_2 + F_3}$, $F_2 + \frac{F_1 F_3}{F_1 + F_3}$, $F_3 + \frac{F_1 F_2}{F_1 + F_2}$, and $\frac{1}{\frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3}}$.

Any of these combinations may be expressed symbolically in the manner before shown; thus, for example, to take the $F_1 + \frac{F_2 F_3}{F_2 + F_3}$ combination, this would be shown thus:—



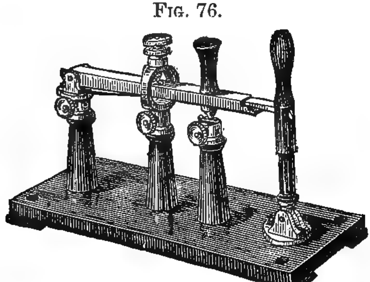
Discharge Keys.

269. To enable the discharge from the condenser to be read on a galvanometer a discharge key is necessary. This, like the other pieces of apparatus, is made in a variety of forms.

Webb's Discharge Key.

270. Fig. 76 shows a pattern (designed by Mr. F. C. Webb), which is in very general use.

Fig. 76.

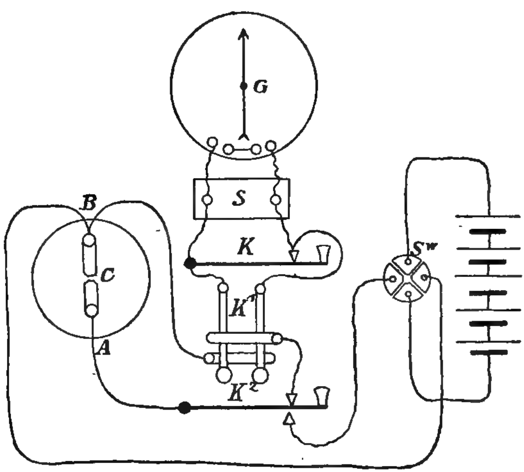


It consists, primarily, of a hinged lever of solid make, pressed upwards by a spring and playing between two contacts. A vertical ebonite lever, hinged at its lower end, is fixed to the base of the instrument in the position shown. This lever has near its upper

end a projecting brass tongue, which, when the lever is pressed forward (by means of a spring), hitches over the extremity of the brass lever. The end of the latter is cut away so as to form two steps; when the brass tongue on the vertical ebonite lever is hitched over the lower step then the brass lever stands intermediate between the top and bottom contacts, and is insulated from both of them, but when the tongue is hitched over the top step then the brass lever is in connection with the lower contact. Again, when the ebonite lever is drawn back the brass lever is freed and springs up against the top contact stop. If we suppose the brass lever to be hitched down on the lower contact stop, then by pulling back the ebonite lever a little the brass tongue unhitches from the top step and hitches on the lower one, thus allowing the brass lever to spring up from the bottom contact but not to come in connection with the upper one; if, however (as before explained), the ebonite lever be pulled completely back then the brass lever rises in connection with the top contact.

271. When using this discharge key for the purpose of measuring the charge in a condenser, the connections to the galvanometer, &c., would be made as shown by Fig. 77. On

FIG. 77.



pressing down the key K_2 the two poles of the battery are put in connection with the two terminals A and B of the condenser C, and on releasing the key so that it comes in contact with the

top contact, the two terminals of the condenser are put in connection with the two terminals of the galvanometer, which thus receives the discharge current through it.

If we so arrange the connections that the top contact of the key, instead of being joined to the condenser through the galvanometer, is connected directly to it, and the galvanometer is placed between the back terminal of the key and the second terminal of the condenser; then on pressing down the discharge key we get the current charging the condenser through the galvanometer, which has its needle deflected to one side of the zero point; and then, on releasing the key, we get the discharge deflection, which will be of the same strength as the charge deflection, but in the opposite direction to it. The first arrangement, given by the figure, is, however, the one generally employed.

The discharge deflection on the galvanometer is only momentary, the needle or spot of light immediately returning towards zero.

272. In using the Thomson galvanometer (which is practically the only instrument of any use for the purpose) for measuring the discharge, the adjusting magnet must be put high up, if it is placed with its poles assisting the earth's magnetism, or low down if it opposes it, so as to make the needle swing slowly enough to enable the deflection to be read on the scale. It is best to avoid making the needle swing very slowly, for then the spot of light will probably not return accurately to zero, but may be three or four degrees out. A little practice will enable a comparatively quick swing to be read to half a degree, or even less.

Kempe's Discharge Key.

273. A form of discharge key designed by the author is shown in plan and elevation by Figs. 78 and 79, and in general view by Fig. 80.

It consists, like Fig. 76, of a solid lever, hinged at one end, and playing between two contacts attached to two terminals. Two finger triggers, near the other end of the lever, marked "Discharge" and "Insulate," are connected to two ebonite hooks. The height of the hook attached to the finger trigger marked "Discharge" is a little higher than the other hook, so that the lever stands intermediate between the two contacts when it is hitched against it. When the lever is pressed down against the bottom contact, the shorter of the hooks hitches it down. If in this position we depress the "Insulate" trigger, the lever is freed from its hook, and springs up against the

second hook, thus insulating the lever from either of the contacts. The "Discharge" trigger now being pressed down, the lever springs up against the top contact.

FIG. 78.

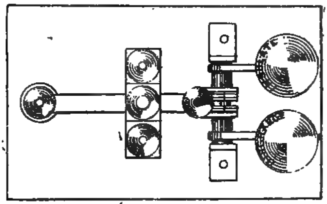


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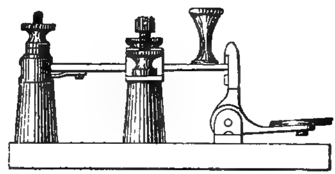
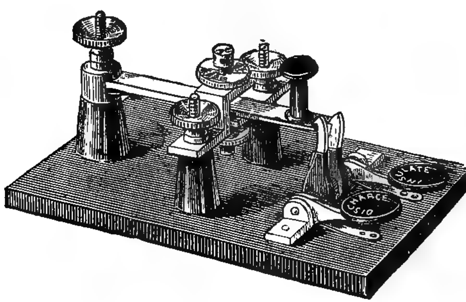
FIG. 79.



To the hook of the "Discharge" trigger there is a small piece of metal fixed which is broad enough to come in front of the second hook, so that if the "Discharge" trigger is depressed first it draws back both the hooks, and thereby, if the

FIG. 80.



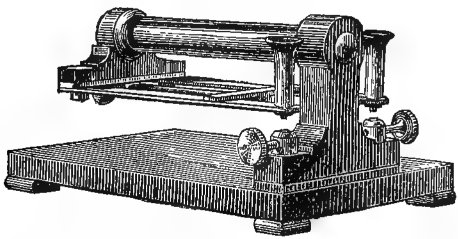
lever at starting be hitched to the bottom contact, allows the lever at once to spring up to the top contact. If, however, the "Insulate" trigger be depressed, only the hook attached to that trigger is drawn back, allowing the lever to spring up against the second hook and be thereby insulated, as at first explained.

Lambert's Discharge Key.

274. The arrangement of discharge key designed by Mr. Lambert and shown by Fig. 81, is a very good one, and possesses the advantage that the principal terminal is highly

insulated when the key is in its normal condition, a point of importance in some tests. The two terminals seen at the front part of the key correspond to the top and bottom contacts of the keys previously described. The ends of two spring levers, provided with ebonite finger-knobs, are set over

Fig. 81.



these contacts; the other ends of the springs are fixed to a brass cross-piece provided with a terminal, the cross-piece being secured to an ebonite bracket fixed at the end of a stout ebonite rod. By this arrangement the terminal connected to the spring levers is insulated by the long ebonite rod as well as by the ebonite bracket by which the rod is supported on the stand. In manipulating the key, the left-hand lever, say, is first depressed, thus putting the back terminal in connection with the contact (corresponding to the *bottom* contact of the other forms of keys) beneath it. This lever is then released, and the right-hand lever depressed, thus putting the back terminal in connection with the contact (corresponding to the *top* contact of the other keys) beneath it. The only objection to this form of key is the fact that it is possible to press both levers down at once, thus connecting together the back and the two front terminals; if this is done accidentally, then, as will be seen by reference to Fig. 77, a direct circuit is formed by the battery through the galvanometer, which may result in the sensibility of the latter being altered through the violence of the deflection. Such an accident obviously cannot possibly occur in the other forms of keys.

Rymer Jones's Discharge Key.

275. An excellent form of discharge key has been devised by Mr. J. Rymer Jones, and is manufactured by the Silvertown Telegraph Works Company. The key is so constructed that (like Lambert's key) the principal terminal is left perfectly free

during the period of "insulation," as shown in Fig. 82. The leakage from this terminal is therefore confined to the ebonite support A B. The form of this support, a vertical section of which is shown by Fig. 83 (page 246), gives a very considerable length of surface over which any leakage must pass, it being in the present case $6\frac{3}{4}$ inches in a height of only $2\frac{1}{2}$ inches; while, since the portion A screws into the outer cap B, the former may be removed, when important tests are about to be made, and scoured with glass-paper, so as to secure the advantage of a fresh surface without disfiguring the outer polished surface.

The movements for "Charge," "Insulate," and "Discharge," will be readily understood from Fig. 82. l l' are ebonite rods; their brass prolongations c c', which move with them as one piece, have the under surfaces, where they rub against the platinum contacts b and g, tipped with platinum.

When l is deflected to the left, the end of the rod r, attached to it, presses against l'—should the latter happen to be turned to that side—and carries it over in the same direction, first breaking contract at c' g, if previously made, and afterwards making contact at b c. Thus the "Battery" and "Cable" terminals are connected together. To "Insulate" the cable terminal it is only necessary to move l back again towards the right, as in Fig. 82. To "Discharge," press l' towards the right. Should l not already be over to the right (as in the last position for "Insulate") it will be carried over with l' and the contact at b c broken before c' and g come together. The rod r in fact prevents the galvanometer and battery terminals from both being put to the cable terminal at the same time.

276. Although not perhaps absolutely necessary, it is advisable to have a second set of resistance coils (which need not, however, be of the bridge form) to act as an adjustable shunt for the galvanometer.

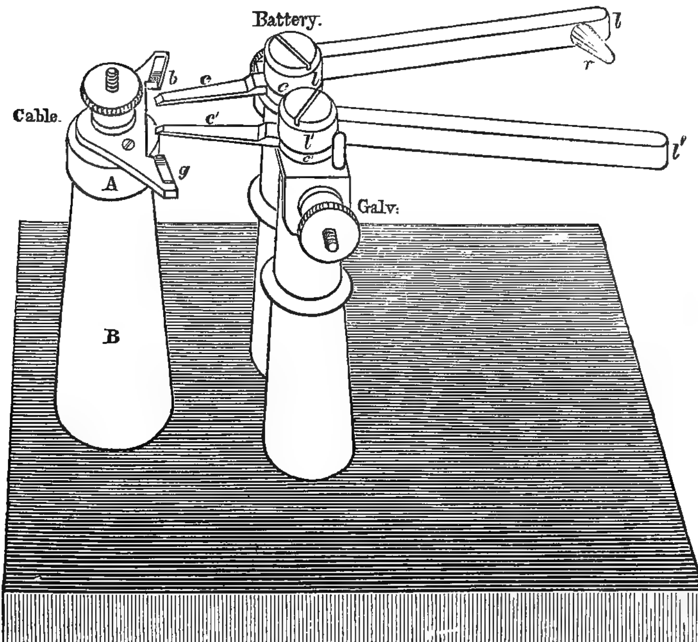
277. A simple form of galvanometer to enable the resistance of the Thomson to be quickly taken, is also useful. This, however, can be dispensed with, as Mr. S. A. Phillips has pointed out that the resistance of the galvanometer can be determined by the very simple device of measuring the resistance of one of the shunts (the $\frac{1}{9}$th preferably). To do this, the shunts will have to be removed from the galvanometer and connected up to the bridge as an ordinary resistance, the galvanometer itself being used in the ordinary manner.

Mr. Phillips suggests that the shunts should be enclosed under the glass shade, so that they may have the same temperature as the galvanometer coils.

As it is preferable to use a set of resistance coils as a shunt, a

single resistance coil of the same wire and resistance as the galvanometer coils, might be permanently fixed to the galvanometer stand under the glass shade; the resistance of this, measured by the help of the galvanometer, would at once give the resistance of the latter. If such a device were adopted, care

FIG. 82.



would have to be taken that the coil is wound double on its bobbin, for otherwise it would affect the galvanometer needle when traversed by a current.

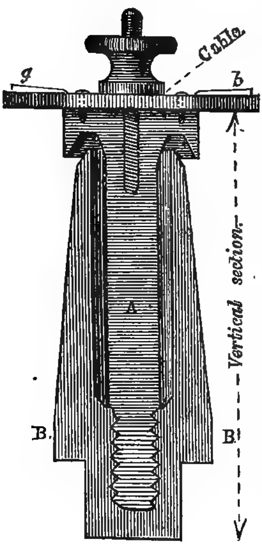
278. The form of bridge coils most generally employed with the Thomson galvanometer is that shown by Figs. 4 and 5 (page 12), the keys attached to the other form not being used.

BATTERIES.

279. Besides the foregoing instruments, a battery of at least 200 cells is necessary. The form known as the Minotto is a convenient one, and is frequently used for testing. It consists of

an earthenware (or more frequently of a gutta-percha) jar, about 8 inches high, at the bottom of which is placed a round plate of copper, resting flat. A strip of copper about three-quarters of an inch wide, coated with gutta-percha, is fixed to this plate, and brought up the side of the jar. Over this plate a layer of coarsely powdered sulphate of copper is placed; the jar is then filled nearly to the top with damp sawdust, and resting on this is placed a thick disc of zinc, provided with a terminal at the top. A series of these cells is coupled up in the ordinary manner.

FIG. 83.



Length of insulating surface 6¾ inches.

The Leclanché battery is used at some cable factories; it has the advantage of high electromotive force, but is not so constant as the Daniell, though if care is taken that it does not become accidentally short circuited through a low resistance it answers very satisfactorily, and requires but little attention. The Chloride of silver battery of Mr. Warren de la Rue (page 121) is also now used to some extent for testing, especially on board ship, as it has the advantage of great compactness and portability.

The batteries should be placed on well-insulated supports, in a dry situation, so as to avoid leakage, which interferes with the constancy of the current.

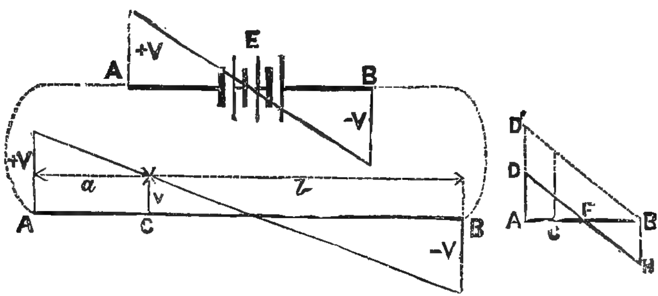
280. Besides the large battery, a single cell placed in a small box, with appropriate terminals outside, is required, whose use will be explained.

CHAPTER XI.

MEASUREMENT OF POTENTIALS.

281. Let E (Fig. 84) be a battery of which A and B are the poles; then the free electricities at those poles, supposing the latter to be free, will have equal but opposite potentials, and the difference of these potentials is the electromotive force

FIG. 84.



of the battery. Thus, if V be the potential at A, then $-$ V will be the potential at B, and the electromotive force E of the battery will be

$$E = V - (-V) = 2V.$$

Although the expression "potentials of the *free* electricities" is, strictly speaking, more correct than "potentials" simply, yet the latter is generally used as an abbreviation of the former, and we shall so use it unless the contrary is indicated.

The potentials diminish regularly from one pole of the battery to the other, the potential at the middle of the battery being zero.

282. If the two poles be connected by a resistance A C B, the potentials will diminish regularly along A C B also, as shown in the figure, the potential at the middle being zero as in the case of the battery. But the potentials at A and B will not

be the same as they were previous to the joining of the poles by A C B.

283. Now, if the two terminals of a condenser be connected to any two points in the circuit, the electromotive force of the charge which the condenser will take will be directly proportional to the difference of the potentials, that is the electromotive force, at those two points. Thus if the condenser were connected to A and B the charge it would take would have an electromotive force, E, of

$$E = V - (-V) = 2V.$$

If the points to which the condenser was connected were A and C, the electromotive force, E_1, of the charge would be

$$E_1 = V - v.$$

Again, if the condenser were connected to C and B, the electromotive force, E_2, of the charge would be

$$E_2 = v - (-V) = V + v.$$

It is easy to see that

$$V - v : V + v :: a : b.$$

If, therefore, we connect two condensers between the points A and C and the points B and C respectively, and adjust the resistances a and b, we could charge the condensers to any relative electromotive forces we please.

284. Although, strictly speaking, the diminution or fall of potential along the resistance A C B is represented by the line D F H (see small figure), the zero point being at F, yet we may generally with perfect correctness assume the zero to be at B and the fall of potential to be represented by the line D′ B, and similarly with the fall from one pole of the battery to the other. In most cases it is convenient to consider the fall as taking place in this way, as we avoid having to consider the potentials as partly + and partly − quantities, which is liable to cause confusion in making calculations.

285. We stated that if the poles of the battery were joined by a resistance, the potentials at those poles will be altered in value; they will, in fact, be reduced in proportion as the resistance is small or large. Now, when a current flows through the galvanometer, it does so in virtue of a difference of potential at its two terminals, and the strength of this current is directly proportional to the value of this difference; conversely, therefore, if we note the difference in the strengths of currents passing through a galvanometer we shall know the

relative values of the differences of potential at its terminals. It may at first sight, therefore, appear sufficient, in order to measure the relative value of the differences, simply to connect the terminals of a galvanometer to the points at which the differences are to be noted, and then to observe the deflections obtained. But by connecting up a galvanometer in this way we should reduce the resistance of the portion of the circuit between its terminals, and the potentials at the poles of the battery would decrease, and therefore the potentials at the points where the galvanometer is connected would decrease also; the current then which would produce a deflection of the galvanometer needle, would be that due to the diminished potentials. If, however, the resistance of the galvanometer be very high compared with the resistances with which it is connected, then its introduction will produce no diminution in the potentials, and consequently its deflection, that is to say, the current passing through it, will be a true index of the value of the difference. If, therefore, we wish to theoretically consider what are the relative differences of potentials at any points in any particular arrangements of batteries and resistances, we have simply to suppose these points to be connected by a galvanometer whose resistance is infinite compared with the other resistances, and then to determine the relative values of the currents which will flow through it in the several cases.

286. From what has been said we can see that practically, if we connect a galvanometer to any two points at which a difference of potential exists, then the deflection obtained will accurately represent that difference of potential, provided the galvanometer has a total resistance in its current very much greater than the resistance between the two points in question.

287. The quantity of electricity in a condenser depends directly upon the electromotive force of the charge, and the deflection obtained upon a galvanometer depends directly upon the quantity discharged through it; the discharge deflection obtained from a condenser, therefore (§ 271, page 240), other things being constant, will represent the electromotive force of the charge in it. It may be mentioned that this is only true if the discharge takes place through a comparatively low resistance, such as would be met with in an ordinary galvanometer, for then the whole discharge practically takes place instantaneously; if, however, the discharge is effected through a very high resistance, such as a megohm (1,000,000 ohms) or more, then the discharge is gradual, and the deflection which would be obtained on the galvanometer would not be an accurate index of the electromotive force of the charge in the condenser.

Measurement of Electromotive Force by Law's Method.

288. The electromotive force of a battery is the difference of the potentials at its poles, when those poles are free (§ 281); by successively charging a condenser, therefore, from two or more batteries, and noting the discharges on a galvanometer by the method described in (§ 271, page 240), we can very simply and quickly determine their comparative electromotive forces.

Discharge deflections on a galvanometer whose deflections are truly proportional to *constant* currents, unless they are nearly equal, are not always proportional to the currents which produce them. It is therefore very desirable, in measurements such as these, in order to ensure accuracy, to adopt the method we mentioned on page 58 (§ 61), viz. to obtain a uniform deflection by means of a variable shunt to the galvanometer. Thus, if we obtain two similar discharge deflections with two electromotive forces E_1 and E_2, using shunts of the respective resistances S_1 and S_2; then, since the deflections are the same, the electromotive forces are in the proportion

$$E_1 : E_2 :: \frac{G + S_1}{S_1} : \frac{G + S_2}{S_2},$$

or as the multiplying power of the shunts, G being the resistance of the galvanometer; for if we multiplied the deflections we obtained, by these quantities, we should get the theoretical deflections we should have had if no shunts had been used.

For example.

With an electromotive force E_1 we obtained a discharge deflection of 300 divisions on a galvanometer whose resistance G was 5000 ohms, using a shunt, S_1, of 1000 ohms, and with a second electromotive force, E_2, also a deflection of 300 divisions, using a shunt, S_2, of 2500 ohms; then

$$E_1 : E_2 :: \frac{5000 + 1000}{1000} : \frac{5000 + 2500}{2500},$$

that is,

$$E_1 : E_2 :: 2 : 1.$$

It is not absolutely necessary that the same deflection be reproduced exactly, although calculation is saved by so doing; as long as the deflections are nearly equal they approximately represent the discharges. It is necessary, of course, that these deflections be multiplied by $\frac{G + S}{S}$ to obtain the relative strengths of the currents.

For example.

With an electromotive force E_1 we obtain a discharge deflection of 300 divisions on a galvanometer whose resistance G was 5000 ohms, using a shunt, S_1, of 1000 ohms, and with a second electromotive force E_2 a deflection of 292 divisions, using a shunt, S_2, of 2400 ohms; then

$$E_1 : E_2 :: \frac{5000 + 1000}{1000} \times 300 : \frac{5000 + 2400}{2400} \times 292;$$

that is,

$$E_1 : E_2 :: 1800 : 900 \cdot 33,$$

or as

$$2 : 1 \text{ very nearly.}$$

This method is very often the best one to employ, not only for discharge, but also for constant deflections, as it is sometimes inconvenient to have to continually adjust until the same deflection exactly is reproduced. In certain cases, indeed, it would be impossible to do so, as will be seen hereafter.

289. It may be here mentioned that, in the case of discharge deflections, the fact that the resistance between the terminals of the galvanometer is varied by the introduction of shunts of different values, does not require to be taken into consideration.

Correction for Discharge Deflections.

290. Mr. Latimer Clark, in a communication addressed to the Society of Telegraph Engineers,* points out an error caused by the use of shunts in measuring discharge deflections.

It was found that if a certain discharge deflection was obtained with a shunt, then on removing the latter the discharge deflection obtained was larger than that given by multiplying the original deflection by $\frac{G + S}{S}$.

After considerable research, the cause of the error was traced to the inductive action of the galvanometer needle on its coils. The movement of this needle set up a slight current, which opposed the discharge current, and consequently reduced its effect. This effect being more marked when the shunt was used, made the discharge deflection without the shunt to appear larger than it should.

The formula for finding what would be the discharge deflection obtained on the removal of the shunt, the discharge

* 'Journal of the Society of Telegraph Engineers,' Vol. ii. page 16.

deflection without the shunt being given, may be thus arrived at :—

First suppose the shunt to be inserted.

Now, in all problems in which a current from a condenser has to be considered, we may suppose the condenser to be a battery with a resistance infinitely great compared with the resistances external to it.

Let E be the electromotive force of the charge, R the resistance of the condenser circuit, G the resistance of the galvanometer, S the resistance of the shunt.

Let the movement of the needle generate an opposing electromotive force e; then calling C, α, and β the respective current strengths in the galvanometer, condenser, and shunt circuits, we get the following equations :

$$\alpha - C - \beta = 0, \text{ or, } \beta = \alpha - C$$
$$\alpha R + C G - E + e = 0$$
$$\alpha R + \beta S - E = 0;$$

therefore

$$\alpha R + C G - E + e = 0$$
$$\alpha R + (\alpha - C) S - E = 0;$$

therefore

$$\alpha R = E - C G - e$$
$$\alpha (R + S) = E + C S;$$

then by division

$$\frac{R}{R + S} = \frac{E - C G - e}{E + C S};$$

by multiplication and changing the signs we get

$$(R + S)(C G + e) - R E - S E = - R E - C R S;$$

therefore

$$(R + S)(C G + e) - S(E - C R) = 0.$$

Next suppose the shunt to be removed, and let the strength of the current be C_1, and the new electromotive force generated by the movement of the needle be e_1, then

$$C_1 = \frac{E - e_1}{R + G}; \text{ therefore } E = C_1 (R + G) + e_1.$$

Substituting this value of E in the last equation, we get

$$(R + S)(C G + e) - S(C_1 (R + G) + e_1 - C R) = 0.$$

Now e and e_1 will be proportional to the deflections of the needle, that is to say, to the strengths of current producing those deflections. They will also be proportional to the strength of the magnetism of the needle, which we will represent by κ.

Then

$$e = \kappa \, C, \qquad e_1 = \kappa \, C_1.$$

Substituting these values we get

$$(R + S) \, C \, (G + \kappa) - S \, (C_1 \, (R + G + \kappa) - C \, R) = 0,$$

or

$$\left(1 + \frac{S}{R}\right) C \, (G + \kappa) - S \left(C_1 \left(1 + \frac{G + \kappa}{R}\right) - C\right) = 0.$$

Now, R is to be infinite as compared with S and G; therefore by putting $R = \infty$, we find that

$$C \, (G + \kappa) - S \, (C_1 - C) = 0,$$

therefore

$$C_1 = C \left(\frac{G + \kappa + S}{S}\right).$$

To make this formula useful, we must determine the value of κ. This can easily be done thus:

Provide two condensers, one having exactly twice the capacity of the other. Charge the larger one with a sufficient battery power to obtain a discharge deflection (a_1) of, say, 200 divisions on the scale, with a shunt inserted equal in resistance to the galvanometer.

Now remove the shunt, and having charged the other condenser from the same battery, note the discharge deflection (a_2); let it be 204 divisions.

It will be seen that the deflection we should have obtained with the larger condenser and no shunt would have been $2 \, a_2$, and this is the theoretical deflection we should obtain when a_1 is multiplied by the multiplying power of the shunt corrected by the constant κ; that is to say,

$$2 \, a_2 = a_1 \left(\frac{G + \kappa + G}{G}\right);$$

therefore

$$\kappa = 2 \, G \left(\frac{a_2}{a_1} - 1\right).$$

To continue the example we have given, let us suppose G = 5000 ohms; then

$$\kappa = 2 \times 5000 \left(\frac{204}{200} - 1\right) = 200.$$

For the particular galvanometer, then, we have been considering, we say that when measuring discharge currents the multiplying power of any shunt (S) which may be used is

$$\frac{G + 200 + S}{S}.$$

Suppose we have given the observed deflection without the shunt and also the observed deflection with the shunt, and we require to know what this latter ought to be in order to give us the *true* deflection compared with the first. Let the true deflection be A; then by the ordinary formula

$$C_1 = A\left(\frac{G + S}{S}\right).$$

But when the error exists,

$$C_1 = C\left(\frac{G + \kappa + S}{S}\right).$$

From these two equations we get

$$C = A\left(\frac{G + S}{S}\right)\left(\frac{S}{G + \kappa + S}\right);$$

therefore

$$A = C\left(\frac{G + \kappa + S}{G + S}\right) = C\left(1 + \frac{\kappa}{G + S}\right),$$

or in words:

$$\text{True deflection} = \text{observed deflection}\left(1 + \frac{\kappa}{G + S}\right).$$

It should be clearly understood that this formula is to be applied to the correction of the deflection obtained *with* the shunt, the deflection *without* the shunt being considered as the index of the current from the condenser.

We may remark that this latter formula corresponds with that obtained by Mr. Charles Hockin, and given by Mr. Latimer Clark in the paper referred to.

For practical use the formula

$$C_1 = C\left(\frac{G + \kappa + S}{S}\right)$$

is the only one we should require, as by it we can at once determine from the deflection obtained *without* the shunt what the deflection *with* the shunt would be, or *vice versâ.*

THE RELATION BETWEEN THE CURRENT, THE RESISTANCE, AND THE ELECTROMOTIVE FORCE, BETWEEN TWO POINTS IN A CIRCUIT.

291. In Fig. 85 let E be a battery of electromotive force, E, and resistance, x, joined up in circuit with a resistance r, and let G be a galvanometer having a resistance very much greater

FIG. 85.

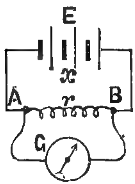
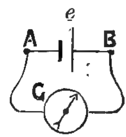


FIG. 86.



than the other resistances, so that it does not affect the flow of the current in the circuit, $x + r$. Now, the current, C_1, flowing through the galvanometer will be

$$C_1 = \frac{E}{x + \frac{r\,G}{r + G}} \times \frac{r}{r + G} = \frac{E\,r}{x\,r + x\,G + r\,G}.$$

It is evident that this current must be due to the existence of an electromotive force, or a difference of potential, in some portion of the circuit in which the galvanometer is placed; and it is, moreover, evident that this electromotive force, or difference of potential, must exist between the points A and B, in the portion of the circuit external to G. Let e be this electromotive force (Fig. 86), then we have (since G is very much larger than the other resistances)—

$$C_1 = \frac{e}{G},$$

but

$$C_1 = \frac{E\,r}{x\,r + x\,G + r\,G},$$

therefore

$$\frac{e}{G} = \frac{E\,r}{x\,r + x\,G + r\,G},$$

that is,

$$\frac{e}{r} = \frac{\mathrm{E\,G}}{x\,r + x\,\mathrm{G} + r\,\mathrm{G}} = \frac{\mathrm{E}}{\dfrac{x\,r}{\mathrm{G}} + x + r};$$

or, since G is very great compared with the other resistances,

$$\frac{x\,r}{\mathrm{G}} = 0;$$

therefore

$$\frac{e}{r} = \frac{\mathrm{E}}{x + r}.$$

But by Ohm's law (§ 2, page 1), the current C, flowing out of the battery—that is, flowing through r—is

$$\mathrm{C} = \frac{\mathrm{E}}{x + r},$$

therefore

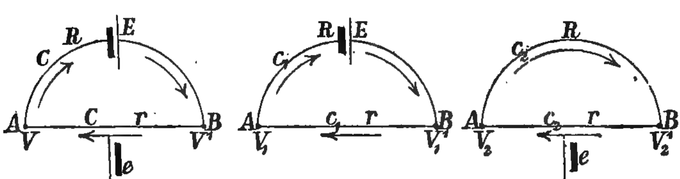
$$\mathrm{C} = \frac{e}{r}, \quad \text{or,} \quad e = \mathrm{C}\,r;$$

that is to say,

(A) *The difference of the potentials at two points in a resistance (in which no electromotive force exists) is equal to the product of the current and the resistance between the two points.*

292. We will next consider the case where an electromotive force exists in the resistance through which the current is flowing, the strength of the latter, and the potentials at the two points, being partly due to an external electromotive force (as in the case just considered.)

FIG. 87. FIG. 88. FIG. 89.



In Fig. 87 let R be a resistance between the points A and B, and let E be an electromotive force in R, also let r be another resistance completing the circuit, this resistance being either a

single one or a combination of several other resistances. Again, let us suppose there to be an electromotive force, e, in some part of r, and let C be the resultant current entering, say, at B and leaving at A.

Let us first suppose, as in Fig. 88, that there is no electromotive force in the resistance (or combined resistances) r, then by Ohm's law we have

$$c_1 = \frac{E}{R + r}, \quad \text{or,} \quad c_1 R + c_1 r = E,$$

that is,

$$c_1 r = E - c_1 R;$$

but by the law (A) we last obtained, we have

$$V_1' - V_1 = c_1 r;$$

therefore

$$V_1' - V_1 = E - c_1 R,$$

or

$$V_1 - V_1' = c_1 R - E. \qquad [1]$$

Next let us suppose, as in Fig. 89, that we have a current, c_2, caused by an electromotive force, e, in some part of the resistance (or combination of resistances), r, then we have

$$V_2 - V_2' = c_2 R. \qquad [2]$$

Now if we take the case shown in Fig. 87, where the current C is produced by the two electromotive forces, then the respective potentials at the points A and B must be

$$V = V_1 + V_2$$

and

$$V' = V_1' + V_2'.$$

Therefore we have

$$V - V' = (V_1 - V_1') + (V_2 - V_2'),$$

and by substituting the values of $V_1 - V_1'$ and $V_2 - V_2'$ given in equations [1] and [2] we get

$$V - V' = c_1 R - E + c_2 R = R(c_1 + c_2) - E;$$

but we can see that

$$C = c_1 + c_2,$$

therefore

$$V - V' = C R - E, \qquad [3]$$

which is similar to equation [1]. In the case we have taken we have supposed the electromotive forces (and consequently the currents c_1 and c_2) to act in the *same* direction, but we should have obtained an equation precisely similar to [3] had the electromotive forces *opposed* one another, provided, however, the current due to the electromotive force e were *less* than the current due to the electromotive force E. If, however, the current due to the electromotive force e were *greater* than the current due to the electromotive force E, that is to say, if the current C acted against E, then we should have

$$\mathrm{V} - \mathrm{V}' = \mathrm{C}\,\mathrm{R} + \mathrm{E}. \qquad [4]$$

293. The result, then, that we have arrived at by the foregoing investigation is, that—

(B) *The difference of the potentials at two points in a resistance in which an electromotive force exists is equal to the product of the current and the resistance between the two points, added to the electromotive force in the resistance, this electromotive force being negative if it acts with the current, and positive if it opposes it.*

This law, we have seen, holds good whether the current in question is due only to the electromotive force in the resistance, or to an external electromotive force also.

Measurement of Battery Resistance by Kempe's Method.

294. Besides determining the electromotive force of a battery, we can also determine its internal resistance with great facility by means of a condenser. To do this, first charge the condenser by means of the battery, and note the discharge deflection, which we will call α; next insert a shunt, S, between the poles of the battery; again charge and discharge the condenser, and note the new deflection, which we will call β. Let e be the electromotive force between the poles of the battery when the shunt S is inserted, and let C be the current flowing, then by law (A) (page 256), we have

$$e = \mathrm{C}\,\mathrm{S}, \quad \text{or,} \quad \mathrm{C} = \frac{e}{\mathrm{S}}.$$

Also, if E be the electromotive force of the battery, and r its resistance, we have

$$\mathrm{C} = \frac{\mathrm{E}}{\mathrm{S} + r};$$

therefore

$$\frac{e}{\mathrm{S}} = \frac{\mathrm{E}}{\mathrm{S} + r},$$

or

$$eS + er = ES;$$

therefore

$$er = S(E - e),$$

or

$$r = S\frac{E - e}{e};$$

but we must also have

$$E : e :: \alpha : \beta;$$

therefore

$$r = S\frac{\alpha - \beta}{\beta}. \qquad [A]$$

To obtain accuracy it is advisable for the value of S to be such that the deflection β is approximately equal to $\frac{\alpha}{3}$.*

For example.

A battery whose resistance (r) was required was joined up with a galvanometer, condenser, discharge key, &c., as shown by Fig. 77, page 240. The condenser being charged and then discharged through the galvanometer (by depressing and then releasing the key K_2), a deflection of 290 divisions (α) was produced. A resistance of 20 ohms (S) was then joined between the terminals of the battery, and the condenser again charged and discharged through the galvanometer, the value of the deflection obtained being 105 divisions (β). What was the resistance of the battery?

$$r = 20\frac{290 - 105}{105} = 35 \cdot 2 \text{ ohms.}$$

It is evident that if S be adjusted till $\beta = \frac{\alpha}{2}$, then

$$r = S. \qquad [B]$$

An error in the foregoing kind of test may possibly arise from one measurement being made with the poles of the battery free, when no action goes on in it, and the second being made with it shunted, which may cause a falling off in its electromotive force, as action would then be taking place; the accuracy of the test depends upon the electromotive force being constant in both cases. If the shunt S be connected to the battery by means of a key, then the second discharge deflection β is best

* The reason of this will be obvious from a consideration of the investigations given in § [illegible], page [illegible], and § [illegible], page [illegible].

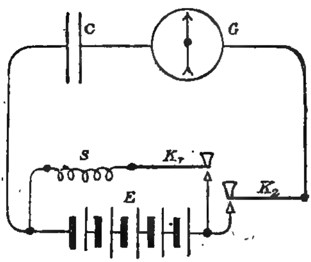
obtained by first pressing down the key K_2 (Fig. 77, page 240), then pressing down the key which connects the shunt to the battery, and then *immediately* afterwards releasing the key K_2, and noting the deflection. Thus as little time as possible is allowed for polarisation to take place.

Measurement of Battery Resistance by Muirhead's Method.

295. A very excellent modification of the foregoing method has been devised by Mr. A. Muirhead; it possesses the great advantage of being free from the source of error just mentioned.

In this test (Fig. 90) the battery, galvanometer, and condenser are joined up in circuit with a key K_2. The condenser C being short circuited for a moment, so as to dissipate any charge which may have been accidentally left in it, key K_2 is depressed; this causes a charge to rush into the condenser through the galvanometer, producing the same deflection as would be produced if the condenser, when charged from the battery direct, were discharged through the galvanometer.

Fig. 90.



The charge deflection (α) being noted, the key K_2 is kept permanently down, so as to keep the condenser charged. By means of key K_1 a shunt (S) is now connected between the poles of the battery; at the moment this takes place the potential at the poles of the latter falls, and a reverse deflection of the needle of the galvanometer is produced. If we suppose this deflection to be due to an alteration of the potential from α to β (the latter being the same quantity as that given in the previous test), its value, ζ, will be

$$\zeta = \alpha - \beta, \quad \text{or,} \quad \beta = \alpha - \zeta.$$

If, then, we substitute this value of β in equation [A] of the previous test, we shall get

$$r = \mathrm{S} \frac{\zeta}{\alpha - \zeta}.$$

For example.

The shunt S having a resistance of 10 ohms, the deflection produced on depressing key K_2 was 310 divisions (α). K_2 being

held down, K_1 was depressed, when a deflection, of 100 divisions (ζ)—in the reverse direction to a—was obtained. What was the resistance of the battery?

$$r = 10 \frac{100}{310 - 100} = 4 \cdot 76 \text{ ohms.}$$

As, in the previous test, it is advisable to give S such a value that ζ is approximately equal to $\frac{a}{3}$.

As no polarisation of any extent takes place in the battery till some seconds after the shunt has been connected to the former by the key, and as the deflection takes place immediately the key is depressed, it follows that very accurate results will be obtained by this test. It may be remarked, however, that Professor Garnett has found that polarisation takes place in a battery in an extremely short space of time—in even the $\frac{1}{1000}$th part of a second; the amount is, however, of course very small. In Muirhead's test the time during which polarisation would tend to affect the accuracy of the test would be that occupied by the galvanometer needle in swinging from zero to the deflection, ζ, consequently the quicker the swing (consistent with accurate reading) the better.

Measurement of Battery Resistance by Munro's Method.

296. A modification of Muirhead's method has been suggested by Mr. J. Munro, which simplifies calculation, inasmuch as it gives the value of $a - \zeta$ by a single deflection.

Key K_1 is first depressed, and then immediately afterwards key K_2 is also depressed; this gives a deflection θ, which is equivalent to the difference between the deflections a and ζ in the last test. Key K_1 is now raised, leaving key K_2 down; and as soon as the galvanometer needle becomes steady, K_1 is depressed again, and the deflection ζ read, then we have

$$R = S \frac{\zeta}{\theta}.$$

As a slight interval of time may elapse between the depression of key K_1 and key K_2, when obtaining the deflection θ (during which time the battery would be partially short circuited), it would be preferable to make the test in the following manner:—Make the connections so that the front contact of key K_1 is joined on to the lever of key K_2 instead of on to the

front contact of the latter, as in Fig. 90; then in order to obtain θ, depress K_1 and keep it down, and immediately afterwards depress K_2; the deflection observed in this case will be θ. Now raise key K_1, keeping key K_2 down, and when the galvanometer needle has become steady, depress K_1, then the deflection obtained will be ζ.

Measurement of Polarisation in Batteries.

297. The amount of polarisation which takes place in a battery when short circuited may, if required, be easily ascertained in the following manner:—In Fig. 90 let S be a short piece of wire of practically no resistance, then having short circuited the condenser C for a moment, depress key K_2 and note the deflection d_1. Keeping K_1 down, depress K_2, and hold it down for a definite time, say one minute; at the end of the interval, release K_2, and note the deflection d_2; then the percentage of polarisation in the one-minute interval will obviously be

$$\frac{100\,(d_1 - d_2)}{d_1}.$$

Measurement of the Resistance of Batteries of Low Resistance.

298. In cases where it is required to measure the total resistance of a number of cells of extremely low resistance (secondary batteries, or accumulators, for example) by any of the foregoing methods, the heating effect produced by the current passing through the shunt S when the latter is connected to the battery by means of the key K_2, would be liable to heat and damage the coils of which the shunt is composed. In such cases the cells should be divided into two sets, one set having, say, one more cell than the other; the two sets should then be joined together so that their electromotive forces oppose one another. By this arrangement we practically obtain a battery whose electromotive force is equal to one cell only, but whose resistance is equal to that of all the cells; consequently the current generated can be but comparatively small, and would have but little heating effect. The contact in key K_2 should be made by means of a mercury cup.

299. When comparing large electromotive forces with small ones—as, for instance, 100 cells with 1 cell—by the condenser discharge method, the smaller force should be taken first; for a large charge usually leaves a residuum in the condenser,

which may be greater than the small force, and which can only be thoroughly dissipated by leaving the condenser short circuited for some time. If the smaller force is measured first, then any residuum it may leave becomes entirely swamped by the larger force, and no increase of charge is added to the condenser beyond what the force itself possesses.

300. Although the condenser practically becomes charged instantaneously, it is best to keep the current on for a definite time, such as half a minute or a minute. We shall then be certain that the charging is complete.

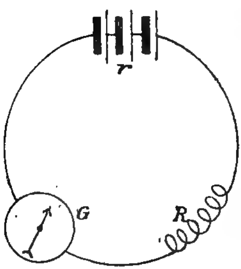
301. When discharge currents are being measured, especial care must be taken to insert a shunt of small resistance in the galvanometer at first, as momentary currents are very liable to weaken the magnetism of the needles when these currents are strong. If this precaution is not taken, a set of measurements for one test may be rendered useless, as a comparison of measurements made before the magnets become weakened, with measurements taken after, would be obviously impossible, and much loss of time would result.

CHAPTER XII.

MEASUREMENT OF CURRENT STRENGTH.

302. If we have a simple circuit, as shown by Fig. 91, then if we know the total resistance, $R + G + r$, of the same, and also the electromotive force, E, of the battery, we can at once determine the strength of the current flowing, for by Ohm's law we have

FIG. 91.



$$C = \frac{E}{R + G + r}.$$

If the resistances are in ohms and the electromotive force of the battery in volts, then the resulting current will be in ampères.

For example.

The electromotive force, E, of a battery which produced a current, C, in a circuit whose total resistance, R_1, was 500 ohms, was found by comparison with a Daniell cell to be 3·5 times as strong as the latter; what was the strength of the current, C, flowing in the circuit?

$$E = 3 \cdot 5 \times 1 \cdot 079 = 3 \cdot 777 \text{ volts.}$$

$$C = \frac{3 \cdot 777}{500} = \cdot 00755 \text{ ampères.}$$

303. In the foregoing method of measurement, in order to determine the strength of the current, it was necessary to know both the resistance of the circuit and also the electromotive force producing the current. A direct determination of the latter can only be made by comparing it with a current of a known strength as follows:—

Direct Deflection Method.

304. In this method a galvanometer is inserted in the circuit through which flows the current whose strength is to be measured. The resistance of the galvanometer should be sufficiently low not to appreciably increase the total resistance of the circuit in which it is inserted. The deflection produced by the current being noted, the galvanometer is removed and joined up in circuit with a standard battery (page 118) and a resistance; the latter is then adjusted until the deflection which was obtained in the first instance is reproduced; in this case, then, the current flowing must be equal to the current whose strength is required. If therefore we divide the electromotive force (in volts) of the standard battery, by the *total* resistance (in ohms) in its circuit, we get at once the required strength of the current, in ampères. The resistance of the standard battery requires of course to be included in the total resistance unless it is so small that it can be neglected.

305. The degree of accuracy attainable in a test of this kind is directly proportional to one-half the degree of accuracy with which the galvanometer deflection can be read, for, since two measurements have to be made, one with the current whose strength is required, and the other with the standard cell, there may be errors made in both of these. If the current to be measured is a strong one, so that it is necessary to shunt the galvanometer when obtaining a deflection, this shunt not being used when the deflection is reproduced with the standard cell, then in this case the result obtained by dividing the electromotive force of the standard cell by the total resistance in its circuit, must be multiplied by the multiplying power of the shunt (§ 56, page 51) in order that the correct strength may be obtained.

For example.

In measuring the strength of a current, the deflection produced on the galvanometer shunted with the $\frac{1}{10}$th shunt, was 50°. The galvanometer (without the shunt) being connected up with a standard Daniell cell of 10 ohms resistance, and a set of resistance coils, it was found necessary to adjust the latter to 560 ohms in order to bring the needle to 50°; what was the strength, C, of the current to be measured?

$$C = \frac{1 \cdot 079}{560 + 10} \times 10 = \cdot 0189 \text{ ampères.}$$

306. When a galvanometer is inserted in a circuit through which a current is flowing whose strength it is required to measure, it is very necessary that the resistance of the instrument be very low compared with the resistance of the circuit itself, otherwise the introduction of the galvanometer will reduce the current flowing, and the result obtained will not be the one required. Before making the test it would of course be necessary to ascertain whether the galvanometer available for use would meet the desired conditions.

To make the test as accurately as possible it would be necessary that the galvanometer needle when deflected be as near to the *angle of maximum sensitiveness* (page 20) as possible. If the strength of current necessary to give this angle be found by joining up a standard cell and a set of resistances, and varying the latter until the required deflection is obtained, then we can always tell whether the instrument would be suitable for a particular purpose. Thus, for example, suppose the galvanometer had a resistance of 1000 ohms, and the angle of maximum sensitiveness were approximately equal to 60°, and suppose further that this deflection was obtained by 1 Daniell cell through a total resistance of 8000 ohms, that is, with a current of $\frac{1 \cdot 079}{8000} = \cdot 000135$ ampères; then to measure such a current we must have the whole resistance of the galvanometer, viz., 1000 ohms, in circuit. If the instrument were shunted with the $\frac{1}{10}$th shunt, then the resistance would be reduced to 100 ohms, and the current corresponding to 60° deflection would be ·00135 ampères; and again, if the $\frac{1}{1000}$th shunt were employed the resistance would be reduced to 1 ohm, and the current corresponding to 60° would be ·135 ampères; thus we see that if it were required to measure a current of about ·0135 ampères and it was necessary that no greater a resistance than 1 ohm should be inserted in the circuit, then it is evident that the galvanometer in question would not answer the purpose required, since a good deflection with ·0135 ampères of current would not be obtained if a lower shunt than $\frac{1}{100}$th were employed, which latter shunt would reduce the galvanometer resistance down to 10 ohms only.

It is preferable, when possible, to employ a galvanometer of high resistance shunted down, rather than one of low resistance not shunted down, since with such a galvanometer it is easier to measure the "constant" of the instrument accurately; for the high resistance of the latter, together with the high resistance which it would be necessary to place in circuit in order to

get a readable deflection with one standard cell only, would swamp, as it were, the resistance of the cell, which resistance need not then be taken into account, or at least need only be known approximately. With a galvanometer of low resistance, however, where a comparatively small resistance only would have to be introduced into the circuit in order to get the required deflection, the resistance of the cell would be required to be known accurately, as it would form an important item in the total resistance of the circuit.

307. The foregoing test has the advantage that it can be made with almost any form of galvanometer, for as only one deflection has to be obtained it is not necessary to know what proportions the various degrees of deflection which it is possible to have, bear to the currents which produce them. If, however, a *tangent* galvanometer is employed to make the test, then it is unnecessary to reproduce the same deflection exactly, though it is advisable to make it approximately near to it.

308. Suppose that in the last example the test had been made with a tangent galvanometer, and the deflection obtained with the standard cell had been 54° instead of 60° (the deflection given by the current whose strength was required), then in this case the actual strength, C_1, of the current would be

$$C_1 = \frac{1\cdot079}{560+10} \times 10 \times \frac{\tan 60^\circ}{\tan 54^\circ} =$$

$$\frac{1\cdot079}{560+10} \times 10 \times \frac{1\cdot7321}{1\cdot3764} = \cdot0238 \text{ ampères.}$$

309. If we use no shunt, or the same shunt when taking both the deflections, and, further, if we make the total resistance in circuit with the standard Daniell cell equal to 1079 ohms, then calling d° the deflection given by the current, and $d_1{}^\circ$ the deflection given by the standard cell, we have

$$C_1 = \frac{1\cdot079}{1079} \times \frac{\tan d^\circ}{\tan d_1{}^\circ} = \frac{\tan d^\circ}{1000 \tan d_1{}^\circ} \text{ ampères ;}$$

or, further still, if by means of an adjusting magnet we can arrange that the deflection given by the standard cell through 1079 ohms equals 45°, then since $\tan 45^\circ = 1$, we must have

$$C_1 = \frac{\tan d^\circ}{1000} \text{ ampères.}$$

Cardew's Differential Method.*

310. This method, devised by Lieut. Cardew, R.E., is a very satisfactory and useful one; its theory is shown by Fig. 92.

The galvanometer G is wound with two wires, g and g_1; the current C_1 whose strength it is required to measure is passed through the coil g_1 (which has a low resistance), and a standard

Fig. 92.

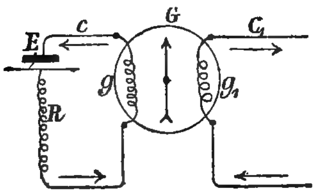
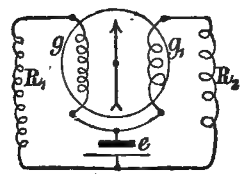


Fig. 93.



battery E is connected in circuit with the second coil g and with an adjustable resistance R. The current being passed through the coil g_1, the resistance R is adjusted until the needle comes to zero.

If we call n and n_1 the relative deflective effects, for the same current, of the two coils g and g_1, and if C_1 and c be the currents flowing through g_1 and g, respectively, then in order to produce equilibrium we must have

$$c : C_1 :: n_1 : n,$$

or

$$C_1 = \frac{c\,n}{n_1}.$$

Now the current through c will evidently be

$$c = \frac{E}{R + g},$$

the resistance of the battery being included in R; therefore

$$C_1 = \frac{E}{R + g} \times \frac{n}{n_1}.$$

For example.

The relative deflective effects of the coils g and g_1 were as 1000 to 1; the resistance of g was 100 ohms. The battery E was a 1-cell standard Daniell. In order to obtain equilibrium

* 'Journal of the Society of Telegraph Engineers,' Vol. XI., page 301.

the resistance R had to be adjusted to 4800 ohms. What was the strength of the current C_1?

$$C_1 = \frac{1 \cdot 079}{4800 + 100} \times \frac{1000}{1} = \cdot 220 \text{ ampères.}$$

311. The relative deflective effects of g and g_1 are easily ascertained by joining up a battery e and two resistances R_1 and R_2, as shown by Fig. 93, and then adjusting until equilibrium is produced; in this case we have

$$n : n_1 : : R_1 + g : R_2 + g_1,$$

or

$$\frac{n}{n_1} = \frac{R_1 + g}{R_2 + g_1}.$$

312. As the accuracy with which a test can be made depends, amongst other things, upon the accuracy with which $\frac{n}{n_1}$ is known, the higher the battery power, e, it is possible to use, the better, since in this case the higher will be the values which can be given to R_1 and R_2, and the higher consequently will be their range of adjustment; thus if we use sufficient battery power to enable a change of $\cdot 1$ per cent., that is to say, 1 ohm in 1000 ohms, or $\frac{1}{10}$th of an ohm in 100 ohms in $R_1 + g$, to produce a perceptible movement of the needle, then we can obtain the value of $\frac{n}{n_1}$ to an accuracy of $\cdot 1$ or $\frac{1}{10}$th per cent.

The resistance of g_1 would have to be very small compared with the resistance of g, so that it would not add appreciably to the resistance of any circuit in which it is inserted.

313. As regards the *Best conditions for making the test*, this will be directly proportional to the relative values of the figure of merit of the coil g_1 and the current to be measured, for it is evident that no matter whether equilibrium exists owing to there being no current flowing through the coils g and g_1, or to equal currents flowing, still the current which will deflect the needle 1 division will be the same in both cases; hence if the reciprocal of the figure of merit of the coil g_1 be, say, $\frac{1}{10,000}$th of an ampère, then an increase of $\frac{1}{10,000}$th of an ampère in the current C_1, no matter what the strength of the latter may be, will produce a deflection of 1 degree. It is evident, therefore, that the greater the strength of the current the greater is the degree of accuracy with which its value can be determined; thus if $\frac{1}{c'}$ be the figure of merit of the coil g_1 and C_1 be the

current to be measured, then the *Percentage of accuracy attainable* will be the percentage which c' is of C_1.

To enable this percentage to be obtained, however, it would be necessary that the total resistance of the circuit of g be adjustable to a similar degree of accuracy; in order that this may be the case E must be of such a value that the number of units in $R + g$ is not less than that which satisfies the equation

$$\frac{1}{R + g} = \frac{c'}{C_1}.$$

Now,

$$C_1 = \frac{E}{R + g} \times \frac{n}{n_1},$$

therefore

$$C_1 = \frac{E\,c'}{C_1} \times \frac{n}{n_1},$$

or

$$E = \frac{C_1^2}{c'} \times \frac{n_1}{n}.$$

For example.

The reciprocal of the figure of merit of coil g_1 of the galvanometer was ·0001 ampères (c'); the current to be measured was approximately ·5 ampères (C_1); and the value of $\frac{n_1}{n}$ was ·001. What was the possible degree of accuracy attainable in making the test, and what would have been the lowest value which should have been given to E in order that this degree of accuracy might be attained?

$$\text{Percentage of accuracy} = \frac{100 \times \cdot 0001}{\cdot 5} = \cdot 02 \text{ per cent.};$$

also

$$E = \frac{\cdot 5 \times \cdot 5}{\cdot 0001} \times \cdot 001 = 2\cdot 5 \text{ volts.}$$

If, therefore, E consisted of 3 Daniell cells, the required value of $R + g$ would have been obtained.

To sum up, then, we have

Best Conditions for making the Test.

314. Make E not less than $\frac{C_1^2}{c'} \times \frac{n_1}{n}$, c' being the reciprocal of the figure of merit of the coil through which the current to be measured is passed.

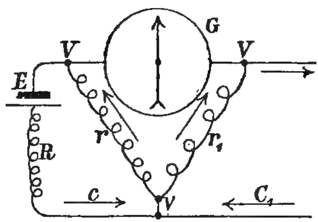
Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100\,c'}{C_1}\,.$$

KEMPE'S BRIDGE METHOD.

315. This method is a modification of the foregoing, and it has the advantage that it does not require a special form of galvanometer for its execution. It is shown in principle by Fig. 94.

FIG. 94.



In making the test the resistance R is adjusted until no deflection is observed on the galvanometer; when this is the case the current c from the battery must also be the current flowing through r, and again the current C_1 must also be the current flowing through r_1. Now since no current flows through the galvanometer, the potentials, V, V, on either side of it must be the same, hence if v be the potential at the junction of r and r_1, then by law (A) (page 256) we have

$$V - v = c\,r,$$

and

$$V - v = C_1\,r_1\,;$$

therefore

$$C_1\,r_1 = c\,r,$$

or

$$C_1 = \frac{c\,r}{r_1}\,;$$

but

$$c = \frac{E}{R + r},$$

therefore

$$C_1 = \frac{E\,r}{r_1\,(R + r)}. \qquad [A]$$

For example

The battery E consisted of a single standard Daniell cell. The resistances r and r_1 were 100 ohms and 1 ohm, respectively. Equilibrium was obtained on the galvanometer G when R was adjusted to 4000 ohms; what was the strength of the current C_1?

$$C_1 = \frac{1 \cdot 079 \times 100}{1\,(4000 + 100)} = \cdot 0263 \text{ ampères.}$$

316. Let us now consider the *Best Conditions for making the Test.*

What we have to determine is,—1st, what should be the values of E and r? and 2nd, what should be the value of R?

The values which E and r should have should be such that the deflection of the galvanometer needle is as large as possible when equilibrium is very nearly, though not quite, produced. Now if we regard R as a constant quantity, then the value which E must have will depend upon the value given to r, consequently we have to determine what the latter quantity should be.

Practically the resistance r_1 would in all cases have to be of a very low value, and if we consider it to be so the problem to be solved becomes a comparatively simple one. We may regard the current c' producing the deflection of the galvanometer needle as due to a difference of two currents, C_1 being one, and the current produced by the electromotive force E, being the other. Let, then, c_1 and c_2 be the portions of these currents which flow in opposite directions through the galvanometer G, then if we suppose the deflection to be due to R being incorrectly adjusted to $R + \delta$, we have (supposing r_1 to be very small),

$$c_1 = \frac{C_1 r_1}{G + \frac{(R + \delta) r}{R + \delta + r}} = \frac{C_1 r_1 (R + \delta + r)}{G R + G \delta + G r + R r + r \delta},$$

and

$$c_2 = \frac{E}{R + \delta + \frac{r G}{r + G}} \times \frac{r}{r + G} = \frac{E r}{G R + G \delta + G r + R r + r \delta};$$

but since from equation [A] (page 271) we have

$$C_1 = \frac{E r}{r_1 (R + r)}, \qquad \text{or,} \qquad E r = C_1 r_1 (R + r),$$

therefore

$$c_2 = \frac{C_1 r_1 (R + r)}{G R + G \delta + G r + R r + r \delta}.$$

Now,

$$c' = c_1 - c_2,$$

therefore

$$c' = \frac{C_1 r_1 (R + \delta + r)}{G R + G \delta + G r + R r + r \delta} - \frac{C_1 r_1 (R + r)}{G R + G \delta + G r + R r + r \delta}$$

$$= \frac{C_1 r_1 \delta}{G R + G \delta + G r + R r + r \delta};$$

Or, since δ is very small, we may say

$$c' = \frac{C_1 r_1 \delta}{G R + G r + R r} = \frac{C_1 r_1 \delta}{R (G + r) + G r}.$$

From this equation we can see that r should be made small, but we can also see that there is but little advantage in making it much smaller than G. In fact, there is an actual disadvantage in making r extremely small, for this would necessitate E being made very large, which would be inconvenient.

We have next to determine what is the best value to give to R. Now, the larger we make the latter, the greater will be its range of adjustment, consequently, as in previous tests, we should give it the *highest value such that a change of* 1 *unit from its correct resistance produces a perceptible deflection of the galvanometer needle.*

We have

$$c' = \frac{C_1 r_1 \delta}{R (G + r) + G r};$$

and if in this equation we put $\delta = 1$ we shall get the current corresponding to a change of 1 unit from the correct value of R, that is

$$c' = \frac{C_1 r_1}{R (G + r) + G r},$$

or, since r must be small, we may practically say

$$c' = \frac{C_1 r_1}{R G},$$

from which

$$R = \frac{C_1 r_1}{c' G}. \qquad [B]$$

If then we make c' the reciprocal of the figure of merit of the galvanometer, the value of R worked out from the equation will show the highest value that the latter quantity should have. But the value of R depends upon the value given to E; we must therefore determine what the latter should be.

We have

$$C_1 = \frac{E r}{r_1 (R + r)},$$

or

$$E = \frac{C_1 r_1 (R + r)}{r}.$$

and substituting the value of R obtained from equation [B] we get

$$E = \frac{C_1 r_1 \left(\frac{C_1 r_1}{c' G} + r\right)}{r};$$

or, as r is small compared with R, that is with $\frac{C_1 r_1}{c' G}$, we may say

$$E = \frac{(C_1 r_1)^2}{c' G r}.$$

For example.

It was required to measure the exact strength, C_1, of a current whose approximate strength was known to be ·03 ampères. A Thomson galvanometer of 5000 ohms resistance (G) was employed for the purpose, its figure of merit being 1,000,000,000 $\left(\frac{1}{c'}\right)$. The resistances of r and r_1 were 100 ohms and 1 ohm respectively. What should be the value of E in order that R may be as high as possible?

$$E = \frac{(\cdot 03 \times 1)^2}{\frac{1}{1{,}000{,}000{,}000} \times 5000 \times 100} = 1\cdot 8 \text{ volts};$$

that is to say, practically, E should consist of 2 Daniell cells.

Assuming E to be equal to 2 volts approximately, then (from equation [B]) the value which R would have in order to obtain balance would be

$$R = \frac{\cdot 03 \times 1}{\frac{1}{1{,}000{,}000{,}000} \times 5000} = 6000 \text{ ohms}$$

approximately.

317. In order to determine the *Possible degree of accuracy attainable*, let us suppose R to be 1 unit out of adjustment, and let λ be the corresponding error produced in C_1, then we have

$$C_1 + \lambda = \frac{E\, r}{r_1 (R - 1 + r)},$$

or

$$\lambda = \frac{E\, r}{r_1 (R - 1 + r)} - C_1 = \frac{E\, r}{r_1 (R - 1 + r)} - \frac{E\, r}{r_1 (R + r)}$$

$$= \frac{E\, r}{r_1 (R - 1 + r)(R + r)};$$

or, since R is large, we may say

$$\lambda = \frac{E\,r}{r_1\,(R + r)^2},$$

but

$$C_1 = \frac{E\,r}{r_1\,(R + r)}, \quad \text{or,} \quad (R + r)^2 = \left(\frac{E\,r}{C_1\,r_1}\right)^2;$$

therefore

$$\lambda = \frac{E\,r}{r_1} \cdot \left(\frac{C_1\,r_1}{E\,r}\right)^2 = \frac{C_1^2\,r_1}{E\,r}.$$

If we call λ' the *percentage* of accuracy, then

$$\lambda = \frac{\lambda'}{100} \text{ of } C_1, \quad \text{or,} \quad \lambda' = \frac{100\,\lambda}{C_1} = \frac{100\,C_1\,r_1}{E\,r}.$$

If we take the values given in the foregoing example we have approximately

$$\lambda' = \frac{100 \times \cdot 03 \times 1}{2 \times 100} = \cdot 015 \text{ per cent.}$$

To sum up, then, we have

Best Conditions for making the Test.

318. Make E the nearest possible value above $\frac{(C_1\,r_1)^2}{c'\,G\,r}$, where c' is the reciprocal of the figure of merit of the galvanometer, and C_1 is the approximate strength of the current to be measured.

The value which R will require to have will be

$$R = \frac{C_1\,r_1}{c'\,G}.$$

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100\,C_1\,r_1}{E\,r}.$$

Difference of Potential Deflection Method.

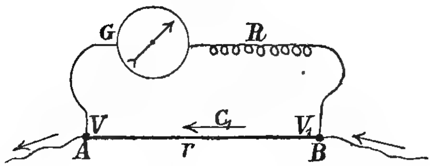
319. Fig. 95 shows the general principle of this method.

A B is a low resistance through which the current, C_1, to be measured passes. A galvanometer, G, in circuit with a high

resistance, R, is connected between the ends of A B as shown, then, calling V and V_1 the potentials at A and B respectively, we have by law (A) (page 256)

$$C_1 = \frac{V - V_1}{r}.$$

FIG. 95.



To determine $V - V_1$ all we have to do is to note the deflection d on the galvanometer G, and then, having disconnected the latter, together with the resistance R, from A B, to join them in circuit with a standard cell of known electromotive force, E, and to obtain a new deflection d_1; we then have

$$V - V_1 : E :: d : d_1,$$

or

$$V_{,} - V_1 = \frac{E\,d}{d_1},$$

so that

$$C_1 = \frac{E\,d}{r\,d_1}.$$

320. In order that the test may be a satisfactory one the resistance G + R should be very high compared with the resistance r, so that the strength of C_1 is practically the same whether G + R is connected to A B or not; also r should be as low as possible, so that it may not appreciably add to the resistance of the circuit in which it is placed. In order, therefore, that a satisfactory deflection may be obtained, the galvanometer G should be one with a high figure of merit; a Thomson galvanometer answers the purpose very satisfactorily.

For example.

In making a measurement according to the foregoing test the resistance r was $\frac{1}{10}$th of an ohm, and the deflection obtained on G was 250 divisions (d). When G and R were connected to a standard Daniell cell in the place of being joined to A B,

a deflection of 230 divisions (d_1) was obtained; what was the strength of the current C_1?

$$C_1 = \frac{1 \cdot 079 \times 250}{\frac{1}{10} \times 230} = 11 \cdot 7 \text{ ampères.}$$

As it is obviously advisable that the deflections obtained should both be as high as possible, the standard electromotive force E may have to be adjusted for the purpose, that is to say, it may have to consist of several cells. Instead of adjusting E only we may make the latter of any convenient high value, and then adjust R so that the required deflection is obtained; in this case if R_1 be the resistance when E is in circuit, we must have

$$C_1 = \frac{E\, d\,(R + G)}{r\, d_1\,(R_1 + G)}$$

For example.

In making a measurement according to the foregoing test the resistance of r was $\frac{1}{10}$th of an ohm and the deflection obtained on G was 270 divisions (d); the resistances of G and R were 5000 ohms and 1000 ohms respectively. When G and R were connected to a standard Daniell cell R had to be adjusted to 7000 ohms (R_1) in order to obtain a deflection of 300 divisions (d_1); what was the strength of the current C_1?

$$C_1 = \frac{1 \cdot 079 \times 270 \times (1000 + 5000)}{\frac{1}{10} \times 300 \times (7000 + 5000)} = 4 \cdot 86 \text{ ampères.}$$

Of course if the value of R_1 is made such that the deflections d and d_1 are equal, then

$$C_1 = \frac{E\,(R + G)}{r\,(R_1 + G)}.$$

321. From the extreme simplicity of the test it must be obvious that the "Best conditions for making the test" and the "Possible degree of accuracy attainable" must be as follows:—

Best Conditions for making the Test.

Make R and R_1 of such values that the deflections obtained are as high as possible.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = 100\,\frac{1}{m}\left(\frac{1}{d} + \frac{1}{d_1}\right)$$

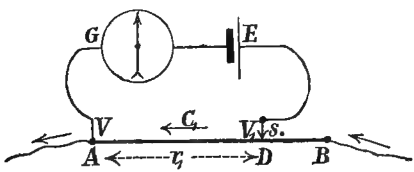
where $\frac{1}{m}$ is the fraction of a division to which each of the deflections can be read.

DIFFERENCE OF POTENTIAL EQUILIBRIUM METHOD.

322. Fig. 96 shows the general principle of this method.

A B is a slide wire resistance, s being the slider. A galvanometer, G, and a standard battery, E, are joined up as shown,

FIG. 96.



so that the latter tends to send a current through r_1 in a direction opposing the current C_1. s is then slid along A B until the point is reached at which no deflection of the galvanometer needle is observed; when this is the case, then by law (A) (page 256) we have

$$V - V_1 = C_1 r_1;$$

and by law (B) (page 258), since no current is flowing through the galvanometer,

$$V - V_1 = E;$$

therefore

$$C_1 r_1 = E,$$

or

$$C_1 = \frac{E}{r_1}.$$

If the resistance of the whole length of wire A B be r ohms, and if it be divided into n divisions, then if the number of divisions between A and D be n_1, the resistance r_1 will be

$$r_1 = \frac{n_1 r}{n};$$

consequently we must have

$$C_1 = \frac{E n}{r n_1}.$$

For example.

The electromotive force E consisted of 1 standard Daniell cell; the wire A B had a resistance of 1 ohm (r), and was

divided into 1000 parts (n). Equilibrium was obtained when the slider was set at the 750th division (n_1); what was the strength of the current C_1?

$$C_1 = \frac{1 \cdot 079 \times 1000}{1 \times 750} = 1 \cdot 44 \text{ ampères.}$$

323. The conditions for making the test in the most satisfactory manner are comparatively simple. The nearer we have the slider to B, that is to say, the larger we make n_1, the smaller will be the percentage of error in the latter due to the slider being, say, 1 division out of position. As the position of the slider for equilibrium depends upon the value of E, the latter must be sufficiently great to enable n_1 to be as large as possible. The greatest theoretical value which E could have must be that which it would possess when $n_1 = n$, in which case we get

$$C_1 = \frac{E}{r}, \text{ or, } E = C_1 r.$$

As it is only possible to adjust E by variations of 1 cell, we must take care that its actual value is less rather than greater than $C_1 r$, otherwise it would be impossible to obtain equilibrium.

It is also necessary that the figure of merit of the galvanometer be sufficiently high to enable a perceptible movement of the needle to be obtained when the slider is moved a readable distance, δ, from the position of exact balance. If we suppose the slider to be at D when equilibrium is produced, then the electromotive force which would tend to send a current through the galvanometer, supposing the slider to be displaced a distance δ, would be

$$E \times \frac{\delta}{n_1},$$

consequently the current c', passing through the galvanometer, will be

$$c' = \frac{E\,\delta}{G\,n_1} = \frac{C_1\,r\,\delta}{G\,n};$$

if, therefore, we require to adjust the slider to an accuracy of δ, the figure of merit $\left(\frac{1}{c'}\right)$ of the galvanometer must not be less than $\frac{G\,n_1}{E\,\delta}$.

The percentage of accuracy, λ', with which C_1 can be obtained must obviously be

$$\lambda' = \frac{100\,\delta}{n_1},$$

or since

$$C_1 = \frac{E\,n}{r\,n_1}, \quad \text{or,} \quad n_1 = \frac{E\,n}{C_1\,r},$$

therefore

$$\lambda' = \frac{100\,C_1\,r\,\delta}{E\,n}.$$

For example.

It being required to measure the strength, C_1, of a current whose approximate value is 1·5 ampères, a galvanometer of 500 ohms resistance (G), whose figure of merit is 1,000,000 $\left(\frac{1}{c'}\right)$, is proposed to be employed for the purpose. The resistance of the whole length of the slide wire, which is divided into 1000 divisions (n), is 1 ohm (r); the position of the slider can be read to an accuracy of $\frac{1}{2}$ a division (δ). What is the highest value that could be given to E? also to what percentage of accuracy could C_1 be determined, and what should be the figure of merit of the galvanometer in order that this percentage of accuracy may be attained?

$$E = 1{\cdot}5 \times 1 = 1{\cdot}5 \text{ volts};$$

therefore we cannot make E greater than, say, 1 Daniell cell (1 volt approximately).

$$\text{Percentage of accuracy} = \frac{100 \times 1{\cdot}5 \times 1 \times \frac{1}{2}}{1 \times 1000} = {\cdot}075 \text{ per cent.}$$

To enable this percentage of accuracy to be obtained, the figure of merit $\left(\frac{1}{c'}\right)$ of the galvanometer must not be less than

$$\frac{1}{c'} = \frac{500 \times 1000}{1{\cdot}5 \times 1 \times \frac{1}{2}} = 670{,}000;$$

the figure of merit, therefore, of the galvanometer in question is sufficient for the required purpose.

To sum up, then, we have

Best Conditions for making the Test.

324. Make E the nearest possible value below $C_1\,r$.

The figure of merit of the galvanometer should not be less than $\dfrac{G\,n}{C_1\,r\,\delta}$.

Possible Degree of Accuracy attainable.

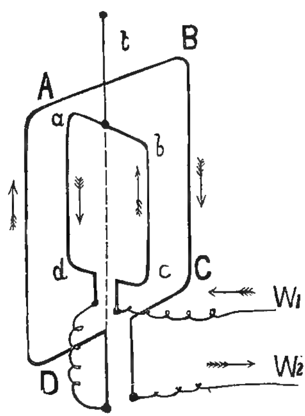
$$\text{Percentage of accuracy} = \frac{100\,C_1\,r\,\delta}{E\,n}.$$

SIEMENS' ELECTRO-DYNAMOMETER.

325. This apparatus, although it can be used for measuring ordinary powerful currents, yet has the special advantage that it enables rapidly *alternating* currents (such as are employed in the Jablochkoff system of electric lighting, for example) to be measured; such currents would give no indications on an ordinary galvanometer.

The principle of the electro-dynamometer is based upon the mutual action of currents upon one another, i. e. upon the fact that currents in the same direction attract, and in opposite directions repel, one another. Fig. 97 shows how the principle is applied.

FIG. 97.



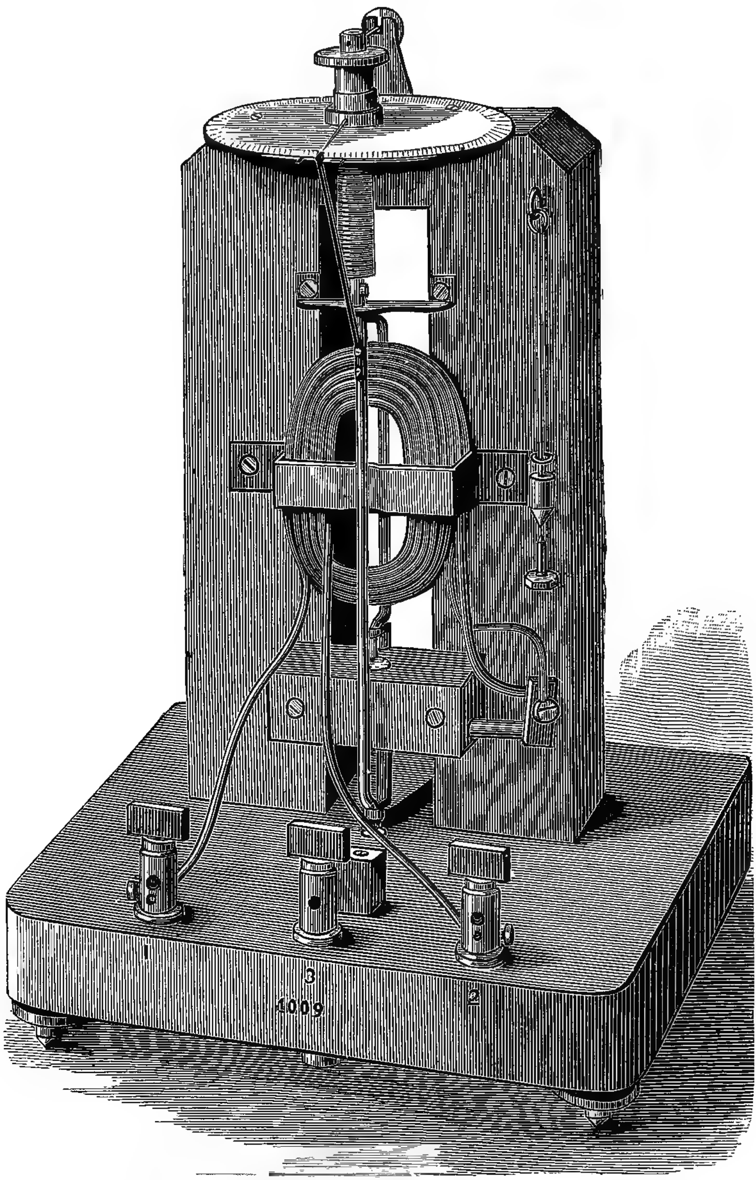
A B C D is a fixed wire rectangle, and *a b c d* a smaller one, suspended by a thread, *t*, within the larger, so that it can turn freely about its axis; the planes of the two are at right angles to each other. Now, if the two rectangles be connected together in the way shown, then a current entering at W_1, and passing out at W_2, will traverse the two, and the current passing from B to C will attract the current passing from *a* to *d*, and will repel the current passing from *c* to *b*. A similar action takes place with reference to the current passing from D to A, consequently the smaller rectangle, under the influence of the forces, will tend to turn about its axis, in the direction in which the hands of a watch rotate. If the current enters at W_2, and leaves at W_1, then, inasmuch as the directions of all the currents in the wires are reversed, the small rectangle must still tend to turn in the direction indicated. If one or both of the rectangles consist of several turns of wire, the turning effect for a given current will be proportionally increased.

As the turning effect on the coil is produced by the action of the current through the fixed coil acting on the current

through the movable coil, and as the two coils are in the same circuit, it follows that if the current passing through the fixed coil is doubled, then the current passing through the movable coil is also doubled, consequently we have one doubled current acting upon another doubled current, and therefore we must have a quadrupled deflective effect—in other words, the deflective force tending to turn the movable coil will vary as the *square* of the current. The way in which this principle is utilised will be best understood by reference to Fig. 98, which shows a general view of the Siemens Dynamometer.

The apparatus consists of a rectangle of wire hung from a fibre whose upper end is fixed to a thumb-screw; the latter is provided with a pointer which can be moved round a graduated dial; one end of a spiral spring is also attached to the rectangle, the other end being fixed to the thumb-screw. In this arrangement the number of degrees to which the pointer is directed evidently indicates the amount of tension given to the spiral spring. To the rectangle also is fixed a pointer, the end of which just laps over the edge of the graduated dial. The rectangle encircles a coil consisting of several turns of thick, and a larger number of turns of thin, wire; the two ends of the thick wire are connected to terminals 2 and 3, and the two ends of the thin wire to terminals 1 and 3. Connection is made between the rectangle and the wire coils by mercury cups, into which dip the ends of the wire forming the rectangle. The base-board has three levelling screws; the level consists simply of a small pointed weight hung at the end of a rod (seen on the right of the figure), the pointed end hangs exactly over a fixed point when the instrument is level.

326. The method of using the instrument is as follows:—The wires leading the current whose strength is to be determined are connected to terminals 1 and 3, or 2 and 3, according as a strong or weak current has to be measured. The current deflects the rectangle; the thumb-screw is now turned in the reverse direction to that in which the rectangle has turned, and torsion being thereby put on the spiral spring the rectangle is forcibly brought back towards its normal position—that is, at right angles to the coils, or to the position at which the pointer attached to the rectangle stands at zero on the scale. The number of degrees of torsion given to the spiral spring being then read off, the strength of the current is found by reference to a table supplied with each instrument. To construct this table a current of a known strength is sent through the instrument, and then the degree of torsion required to bring the rectangle back to zero is carefully noted. This being done,


3
1009

the currents corresponding to other degrees of torsion are easily calculated. The force of torsion varies directly as the number of degrees through which the spiral spring is twisted, whilst, as has been before explained, the deflective effect of the current varies directly as the *square* of the latter. In other words, if ϕ° be the number of degrees of torsion required to bring the rectangle back to zero when it is traversed by a current of C ampères, then if C_1 be the current which will correspond to any other degree of torsion ϕ_1°, we have

$$\phi^\circ : \phi_1^\circ :: C^2 : C_1^2 ;$$

or

$$C_1 = \sqrt{\frac{\phi_1^\circ C^2}{\phi^\circ}}.$$

For example.

If 180° (ϕ°) of torsion were required to bring the rectangle back to zero when it was traversed by 47·57 (C) ampères of current, what current (C_1) would be represented by 80° (ϕ_1°) of torsion?

$$C_1 = \sqrt{\frac{80 \times 47 \cdot 5 \times 47 \cdot 5}{180}} = 31 \cdot 7.$$

327. Like galvanometers, the Siemens electro-dynamometer is not susceptible of great accuracy when the readings are very low, in fact the higher the readings are, the more accurate are the results obtainable. Thus, for example, 5° of torsion of the spring represents a current (in the instrument (No. 1009) shown by Fig. 98) of 7·93 ampères, whilst 5° more, that is 10° in all, represents a current of 11·23 ampères. In other words, a range of 5° of torsion only, represents a difference in the current of

$$\frac{(11 \cdot 23 - 7 \cdot 93)\,100}{7 \cdot 93} \text{ per cent.} = 42 \text{ per cent.}$$

If, however, the current had been 66·38 ampères, which corresponds to a torsion of 350°, then 5° more of torsion, or 355° in all, represents a current of 66·86 ampères, consequently the range of 5° of torsion in this case represents a difference in the current of

$$\frac{(66 \cdot 86 - 66 \cdot 38)\,100}{66 \cdot 38} \text{ per cent.} = \cdot 72 \text{ per cent.};$$

and a greater degree of torsion would have rendered the error still less.

Every instrument is supplied with a table which shows the

current strengths corresponding to various angles of torsion; practically this table is different for every instrument, as it is almost impossible (nor is it necessary) to make two dynamometers alike. The table supplied with the instrument (shown by Fig. 98 (No. 1009)) is calculated so that the latter can theoretically be used for measuring currents varying from 1·05 to 66·86 ampères in strength. The thin wire coil is to be employed when currents of from 1·05 to 19·87 ampères are to be measured, and the thick wire coil for currents of from 3·54 to 66·86. The numbers of degrees of torsion representing various currents are all multiples of 5; thus the first calculation on the table (thick wire coil) is 1°, which represents 3·5 ampères of current; the next is 5°, representing 7·93 ampères; the next, 10°, representing 11·28 ampères; and so on. Practically the instrument cannot well be adjusted to a closer degree of accuracy than 5°.

The thin wire coil, having about three times the magnetic effect of the thick one, requires, for a definite current, that the number of degrees of torsion to bring the needle back to zero be about three times that which is required in the case of the thick coil; in other words, with the thin wire coil we can practically measure currents to about three times the degree of accuracy which is possible with the thick coil; but, on the other hand, the highest current which we can practically measure with the thin coil is about one-third only of the highest current which can be measured with the thick coil.

The lowest current which can be measured consistent with a degree of accuracy equal to 10 per cent. is 5·76; for the next current below this on the table is 5·25, and therefore we have

$$\frac{(5 \cdot 76 - 5 \cdot 25)\,100}{5 \cdot 25} \text{ per cent.} = 10 \text{ per cent. nearly.}$$

If we require to be accurate within 1 per cent., then the lowest current we could measure would be 16·77, as the next current below this on the table is 16·60, and we therefore have

$$\frac{(16 \cdot 77 - 16 \cdot 60)\,100}{16 \cdot 70} \text{ per cent.} = 1 \text{ per cent. nearly.}$$

Since the percentage of accuracy is equal to

$$\frac{(C - C_1)\,100}{C_1} = \left(\frac{C}{C_1} - 1\right)100$$

where C is a particular current, and C_1 the current next below it on the table, and since

$$C^2 : C_1^2 : : \phi^\circ : \phi_1^\circ$$

where ϕ° and ϕ_1° are the degrees of torsion corresponding to the currents C, C_1, therefore

$$\text{Percentage of accuracy} = \left(\sqrt{\frac{\phi^\circ}{\phi_1^\circ}} - 1\right) 100\,;$$

and as the smallest difference to which we can practically read is 5°, therefore

$$\text{Percentage of accuracy} = \left(\sqrt{\frac{\phi_1^\circ + 5^\circ}{\phi_1^\circ}} - 1\right) 100 = \lambda', \text{ say.}$$

Therefore

$$\sqrt{1 + \frac{5^\circ}{\phi_1^\circ}} = \frac{\lambda'}{100} + 1\,;$$

therefore

$$1 + \frac{5^\circ}{\phi_1^\circ} = \frac{\lambda'^2}{10{,}000} + 1 + \frac{\lambda'}{50}\,;$$

therefore

$$\frac{5^\circ}{\phi_1^\circ} = \frac{\lambda'^2}{10{,}000} + \frac{\lambda'}{50}\,;$$

or,

$$\phi_1^\circ = \frac{50{,}000}{\lambda'^2 + 200\,\lambda'},$$

which shows us the smallest number of degrees of torsion which must be given to the spiral spring when measuring a current, in order that the latter may be measured to an accuracy of x per cent.

For example.

It was required to be able to measure currents of 10 ampères and upwards to an accuracy of 1 per cent., by means of an electro-dynamometer; how many degrees of torsion would the spiral spring be required to make?

$$\phi_1^\circ = \frac{50{,}000}{1 + 200} = 248^\circ\,;$$

showing that the electro-dynamometer must be so constructed that when currents of 10 ampères and upwards have to be measured, not less than 248° of torsion have to be given to the spiral spring in order to bring the needle back to zero.

328. From the construction and principle of the electro-dynamometer it must be evident that the accuracy of the

absolute results obtained by its means must depend entirely upon the torsion of the spiral spring remaining constant. It seems possible that change of temperature and frequent use might alter the value of the torsion, but this point does not appear to have been satisfactorily settled. The instrument might probably be made of more value if its coil were composed of a large number of turns of thin wire, shunted by a thick wire shunt. The latter would be used when measuring the strong currents, whilst the correctness of the instrument could be verified by sending a comparatively weak current through the unshunted coil. It is not often that powerful currents of an accurately known value can be had for the purpose of verifying the correctness of an instrument, though weaker currents are almost always obtainable.

CHAPTER XIII.

MEASUREMENT OF ELECTROSTATIC CAPACITY.

Direct Deflection Method.

329. The simplest way of measuring electrostatic or inductive capacities is, with the same battery power, to compare the discharges from the unknown capacities with the discharge from a condenser of a known capacity; thus we note the discharge deflection a given by the standard condenser F, and then the discharges a_1, a_2, &c., given by the cables or condensers whose capacities F_1, F_2, &c., are required, in which case

$$F : F_1 : F_2 :: a : a_1 : a_2.$$

For example.

A standard condenser had a capacity of $\frac{1}{3}$ microfarad, and gave a discharge deflection of 300, and two other cables or condensers, F_1, F_2, gave discharge deflections of 225 and 180 respectively, then

$$\tfrac{1}{3} : F_1 : F_2 :: 300 : 225 : 180;$$

that is,

$$F_1 = \tfrac{1}{3} \cdot \frac{225}{300} = \tfrac{1}{4} \text{ microfarad,}$$

and

$$F_2 = \tfrac{1}{3} \cdot \frac{180}{300} = \tfrac{1}{5} \text{ microfarad.}$$

If we use shunts and obtain the *same* deflection, then

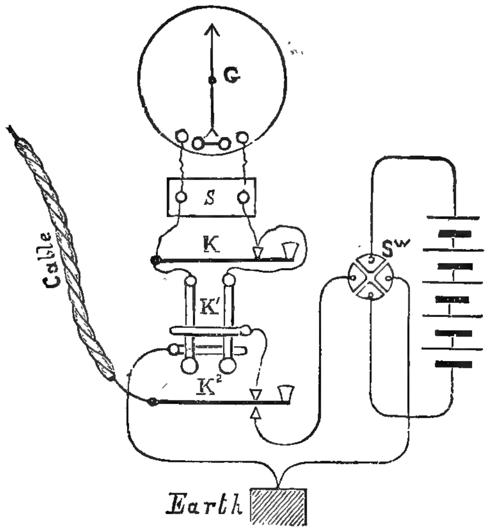
$$\tfrac{1}{3} : F_1 : F_2 :: \frac{G + S}{S} : \frac{G + S_1}{S_1} : \frac{G + S_2}{S_2}.$$

330. In measuring the electrostatic capacity of a cable by this method, the connections for measuring the discharge from the cable would be made in the manner shown by Fig. 99. The arrangements for measuring the discharge from the condenser would be those indicated by Fig. 77 (page 240).

Then, as before, the capacity of the cable will be to the

capacity of the condenser as the discharge deflection of the one is to the discharge deflection of the other, or obtaining the same deflection by means of shunts, as the multiplying power of the shunts.

FIG. 99.



331. The capacity per mile will be the result divided by the mileage of the cable.

332. When a number of capacities of about the same value have to be measured, as, for instance, the capacities of two-knot lengths of cable core, a device may be adopted which considerably simplifies the operation. Let F be the capacity of the standard condenser whose discharge is D divisions, and let f be the capacity of one of the lengths of cable, and d the discharge from the same. Then we have

$$F : f :: D : d,$$

or

$$f = \frac{F\,d}{D}.$$

Now if we make $\frac{F}{D}$ a submultiple of 10, then the value of d read off from the scale will give at once the value of f. Thus

if F were a condenser of $\frac{1}{3}$ microfarad capacity, and we so adjusted the galvanometer that this capacity gave a discharge deflection of a little over 333 divisions, then we should have

$$f = \frac{\frac{1}{3}\,d}{333\frac{1}{3}} = \frac{d}{1000};$$

so that if the discharge deflection reading from the cable consisted of three figures, a decimal point put before the latter would give at once the capacity of the cable; or if the reading consisted of two figures, then we must put a decimal point and a cypher. In the same way, if we had a condenser of 1 microfarad capacity, we should adjust the galvanometer so as to obtain a deflection of 100 divisions, for then

$$f = \frac{1\,d}{100} = \frac{d}{100}.$$

SIEMENS' LOSS OF CHARGE DISCHARGE METHOD.

333. The principle of this method of measurement is that of observing the rate at which the charged condenser or cable, whose capacity is required, discharges itself through a known resistance, and calculating the capacity from a formula which we will now consider.

The elements with which we have to deal are: capacity (farad), resistance (ohm), quantity (coulomb), time (second), and potential (volt).

Let us suppose the cable or condenser has an electrostatic capacity of F farads, and is charged to a potential of V volts, so that it contains Q coulombs (equal to V F) of electricity, and is discharging itself through a resistance of R ohms during one second.

The quantity of electricity in the condenser or cable at starting is Q coulombs.

If now we take a very short interval of time t, we may consider the discharge, which really varies continually, to flow throughout that time t, at the same rate as it had at the commencement, and the smaller t is taken, the more accurate will be the result.

Thus, since the quantity escaping is directly proportional to the potential driving it out, and to the time during which the escape occurs, and inversely proportional to the resistance

through which the escape takes place, the quantity escaping will vary as

$$\frac{V\,t}{R}, \text{ that is, it equals } \frac{V\,t}{R}\,K$$

where K is a constant to be determined.

Now the units are so made that a condenser of 1 farad electrostatic capacity charged to a potential of 1 volt, that is, containing 1 coulomb of electricity, will commence to discharge itself through a resistance of 1 ohm, at the rate of 1 coulomb per second. That is to say,

$$1 = \frac{1 \times 1}{1}\,K, \text{ therefore } K = 1\,.$$

The quantity escaping during the interval of time t in our problem is therefore

$$\frac{V\,t}{R}.$$

The quantity remaining in the condenser will be

$$Q - \frac{V\,t}{R} = Q - \frac{V\,F\,t}{F\,R} = Q\left(1 - \frac{t}{F\,R}\right).$$

Again, since this is the quantity at the commencement of the second interval, that at the end will be

$$\left[Q\left(1 - \frac{t}{F\,R}\right)\right]\left[\left(1 - \frac{t}{F\,R}\right)\right] = Q\left(1 - \frac{t}{F\,R}\right)^2,$$

and that at the end of the nth interval will be

$$Q\left(1 - \frac{t}{F\,R}\right)^n = q\,.$$

Let these n intervals of t seconds equal T, so that $n\,t = T$.

Now we have seen that the smaller t is, the more accurate will our results be. Let us therefore make t infinitely small, and n infinitely great, so that $n\,t$ still $= T$, we shall then get a perfectly accurate result, and the amount remaining at the end of time T will be

$$q = Q\left(1 - \frac{T}{n\,F\,R}\right)^n$$

where $n = \infty$.

To evaluate q put

$$\frac{\mathrm{T}}{n\,\mathrm{F}\,\mathrm{R}} = -\frac{1}{x},$$

so that

$$x = \infty \text{ when } n = \infty\ ;$$

then

$$q = \mathrm{Q}\left[\left(1 + \frac{1}{x}\right)^x\right]^{-\frac{\mathrm{T}}{\mathrm{F}\,\mathrm{R}}}$$

when $x = \infty$; but when this is the case the expression within the square brackets is known to be equal to e,* thus

$$\frac{q}{\mathrm{Q}} = e^{-\frac{\mathrm{T}}{\mathrm{F}\,\mathrm{R}}},$$

therefore

$$\frac{\mathrm{T}}{\mathrm{F}\,\mathrm{R}} = \log_e \frac{\mathrm{Q}}{q},$$

therefore

$$\mathrm{F} = \frac{\mathrm{T}}{\mathrm{R} \log_e \dfrac{\mathrm{Q}}{q}};$$

but

$$\frac{\mathrm{Q}}{q} = \frac{\mathrm{V}\,\mathrm{F}}{v\,\mathrm{F}} = \frac{\mathrm{V}}{v},$$

where v is the value of the potential corresponding to the value q of the quantity, thus

$$\mathrm{F} = \frac{\mathrm{T}}{\mathrm{R} \log_e \dfrac{\mathrm{V}}{v}} = \frac{\mathrm{T}}{2 \cdot 303\, \mathrm{R} \log \dfrac{\mathrm{V}}{v}}$$

where, as stated at first, T is measured in seconds, F in farads, and R in ohms.

Since V and v now appear in the form of a proportion, the unit in which they are measured is immaterial, although they were measured at the outset in volts.

In practice R is usually measured in megohms (1,000,000 ohms), and consequently F will, in such a case, be measured in microfarads ($\frac{1}{1{,}000{,}000}$ farad).

* Todhunter's Algebra, Fifth Edition, Chapter XXXIX.

For example.

A fully charged condenser gave a discharge deflection of 300 divisions (V); after being recharged and allowed to discharge itself through a resistance of 500 megohms for 60 seconds (T), the discharge deflection obtained was 200 divisions (v). What was the capacity of the condenser?

$$F = \frac{60}{2 \cdot 303 \times 500 \log \frac{300}{200}} = \cdot 295 \text{ microfarads.}$$

334. In executing this test it is advantageous to make V and v bear a certain proportion to one another, for this will cause any small error in reading the value of v to produce as small an error as possible in the value of F when the latter is worked out from the formula. This may be proved thus :—

Let us assume R to be constant, and let there be an error λ in F caused by an error δ_1 in v and an error δ_2 in V, the error δ_1 being plus and δ_2 minus, so that the total resulting error is as great as possible, we then have

$$F + \lambda = \frac{T}{R \log_e \frac{V - \delta_2}{v + \delta_1}}, \quad \text{or,} \quad \lambda = \frac{T}{R \log_e \frac{V - \delta_2}{v + \delta_1}} - F$$

but

$$F = \frac{T}{R \log_e \frac{V}{v}}, \quad \text{or,} \quad \frac{T}{R} = F \log_e \frac{V}{v},$$

therefore

$$\lambda = F \frac{\log_e \frac{V}{v}}{\log_e \frac{V - \delta_2}{v + \delta_1}} - F = F \frac{\log_e \frac{V}{v} - \log_e \frac{V - \delta_2}{v + \delta_1}}{\log_e \frac{V - \delta_2}{v + \delta_1}}$$

$$= F \frac{\log_e \left(1 + \frac{\delta_1}{v}\right) - \log_e \left(1 - \frac{\delta_2}{V}\right)}{\log_e \frac{V - \delta_2}{v + \delta_1}};$$

but if δ_1 and δ_2 are very small,* we get

$$\lambda = F \frac{\frac{\delta_1}{v} - \left(- \frac{\delta_2}{V}\right)}{\log_e \frac{V}{v}}.$$

* Todhunter's Trigonometry, Third Edition, Chapter XII.

If the deflections are taken on a Thomson galvanometer (as would practically be always the case), then $\delta_1 = \delta_2$, so that we get

$$\lambda = \mathrm{F}\frac{\delta_1\left(\frac{1}{v}+\frac{1}{\mathrm{V}}\right)}{\log_e \frac{\mathrm{V}}{v}}.$$

Now the value of v, which makes λ a minimum,* is

$$v = \frac{\mathrm{V}}{3 \cdot 59},$$

* This may be determined in the following manner:—
To make λ a minimum we must make

$$\frac{\log_e \frac{\mathrm{V}}{v}}{\frac{1}{v}+\frac{1}{\mathrm{V}}}$$

a maximum.
Let the above expression equal u, and let

$$v = \frac{\mathrm{V}}{n}.$$

we then get

$$u = \mathrm{V}\frac{\log_e n}{n+1};$$

then

$$\frac{d\,u}{d\,n} = \left(\frac{1}{n+1}\right)^2\left\{(n+1)\frac{1}{n} - \log_e n\right\} = 0$$

at a maximum; therefore

$$\log_e n = \frac{n+1}{n},$$

or

$$\log n = \left(\frac{n+1}{n}\right) \cdot 4343.$$

The solution of this equation is best effected by the "trial" method, viz. by giving n various values until one is found which approximately satisfies the equation. If we make $n = 3 \cdot 59$, we get

$$\cdot 55509 = \left(\frac{3 \cdot 59 + 1}{3 \cdot 59}\right) \cdot 4343 = \cdot 55527,$$

which is sufficiently close for the purpose.
We have therefore

$$\mathrm{V} = v\,n, \quad \text{or,} \quad v = \frac{\mathrm{V}}{n} = \frac{\mathrm{V}}{3 \cdot 59}.$$

so that practically we may say—make $v = \frac{V}{3\cdot 5}$.

We need not be particular, however, about making v *exactly* equal to $\frac{V}{3\cdot 5}$, as we could make it 50 per cent. greater or less than this value without materially increasing λ. If the rate of fall were comparatively quick, there would be a positive advantage in making v less than $\frac{V}{3\cdot 5}$, as the greater we make T, the less will any small error in its value affect the correctness of F, as must be self-evident.

Now, if R is adjustable, it is clear that by making it large enough, we could make T large without reducing v too much. In the case of a cable, R, being the insulation resistance, is of course a fixed quantity, but when the measurement is being made with a condenser, any value may be given to R that is considered convenient. We therefore have

Best Conditions for making the Test.

335. Make v as nearly as possible equal to $\frac{V}{3\cdot 5}$. When it is possible to adjust R, make the latter as high as convenient.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = F\,\frac{100\,(\delta_1 + \delta_2)}{2\cdot 303\, v \log \frac{V}{v}}.$$

If the deflections are read on a Thomson galvanometer (as would usually be the case) then

$$\text{Percentage of accuracy} = F\,\frac{200\,\delta}{2\cdot 303\, v \log \frac{V}{v}}$$

where δ is the fraction of a division to which each of the deflections V and v can be read.

329. When it is an ordinary condenser (whose insulation resistance would practically be infinite) that is to be measured, the connections would be the same as those given in Fig. 77,

page 240, with the addition of the resistance, which would be inserted between the terminals of the condenser.

The instantaneous discharge (V) can be taken without removing the resistance; for, since the latter would be extremely high, there would be no time for any of the charge to have leaked out through it during the small interval occupied by the lever of the key in passing from the bottom to the top contact. To take the discharge after the interval of time, having charged the condenser by pressing down the lever of the discharge key (Fig. 80, page 242), we should depress the "Insulate" trigger, which would take the battery off but not discharge the condenser; then, after the noted interval of time, we should depress the "Discharge" trigger, which would allow the charge remaining to flow out, the deflection obtained from which gives us v.

337. To measure the capacity of a cable by this method, the connections would have to be those given in Fig. 99, page 289, and the way of making the test would be the same as has just been explained. R in this case would be the insulation resistance of the cable, which in this and the following method would have to be determined beforehand in the manner described in Chapter XV., page 331. Inasmuch as R in a cable is a variable quantity and is dependent upon the time a charge is kept in the cable, a mean value only can be given to it, and therefore this and the following test can only give the value of F approximately.

Siemens' Loss of Charge Deflection Method.

338. If the two terminals of a condenser are connected by a high resistance in the circuit of which a galvanometer is placed, and if the two terminals be also connected to a battery, then the condenser will become charged up, and the permanent deflection obtained on the galvanometer will represent the potential of the charge. If now the battery be taken off, a current will flow from the condenser through the resistance and the galvanometer, which current will continually decrease in strength as the condenser empties itself. But the current flowing at any particular moment will be represented by the deflection obtained at that moment, and this deflection will be the same as that which would be obtained if the condenser were kept continuously charged to the potential it had at that moment.

The deflection obtained therefore on the galvanometer when the battery is connected to the condenser indicates the potential which the latter has when fully charged, and the deflection after any interval of time after the battery has been taken off,

indicates the potential of the charge remaining; the capacity therefore is given by the formula

$$F = \frac{T}{2 \cdot 303 \; R \log \frac{D}{d}} \text{ m.f.}, \qquad [A]$$

in which D is the deflection obtained when the battery is on, and d the deflection obtained after T seconds, the battery being off during that time. R is the resistance through which the charge flows.

It may be remarked that the deflection obtained when the battery is on is not affected by the presence of the condenser; it would be the same whether the condenser were connected up or not.

339. The connections for making a test of this kind would be as follows:—Referring to Fig. 77, page 240, the terminal of K_1, which is connected to the top contact of K_2, should in the present case be connected through the resistance R to terminal A of the condenser; the other connections remain the same.

340. In the case of a cable where the flowing out of the charge takes place through the insulating sheathing, a galvanometer cannot be put in the circuit of the flow. To enable the fall of charge to be observed, therefore, a high resistance in circuit with the galvanometer is connected to the cable, and through this resistance a part of the charge passes. As it is only the *rate* at which the fall takes place that is required, it is quite sufficient, in order to observe this fall, that a part only of the charge be allowed to flow through the galvanometer.

If we call R_1 the insulation resistance of the cable, and R_2 the resistance connected to it, then the total resistance through which the charge flows will be

$$\frac{R_1 \; R_2}{R_1 + R_2}.$$

This quantity must be substituted in the place of R in equation [A]; so that we have

$$F = \frac{T}{2 \cdot 303 \frac{R_1 \, R_2}{R_1 + R_2} \log \frac{D}{d}} \text{ m.f.}$$

The resistance R_2, it may be remarked, includes the resistance of the galvanometer.

As in the first test, it is necessary that R_2, through which the discharge has to pass, be sufficiently great to prevent the flow from being too rapid.

For example.

A cable 30 knots in length being connected up, for making the test just described, with a galvanometer, and a resistance R_2, of 4 megohms, the deflection obtained was 300 divisions (D). On taking off the battery the deflection after 30 seconds (T) fell to 100 divisions (d); the mean insulation resistance R_1 of the cable was 10 megohms. What was the electrostatic capacity (F) of the cable?

$$F = \frac{30}{2 \cdot 303 \times \frac{10 \times 4}{10 + 4} \log \frac{300}{100}} = 9 \cdot 55 \text{ m.f.}$$

or

$$\frac{9 \cdot 55}{30} = \cdot 318 \text{ m.f. per knot.}$$

341. The connections for making this test would be as follows:—Referring to Fig. 99, page 289, the terminal of key K_1, instead of being connected to the top contact of the discharge key, would in the present case be connected to the cable through the resistance R_2.

342. A great advantage which this test possesses over the first method (page 289) lies in the fact that it is correct either for long or short cables. Discharge deflections from long cables, or cables coiled in tanks, do not correctly represent their capacity, in consequence of a retardation which takes place in them and which causes the deflection of the galvanometer needle to be less than it would be if this retardation did not exist. By adopting the fall of deflection plan we avoid this cause of error; but, as we pointed out at the conclusion of the last test, since R_1 can only have a mean value, the value of F obtained from the formula will only be approximate.

Thomson's Method.

343. This is a very good method, and it can be applied to long cables, &c., with very accurate results.

The following is its principle:—

If we have two condensers containing equal charges of opposite potentials, and we connect the two together, the two charges will combine and annul one another, and if we then connect the two condensers, so joined, to a galvanometer, no deflection will

be produced, there being no charge left in either of the two. If, however, the charge in one condenser exceeds that in the other, then the union of the two condensers will not entirely annul their charges, but an amount will remain equal to the difference of the two quantities. This quantity will deflect the needle if the joined condensers be now connected to the galvanometer, the deflection being to the right or left, according as the charge in the one or other of the condensers had the preponderance in the first instance.

If then we know the capacity of one condenser, and we so adjust the potentials of the two that no charge remains when they are joined together, we can determine the capacity of the other condenser.

Let Q_1 and Q_2 be the charges in each; then

$$Q_1 : Q_2 :: V_1 F_1 : V_2 F_2$$

where F_1 and F_2 are the capacities of the two, and V_1 and V_2 the potentials of their charges.

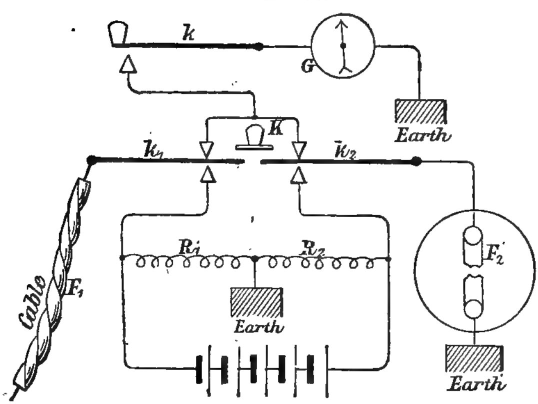
When $Q_1 = Q_2$ then

$$V_1 F_1 = V_2 F_2,$$

or

$$F_1 = \frac{V_2}{V_1} F_2.$$

FIG. 100.



344. An important element in this test is the adjustment of the potentials V_1 and V_2. Fig. 100 shows a method of making the test when it is a cable whose capacity has to be measured.

The poles of the battery are joined together by two resistances R_1 and R_2 connected to earth as shown. Then the potentials at the points of junction of the battery with the resistances will be in the proportion

$$V_1 : V_2 :: R_1 : R_2 ;^*$$

and since

$$F_1 = \frac{V_2}{V_1} F_2 ,$$

therefore

$$F_1 = \frac{R_2}{R_1} F_2 . \qquad [A]$$

345. In making the test practically, R_1 and R_2 are first adjusted as nearly as can be guessed in the proportion of F_1 to F_2, keys k_1 and k_2 are then depressed by means of the knob K; this charges the cable and condenser.

K is now released so as to allow k_1 and k_2 to come in contact with their upper stops; as the two latter are joined together, the cable and condenser become connected to each other.

Key k is now pressed, which allows any charge which may remain uncancelled to be discharged through the galvanometer G. If no deflection is produced, then R_1 and R_2 are correctly adjusted, but if not they must be readjusted until no discharge is obtained; F_1 is then calculated from the formula.

For example.

A cable 500 knots long was joined up with a condenser of 20 microfarads capacity, and with resistance coils, according to Thomson's method of measuring electrostatic capacities. When R_1 and R_2 were adjusted to 500 and 4400 ohms respectively, no charge remained in the cable and condenser when the two were connected together. What was the capacity of the cable?

$$F_1 = \frac{4400}{500} \times 20 = 176 \text{ m.f.},$$

or

$$\frac{176}{500} = \cdot 352 \text{ m.f. per knot.}$$

346. Fig 101 shows a very convenient form of key, designed by Mr. Lambert, which enables the test to be made with the greatest facility. By pushing forward key button K the two

* Page 248.

keys k_1, k_2 (Fig. 100) are depressed, so that F_1 and F_2 become charged, and upon drawing K back, k_1 and k_2 are allowed to rise, thus causing the charges to mix; finally by depressing k the galvanometer is brought into circuit.

347. If it were the capacity of a condenser which was to be measured, then the connections would be similar to those in

FIG. 101.

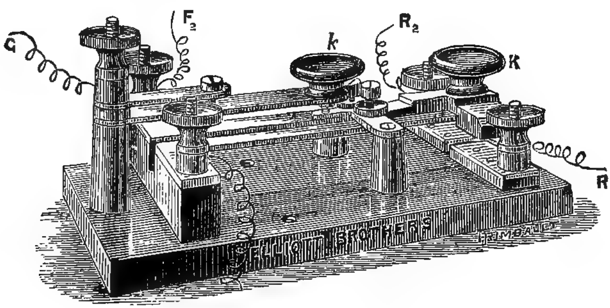


Fig. 100, with the exception that the points there put to earth would in the present case be connected to the second terminal of the condenser.

The resistances R_1 and R_2 may be formed of a slide resistance, the slider being to earth in the case of a cable test, or connected to the second terminal of the condenser in the case of a condenser test.

348. As in the "Direct deflection method" (page 288), the test can be considerably simplified if we make $\frac{F_2}{R_1}$ (equation [A]) a submultiple of 10, for then the value of R_2 read off from the resistance box will at once give the value of F_1. Thus if F_2 were a condenser of, say, ·5 of a microfarad, and if R_1 were 5000 ohms, then the capacity of F_2 can be read off directly from R_2 to four places of decimals.

349. When a long cable has to be tested by this method Mr. A. Jamieson recommends* that K be depressed for five minutes to charge, and then raised for ten seconds for mixing previous to depressing k. It is also advisable to take the mean of several tests made alternately with zinc to line and copper to line.

350. With regard to the "Best conditions for making the test" it is advisable that the capacity of the condenser F_2 be

* Rankine's 'Useful Rules and Tables,' page 318.

as nearly equal to F_1 as possible, so that the potentials to which the two have to be charged may not differ to any very great extent. For if a long cable has to be tested, then inasmuch as the latter would have to be charged to a potential of at least 5 Daniells so as to swamp, as it were, any local charge, the potential to which the condenser (if small) would have to be charged would be very great; this would be liable to cause an error, from the fact that with a very high potential a certain amount of the charge becomes absorbed, and this charge would cause a deflection of the galvanometer needle over and above that due to the simple inequality between the actual free quantities in the two capacities. This abnormal deflection might of course be mistaken as being due to an incorrect adjustment of R_1 and R_2. If F_2 is about a fifth of F_1 it will not be too small for the purpose of the test.

The values given to R_1 and R_2 should be as high as possible so that their range of adjustment may be sufficiently wide. The battery power should be sufficiently high to enable a perceptible discharge deflection to be obtained when R_2 (the larger of the two resistances) is 1 unit out of exact adjustment; this is best determined by experiment.

We have therefore

Best Conditions for making the Test.

351. Make F_2 as nearly equal to F_1 as possible.
Make R_1 and R_2 as high as possible.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100}{R_2}.$$

GOTT'S METHOD.*

352. This method, devised by Mr. J. Gott, is shown by Fig. 102; it is executed as follows:—

The key K is first depressed and clamped down; this causes both the cable and condenser to become charged, since they are connected together in "cascade." R_1 or R_2 is now adjusted until it is found that on the depression of key k no deflection of the galvanometer G is produced. When this is the case then

$$F_1 : F_2 :: R_1 : R_2,$$

* 'Journal of the Society of Telegraph Engineers,' Vol. X., p. 278.

or

$$F_1 = \frac{R_1}{R_2} F_2.$$

It is obvious that we must have

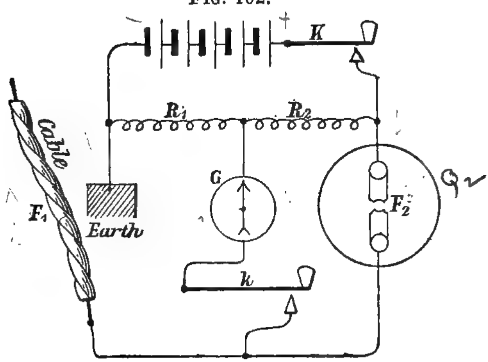
Best Conditions for making the Test.

353. Make F_2 as nearly equal to F_1 as possible. Make R_1 and R_2 as high as possible.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100}{R_1}.$$

FIG. 102.



DIVIDED CHARGE METHOD.

354. If a charged condenser has its two terminals connected to the two terminals of a second condenser which contains no charge, then the charge will become distributed over the two; and if the condensers be then separated, the quantities held by them will be directly proportional to their respective capacities. Thus, if Q_2 be the charge contained in a condenser whose capacity is F_2, then if it is connected to a condenser or cable whose capacity is F_1, the quantity Q which will remain in F_2 will be

$$Q = Q_2 \frac{F_2}{F_1 + F_2}.$$

From this we get

$$F_1 = F_2 \frac{Q_2 - Q}{Q}.$$

If therefore Q_2 be the discharge obtained from a condenser F_2 when full, and Q the discharge obtained from it when, after being charged from the same battery, it is connected for a few seconds to F_1, then the capacity of F_1 is given by the above formula.

For example.

A condenser of $\frac{1}{3}$ microfarad capacity (F_2), when fully charged, gave a discharge of 300 (Q_2). After being recharged and connected a few seconds to a piece of cable whose capacity F_1 was required, the quantity of charge remaining gave a discharge of 140 (Q). What was the capacity of the piece of cable?

$$F_1 = \frac{1}{3} \times \frac{300 - 140}{140} = \cdot 381 \text{ m.f.}$$

355. The capacity which the condenser F_2 should have in order that the test may be made as accurately as possible, may be thus arrived at:—

Let there be an error λ in F_1 caused by an error $-\delta$ in Q and a $+\delta$ error in Q_2, so that λ is as great as possible; we then have

$$F_1 + \lambda = F_2 \frac{Q_2 + \delta - (Q - \delta)}{Q - \delta} = F_2 \frac{Q_2 - Q + 2\,\delta}{Q - \delta};$$

but we know that

$$F_1 = F_2 \frac{Q_2 - Q}{Q}, \text{ or, } F_2 = F_1 \frac{Q}{Q_2 - Q};$$

therefore

$$F_1 + \lambda = F_1 \frac{Q}{Q_2 - Q} \times \frac{Q_2 - Q + 2\,\delta}{Q - \delta};$$

that is,

$$\lambda = F_1 \left\{ \frac{Q}{Q_2 - Q} \times \frac{Q_2 - Q + 2\,\delta}{Q - \delta} - 1 \right\} = F_1 \frac{(Q_2 + Q)\,\delta}{(Q_2 - Q)(Q - \delta)};$$

or, since δ is a very small quantity, we may say

$$\lambda = \frac{(Q_2 + Q)\,\delta}{(Q_2 - Q)\,Q}. \qquad [A]$$

We have then to find the value of Q which makes λ as small as possible. Now

$$\frac{(Q_2+Q)\,\delta}{(Q_2-Q)\,Q} = \frac{\delta}{Q_2}\left\{\frac{Q_2-Q}{Q} + \frac{2\,Q}{Q_2-Q} + 3\right\} =$$

$$\frac{\delta}{Q_2}\left\{\frac{Q_2-Q}{Q}\left[1-\frac{Q\sqrt{2}}{Q_2-Q}\right]^2 + 2\sqrt{2} + 3\right\},$$

and to make the latter expression as small as possible we must make

$$\left[1-\frac{Q\sqrt{2}}{Q_2-Q}\right]$$

as small as possible; that is to say, we must make

$$1-\frac{Q\sqrt{2}}{Q_2-Q} = 0,$$

or

$$\frac{Q\sqrt{2}}{Q_2-Q} = 1\,;$$

therefore

$$Q\sqrt{2} = Q_2 - Q,$$

or

$$Q\,(\sqrt{2}+1) = Q_2\,;$$

that is

$$Q = \frac{Q_2}{\sqrt{2}+1} = \frac{Q_2}{2\cdot 4142}.$$

It was pointed out, however, in a similar investigation which we made in (§ 94, page 84), that practically we may say, make—

$$Q = \frac{Q_2}{3},$$

or, in other words, the capacity of F_2 should be such that when it is connected to F_1 it should lose two-thirds of its charge. This is obtained, of course, by making F_2 equal to $\frac{F_1}{2}$.

356. The connections for the practical execution of the test would be very similar to those shown in Fig. 77, page 240, but the condenser or cable under trial would be substituted in the place of the battery. When it is a cable whose capacity is being measured, then terminal B would be put to earth, and the wire

shown as leading from B to the battery would be removed. The test would then be made in the following manner:

Key K_2 being pressed down so as to hitch on the "Insulate" trigger (Fig. 80, page 242), the condenser C would be charged by touching the terminals A and B with the wires from the two poles of the battery. The "Discharge" trigger of the key then being depressed, the discharge Q_2 is noted. The key then being again placed at "Insulate," and the condenser again charged up by the battery, the key would be pressed down on to its bottom contact; this puts the condenser C in connection with the trial condenser or cable. The "Discharge" trigger then being pressed, the discharge Q is noted.

The "Divided Charge" method, like that of Thomson or Gott, is very accurate when employed for measuring the capacity of long cables, or short cables coiled in tanks.

Best Conditions for making the Test.

357. Make F_2 as nearly equal to $\frac{F_1}{2}$ as possible.

Possible Degree of Accuracy attainable.

From equation [A] (page 304) it follows that

$$\text{Percentage of accuracy} = \frac{100\,(Q_2 + Q)\,\delta}{F_1\,(Q_2 - Q)\,Q},$$

where δ is the fraction of a division to which each of the deflections Q and Q_2 can be read.

358. By a modification of the foregoing method, due to Dr. Siemens, a comparatively small condenser may be used for measuring the capacity of long cables, or of condensers of high capacity. It may be called

SIEMENS' DIMINISHED CHARGE METHOD.

If we connect a condenser to a charged cable, the latter loses the amount which the condenser takes up, and if the condenser be discharged and then again connected to the cable, and again discharged, and this process be repeated several times, the quantity in the cable can be definitely diminished as much as we like. The quantity removed each time, however, is not the same, but becomes less and less after each discharge.

Let Q_2 be the quantity contained in the condenser, and Q_1 the quantity contained in the cable, when the two are charged full from the same battery. Then

$$Q_2 : Q_1 :: F_2 : F_1,$$

or

$$Q_1 = Q_2 \frac{F_1}{F_2}.$$

Supposing now the cable to be completely charged, and the battery taken off, and the condenser to be empty, then, on connecting the condenser to the cable, the charge the former will take will be

$$Q_1 \frac{F_2}{F_1 + F_2} = Q_2 \frac{F_1}{F_2} \times \frac{F_2}{F_1 + F_2} = Q_2 \frac{F_1}{F_1 + F_2},$$

whilst the quantity remaining in the cable will be

$$Q_1 \frac{F_1}{F_1 + F_2}.$$

On discharging the condenser and connecting it a *second* time to the cable, the charge it will take will be

$$Q_1 \frac{F_1}{F_1 + F_2} \times \frac{F_2}{F_1 + F_2} = Q_2 \frac{F_1}{F_2} \times \frac{F_1}{F_1 + F_2} \times \frac{F_2}{F_1 + F_2} = Q_2 \left(\frac{F_1}{F_1 + F_2}\right)^2;$$

consequently, after the *n*th application, the charge Q it will take will be

$$Q = Q_2 \left(\frac{F_1}{F_1 + F_2}\right)^n;$$

therefore

$$\frac{F_1}{F_1 + F_2} = \frac{\sqrt[n]{Q}}{\sqrt[n]{Q_2}};$$

from which

$$F_1 = F_2 \frac{\sqrt[n]{Q}}{\sqrt[n]{Q_2} - \sqrt[n]{Q}} = \frac{F_2}{\sqrt[n]{\frac{Q_2}{Q}} - 1}.$$

For example.

A condenser of 1·0 microfarad capacity (F_2), when full, gave a discharge equal to 300 (Q_2). A cable whose capacity was required was charged from the same battery which was employed to charge the condenser. The latter was then alternately connected to the cable, removed and discharged 16 times (n); on

the sixteenth occasion the discharge was noted, and it was found equal to 83 (Q). What was the capacity of the cable?

$$F_1 = \frac{1 \cdot 0}{\sqrt[16]{\frac{300}{83}} - 1} = 11 \cdot 97 \text{ m.f.}$$

359. In order to make this test as accurately as possible when it is applied to a cable, the repeated charges and discharges must be made with as little loss of time as possible, as during that time a leakage of the charge will be going on through the insulating sheathing of the cable; the accuracy of the test depends upon this leakage being nothing, or at least very small.

360. The connections for making the test would be similar to those employed in the foregoing one, and the practical execution would be the same with the exception that the trial condenser or cable, and not the standard condenser, would be charged from the battery, and in taking the repeated discharges the galvanometer would have to be short circuited.

Best Conditions for making the Test.

361. Make n equal to $\dfrac{\cdot 5552372}{\log. \left(\dfrac{F_1 + F_2}{F_1}\right)}$, approximately.*

* This may be proved as follows:—

In order to determine F_1 as accurately as possible from the equation

$$F_1 = \frac{F_2}{\sqrt[n]{\frac{Q_2}{Q_1}} - 1}, \quad \text{that is,} \quad F_1 = \frac{F_2}{\left(\frac{Q_2}{Q}\right)^{\frac{1}{n}} - 1},$$

we must determine $\left(\frac{Q_2}{Q}\right)^{\frac{1}{n}}$ as accurately as possible.

Let $\left(\frac{Q_2}{Q}\right)^{\frac{1}{n}}$ equal $\frac{1}{k}$, and let there be a small plus error δ in Q_2, and a small minus error δ in Q, and let there be a corresponding error λ_1 in k, that is, let

$$k + \lambda_1 = \left(\frac{Q_1 - \delta}{Q_2 + \delta}\right)^{\frac{1}{n}};$$

therefore

$$\lambda_1 = \left(\frac{Q_1 - \delta}{Q_2 + \delta}\right)^{\frac{1}{n}} - k.$$

Now

$$\left(\frac{Q_2}{Q_1}\right)^{\frac{1}{n}} = \frac{1}{k}, \text{ or, } \frac{Q_1}{Q_2} = k^n, \text{ or, } Q_1 = Q_2 k^n;$$

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{\delta}{n} \cdot \frac{(Q_2 + Q)\, Q_2^{\frac{1}{n}-1}}{Q\,(Q_2^{\frac{1}{n}} - Q^{\frac{1}{n}})}\, 100\,,$$

where δ is the fraction of a division to which each of the deflections Q and Q_2 can be read.

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therefore

$$\lambda_1 = \left(\frac{Q_2\, k^n - \delta}{Q_2 + \delta}\right)^{\frac{1}{n}} - k = k\left[\left(\frac{Q_2 - \frac{\delta}{k^n}}{Q_2 + \delta}\right)^{\frac{1}{n}} - 1\right],$$

But since δ is very small, we get

$$\lambda_1 = k\, \frac{Q_2^{\frac{1}{n}} - \frac{\delta}{n\, k^n} Q_2^{\frac{1}{n}-1} - Q_2^{\frac{1}{n}} - \frac{\delta}{n} Q_2^{\frac{1}{n}-1}}{Q_2^{\frac{1}{n}}} = -\frac{\delta\, k}{Q_2}\left(\frac{k^{-n}+1}{n}\right).$$

To make λ_1 a minimum we must make $\dfrac{\text{“}\,k^{-n}+1\,\text{”}}{n}$ a minimum.

Let

$$u = \frac{k^{-n}+1}{n},$$

then

$$\frac{du}{dn} = \frac{1}{n^2}\left[-\, n\, k^{-n} \log_e k - (k^{-n} + 1)\right] = 0$$

at a minimum; therefore

$$n \log_e k + 1 + k^n = 0\,,$$

or

$$\log_e k^n + 1 + k^n = 0\,,$$

or

$$\log k^n + (1 + k^n) \cdot 4343 = 0\,.$$

The solution of this equation may be obtained by the “trial” method, i. e. giving k^n various values until one is found which approximately satisfies the equation. If we make k^n equal to $\cdot 27846$ the equation will be very nearly satisfied, for

$$\log \cdot 27846 = \bar{1}\cdot 4447628 = -\cdot 5552372$$

and

$$(1 + \cdot 27846) \cdot 4343 = \cdot 5552352\,.$$

Now

$$F_1 = \frac{F_2}{\left(\frac{Q_2}{Q}\right)^{\frac{1}{n}} - 1}\,, \quad \text{or,} \quad \frac{F_1 + F_2}{F_1} = \left(\frac{Q_2}{Q}\right)^{\frac{1}{n}} = \frac{1}{k}\,;$$

therefore

$$\left(\frac{F_1}{F_1 + F_2}\right)^n = k^n = \cdot 27846\,;$$

362. It may be remarked that when a cable is tested for electrostatic capacity at the factory, it is immaterial whether the test be made by charging the cable positively or negatively; but in the case where the cable is laid, it is advisable to make two tests (or sets of tests), one with a positive and the other with a negative charge, and to take the arithmetic mean of the two results. It is rarely, however, that the two latter differ to any material extent.

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hence

$$n = \frac{\log \cdot 27846}{\log \dfrac{F_1}{F_1 + F_2}} = \frac{- \cdot 5552372}{\log \dfrac{F_1}{F_1 + F_2}} = \frac{\cdot 5552372}{\log \left(\dfrac{F_1 + F_2}{F_1}\right)}.$$

For example.

It being required to measure the exact electrostatic capacity of a cable whose capacity was 12 microfarads (F_1) approximately, a condenser of 1 microfarad (F_2) was used for the purpose. How many times (n) should the condenser be applied to the cable in order that the test may be made with the greatest chance of obtaining an accurate result?

$$n = \frac{\cdot 5552372}{\log \left(\dfrac{12 + 1}{12}\right)} = \frac{\cdot 5552372}{\cdot 0347622} = 16.$$

CHAPTER XIV.

THE THOMSON QUADRANT ELECTROMETER.

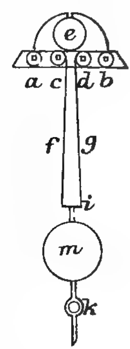
363. This is a most valuable and useful instrument for accurately measuring potentials.

Description.

Fig. 104 (page 312) gives a general view of the instrument.

In the small figure to the right, *n n* is a thin needle of sheet aluminium, shaped like a double canoe-paddle. It is rigidly fixed at its centre to an axis of stiff platinum wire *k* (Fig. 103), in a plane perpendicular to it. At the top end of the wire a small cross-piece *i* is fixed, to the extremities of which single cocoon fibres are attached. These fibres are fixed to small screws *c* and *d*, by the turning of which the length of the former can be altered. The small screws *a* and *b* enable the screws *c* and *d* to be shifted either to the right or left. Finally, by turning *e*, the screws *a* and *b* can be parted more or less, thereby separating the threads of suspension, and rendering the tendency of the needle to lie in its normal position more or less powerful.

Fig. 103.



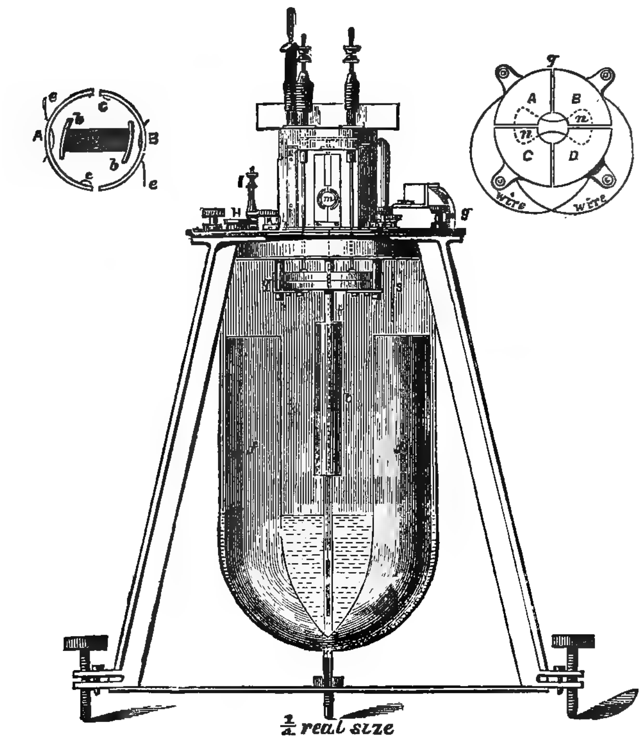
A little below the cross-piece *i* is fixed the mirror *m*, whose movements are reflected on a scale, as in a Thomson galvanometer (page 31). The platinum wire below the mirror passes through a guard tube *t* (Fig. 104), to prevent any great lateral deviation of the needle and its appendages, which might cause damage should the instrument receive any rough usuage. The guard tube itself is fixed to the framework from which the needle is suspended.

It will be seen in the figure that the needle is suspended, apparently, beneath four quadrants (*q*), A, B, C, and D. There are, however, four quadrants also below the needle, united to the top ones at their circumferences. The arrangement is in fact a round, flat, shallow box, cut into four segments.

The alternate segments are connected together by wires as shown in the figure.

Now, if the needle is electrified and the quadrants are in their normal unelectrified condition, and are placed symmetrically with reference to it, no effect will be produced on the needle. That is to say, the spot of light on the scale will be stationary exactly at the centre line.

FIG. 104.



But if the quadrant D, and consequently A, be electrified, then an attraction or repulsion will be exerted on the needle, causing it to turn through an angle proportional to the potential of the electricity.

As the angular movements are very small, the number of divisions of deflection on the scale will directly represent the degree of potential which the quadrants possess.

We can also connect another electrified body to C and B; the needle will then move under the influence of both forces.

To render the instrument of practical value, several conditions must be assured.

Let us suppose the needle to be electrified.

We stated that, at starting, the ray of light should point to the centre line on the scale. To ensure this, the quadrants must be symmetrically placed. This can be roughly done by hand, as means are provided for enabling the quadrants to slide backwards or forwards, and to be fixed by means of small screws, shown in the large figure. For obtaining the final position, one of the quadrants (B) is provided with a micrometer screw (*g*), which enables a fine adjustment to be given to it.

We must also have means of keeping the needle at one uniform potential for a considerable time.

The needle itself could only contain a very small amount of electricity, and a slight escape of this would seriously lower the potential, and make comparative measurements useless; for it is evident that the whole principle of the instrument depends upon the potential of the needle remaining constant during the time a set of experiments are being made.

To get over this difficulty a large glass jar, like an inverted shade, is provided, partially coated with strips of tin-foil (*f*) outside. Inside the jar, to about a third of its height, strong sulphuric acid is placed. This answers a threefold purpose. It enables the air inside it to be kept quite dry, thereby very perfectly keeping those parts insulated which require to be so; secondly, it holds a charge of electricity (acting as the inner coating of the jar); and thirdly, it allows the charge to be communicated to the needle without impeding its movements. This latter is effected by means of a fine platinum wire, which is attached to the lower end of the thick wire which supports the needle and mirror.

The fine wire dips into the acid, whose charge is thereby communicated to the needle.

To keep this wire from curling up out of the acid, and also to steady the movements of the needle, a small plummet of platinum is attached to the end of the wire, as will be seen in the figure.

A thick platinum wire, fixed to the lower extremity of the guard tube *t*, and reaching nearly to the bottom of the jar, is for the purpose of enabling the latter to be charged, in a manner to be explained.

So far, the jar answers the purpose of keeping the needle

supplied with electricity; but although this may prevent the potential from falling very rapidly, it will not prevent its doing so entirely.

The Replenisher.

364. As the instrument is extremely sensitive to very slight changes of potential, some means are requisite by which any small loss can be easily supplied without there being any fear of putting in too much.

This is effected by means of the "*replenisher*," whose principle we can explain by the help of the small cut to the left, in Fig. 104.

A and B are two curved metal shields, one of which (say A) is connected to the acid in the jar and the other, B, to the framework of the instrument, and through it to the foil outside the jar.

b and *b* are two metal wings insulated from one another by a small bar of ebonite, which is centred at *s*, so that it turns in a plane represented by the paper. The spindle is represented in the large figure by *s*, other parts being omitted for simplicity.

It will be observed that the wings curve outwards. This is done in order that they may make a short contact in their revolution with springs *c c* and *e e*. *c* and *c* are connected together permanently, but are insulated from the rest of the apparatus. *e* and *e* are connected to the shields A and B respectively.

Now let us suppose the wings to be rotated in the reverse direction to that in which the hands of a watch turn.

As soon as the left-hand wing comes in contact with the spring *c* at the lower part of the figure, the right-hand wing comes in contact with the other spring. The two wings being thus connected together, and under the influence of the shields, the electricity in A, which we will call positive, draws negative electricity to the wing close to it, and drives the positive to the other wing.

On being rotated a little farther the wings clear the springs, and being thus disconnected, each retains its charge.

Continuing the rotation, the right-hand wing, which had the positive charge communicated to it, comes in contact with the spring of shield A, and the charge is communicated to the jar, the negative electricity in like manner on the other wing running to the outer coating of the jar. The shields are now in a neutral condition, as at first, and on continuing the rotation the process is repeated.

Thus every turn increases the potential of the charge in the

jar, and by continuing the rotation we can augment this as much as we please.

By reversing the motion we can diminish the charge, if we require to do so.

The axis of the replenisher projects above the main cover, and is easily turned by the finger.

The Gauge.

365. But we still require some arrangement by which we can see whether we have kept the potential constant. This is done by means of a small "*gauge.*"

The gauge consists of two metallic discs having their planes parallel and close to each other. The lower of these planes, which will be seen dotted at the upper part of the figure, is in electrical connection with the acid of the jar from which it takes its potential. The upper disc is perforated with a square hole immediately over the centre of the lower disc.

A light piece of aluminium, shaped like a spade, has the part corresponding to the blade fitting in this square hole. At the point where the handle would be joined to the blade this spade is hinged, by having a tense platinum wire fixed to it, which runs at right angles on each side of the handle and blade, and lies in the same plane as the latter.

When the lower plate is electrified, it would attract the blade, thereby raising the end of the handle. So that if we notice the position of the end of the handle with respect to a mark, and see that it moves above or below it, we know that the electricity of the lower plate is either overcoming the tendency of the light platinum wire to keep it up, or is unable to do so.

If then we charge our jar to such a potential that the handle is situated close to the mark, and we keep it so, we know that the potential of the jar is constant. When we notice the handle sinking below the mark, we know that the potential of the electricity in the jar is falling; but a few turns of the replenisher will bring it up again.

In the actual arrangement, the rung of the handle is formed of a fine black hair.

Inside the handle there rises a small pillar, with two black dots on it. The sign of division $\div$ represents this, the line being the hair which, by the movement of the spade blade, rises above or below the two dots, which of course would be almost quite close together.

To enable the hair and spots to be seen distinctly, a plano-convex lens is placed a little distance off. Care must be taken,

in order to avoid parallax error, to keep the line of sight a normal to the centre of the lens.

We spoke of the lower disc, which becomes electrified by the jar, and which acts on the spade blade. Now it is evident that if the distance between the plates be always the same, and the elasticity of the platinum axial wire be also the same, to get the hair between the two spots is to obtain the jar at a particular fixed potential.

But we may require to get this potential, although the same whilst a certain set of experiments are being made, yet different for different series of experiments. This is provided for by enabling the lower disc to be lowered by screwing it round.

The Induction Plate.

366. To enable high potentials to be measured, an "*induction plate*" is added. It consists of a thin brass plate, smaller in area than the top of the quadrant beneath it, and supported from the main cover by a glass stem. It is provided with an insulated terminal I. The use of the plate will be explained later on.

367. A flat brass plate covers the mouth of the jar, and is secured to it so as to be air-tight and prevent the entrance of moisture.

A kind of lantern rises from the middle, which covers the mirror and its suspending arrangements, and above this a box with a glass lid protects the gauge.

The front of the lantern is of glass, which allows the ray of light to fall on the mirror and be reflected back on the scale.

Terminal rods or electrodes, in connection with each set of quadrants, pass through ebonite columns to the outside of the case, and have terminals attached to them. These electrodes can be pulled up and disconnected from the quadrants if necessary.

A charging rod (seen in Fig. 104 to the left of the left-hand quadrant terminal) also is provided, which can be turned round on its axis. It has at its lower end a small spring, fixed at right angles to it. By turning this terminal rod round, the spring can be brought in contact with the framework from which the needle is suspended, and thereby, through the medium of the guard tube and the platinum wire attached to it, the acid in the jar can be charged. When this is done, the spring is moved away, so that no accidental leakage can take place through it.

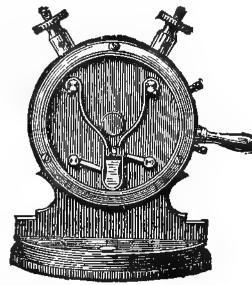
Various insulating supports are provided inside the jar and

lantern. One supports the guard tubes and the adjusting screws of the needle; others support the quadrants.

The whole arrangement is supported by a kind of tripod on a metal base, to keep it steady. There are also levelling screws, and a level on the brass cover, to enable the instrument to be properly levelled, so that the axis of the needle may swing clear of the guard tube.

H is a screw-capped opening through which acid can be introduced into the glass jar.

FIG. 105.



Reversing Key.

368. Fig. 105 represents a reversing key which is specially adapted for use with the instrument.

TO SET UP THE ELECTROMETER.

369. In setting up the instrument for use the following instructions * should be followed:—

The cover being unscrewed and lifted off and supported about 18 inches above the table, it will be observed that the stiff platinum wire to which the needle is attached just appears below the narrow guard tube enclosing it in the centre of the quadrants, and terminates in a small hook. The loop at the end of the fine platinum wire is to be slipped over this hook, so that the fine wire and plummet may hang from it. The wide guard tube, when in its proper position, forms a continuation of the upper guard tube, so as to enclose the fine platinum wire just suspended. It must therefore be passed upwards over the suspended wire, and neck foremost, until the neck embraces the lower part of the upper guard tube, where it must be fixed by the screw pin provided for the purpose; this pin is screwed in by means of one of the square-pointed keys, supplied with the instrument, fitting the square hole in its head. This being done, replace and fasten the cover, place the instrument on a sheet of ebonite or block of paraffin wax so as to insulate it, and level up by means of the circular spirit level on the cover.

Next unscrew and lift off the lantern and, if necessary, adjust the four quadrants so that they hang properly in their places, with their upper surfaces in one horizontal plane. The needle and mirror which have been secured during transit by a pin passing through the ring in the platinum wire just above the

* From instructions drawn up by the late Mr. W. Leitch.

guard tube, and screwed into the brass plate behind, must now be released by unscrewing this pin with the long steel square-pointed key, and placing it in the hole made for it in the cover just behind the main glass stem to prevent its being lost. The needle will now hang by the fibres.

The two quadrants in front of the mirror should now be drawn outwards from the centre as far as the slots allow, by sliding outwards the screws from which they hang, and which project above the cover of the jar with their nuts resting upon flat oblong washers; a better view will thus be obtained of the needle. The surfaces of the latter ought to be parallel to the upper and under surfaces of the quadrants, and midway between them. This will be best observed by looking through the glass of the jar just below the rim. If the needle requires to be raised or lowered, it is done by winding up or letting down the suspending fibres, that is, by turning the proper way the small pins *c*, *d*. The suspending wire which passes through the centre of the needle should also be in the centre of the quadrants. This is best observed when the quadrants have been moved to their closest position. The fourth quadrant is moved out or in by the micrometer screw *g* with the graduated disc overhanging the edge of the cover. A deviation of the suspending wire from its proper central position, as was explained at the beginning of the chapter, may be corrected by means of the small screws *a*, *b*, *c*, and *d*. When proper adjustment is attained the black line on the top of the needle should be parallel to the transverse slit made by the edges of the quadrants when these are symmetrically arranged.

The sulphuric acid may now be put into the jar. For this purpose, the strongest sulphuric acid of commerce is to be boiled with some crystals of sulphate of ammonia, in a florence-flask supported on a retort-stand over a jet of gas or other convenient source of heat. It is recommended to boil under a chimney, so that the noxious fumes rising from the acid may escape. To guard against the destructive effects of the acid in the event of the flask breaking by the heat, there should be placed beneath it a broad pan filled with ashes, or it should stand above a fire-place containing a sufficient quantity of cold ashes. A little sand put into the flask will lessen the risk of breaking. The object of boiling the acid is to expel the volatile acid impurities which will otherwise impregnate the air inside of the jar and tarnish the works. When cool, the acid may be best poured into the jar through a glass filler with a long stem inserted through the screw opening H provided for the purpose. The stem of the filler should reach the bottom of the jar, to avoid

splashing upon its sides or upon the works, and in removing it care should be taken that it is drawn out without its end touching any of the brasswork. The acid may be poured in till the surface is about an inch below the lower end of the wide brass tube which hangs down the middle of the jar. It must at least reach the three platinum wires hanging from the works.

370. The instrument thus adjusted and charged with acid should be allowed to rest for some little time so that any films of moisture on the insulating portions of the apparatus may become absorbed.

The scale should now be placed at the proper distance so that the reflected image is sharply defined and stands at the middle of the scale, that is, at 360; for the electrometer scale (unlike that of a galvanometer) is graduated from 0 to 720, 360 being the middle point. Care must be taken that the two ends of the scale are equidistant from the centre of the mirror.

Next connect together the two electrodes of the quadrants and the induction plate electrode, by means of a piece of thin wire joined to the cover of the jar; also turn the charging rod so that it touches the framework of the platinum wire of the needle.

Now charge the jar positively by means of a few sparks from a small electrophorus, the frame of the instrument being put to earth for the purpose, and afterwards disconnected. When the proper potential is reached, it is indicated by the lever of the aluminium balance rising; the charging rod should then be turned so as to disconnect the latter from the needle. The replenisher must now be used to adjust the charge exactly, so that the hair may stand between the black spots when observed through the lens. When the lever carrying the hair is at either extremity of its range, it is apt to adhere to the stop; in using the replenisher to bring it from either limit, therefore, it is necessary to free it from the stop by tapping the cover of the jar with the fingers.

If the charge has caused the reflected image to be deflected from the middle of the scale, it may be brought back to that position by turning the micrometer screw which moves the fourth quadrant, and, if necessary, sliding out or in one or more of the other quadrants.

The small percentage of the charge lost from day to day may be recovered by using the replenisher. Under ordinary conditions this loss will not amount to more than ½ per cent. per day.

The charge may suffer loss from several causes, the most prevalent being the presence of dust on portions of the apparatus inside the jar. Every portion should be carefully dusted

with a camel-hair brush, and especially the round induction plate beneath the aluminium balance.

Loss may occur by shreds inside of the quadrants drawing the charge from the needle. It should be ascertained whether this takes place. Insulate alternately each pair of quadrants by raising the corresponding electrode, while the other pair are connected through their electrode with the cover. If the reflected image in either case keeps moving slowly along the scale, for instance over 20 scale divisions in half an hour, the charge in the jar being at the same time kept constant by the use of the replenisher if necessary, the insulated pair of quadrants is receiving a charge from the needle. In that case the inside of the quadrants may be brushed with a light feather, or camel-hair brush, after sliding them outwards as far as the slots allow, and securing the needle in the position in which it was fixed during transit; care being taken not to press upon the needle so as to bend it or the suspending wire. Without securing the needle, each quadrant may be drawn outwards and brushed, while the needle is deflected away from it by the screws *a*, *b*, or by any obvious means of keeping the needle deflected, care being taken not to strain the fibres.

Another possible source of loss of charge is want of insulation over the portion of the glass jar above the acid. If the percentage of the charge lost from day to day be so considerable as to require much use of the replenisher to recover it, the glass should be cleaned with a wet sponge, rubbed with soap at first, or with a piece of hard silk ribbon, wet and soaped at first, then simply wet with clean water, which may be drawn round the glass to clean every part of it. The ribbon being dried before a fire, may be used in the same manner to dry the glass.

If everything fails to make the apparatus keep its charge, the cause is probably due to a defective glass jar, and this can only be remedied by the manufacturers.

371. The good insulation of the instrument being satisfactorily accomplished, the symmetrical suspension of the needle by the fibres should be tested. The conditions sought to be realised are, that in the level position of the instrument the needle may hang with equal strain on the two fibres, and in a symmetrical position with regard to the four quadrants. It is plain that if these conditions be fulfilled the deflection produced by the same electric force in the level position of the instrument, will be less than it will be in any position of the instrument which throws the greater part of the weight on one fibre, or brings the needle nearer to any part of the inner surface of the quadrants than it is in its symmetrical position, which is its position of greatest

distance from all the quadrants. To make the test, the two quadrant terminals should be connected to the two poles of a single-cell battery, and the deflections produced upon the scale compared, while the instrument is set at different levels, by screwing one or more of the three feet on which it is supported. At each observation the extreme range, or difference of readings got by reversing the battery, should be noted. If the range diminishes as one side of the instrument is raised, the suspending fibre on that side must be drawn up, by turning very slightly the small pin *c* or *d*, round which it is wound, and another series of observations taken in the same manner, beginning with the instrument levelled. Instead of drawing up one fibre, the other may be let down, to keep the needle midway between the upper and under surfaces of the quadrants, and after each alteration of the suspension it will be necessary to readjust the screws *a*, *b*, to make the black line on the needle hang exactly midway between the quadrants when the needle is undisturbed by electricity. It will be observed also that the charge of the jar is lost by touching these screws, unless the insulated key is used. They are reached without taking off the lantern by screwing out a vulcanite plug in the glass window in front of them.

In deflecting the instrument much from its level position, the guard tube may be brought into contact with the wire hanging from the needle, and the movements of the latter be thus interfered with by friction. When the needle vibrates freely, it will be observed that the image comes to rest in any position to which it may be deflected, after vibrating with constant period and gradually diminishing range on each side of this position of rest. The occurrence of friction is shown by the needle coming to rest abruptly, or vibrating more quickly than proper. The reading obtained under these circumstances is, of course, of no value. The quicker vibrations obtained in using the induction plate must not be mistaken for vibrations indicating friction, from which they may be easily distinguished by their regularity.

If, as may possibly happen, the process of observing the deflections at different levels, and drawing up the fibre on that side which is being raised while getting less sensibility, should only lead the operator to draw up one fibre till it bears the whole weight, while the other is seen to hang loosely, he should adjust them as nearly as he can by the eye to bear an equal share of the weight, and examine the position of the needle by looking through the glass of the jar just below the rim, the two quadrants in front of the mirror being drawn out, and the

lantern taken off to let in plenty of light. He will probably find that the needle leans slightly downwards relatively to the quadrants on that side which he was drawing up while getting smaller deflections. To correct this is a delicate operation, which should only be attempted by a very careful operator. Though perfect symmetry of suspension is aimed at, it is not essential to the utility of the instrument. If it be desired to make the correction, first secure the needle as during transit; take off the cover, and while it is held by a careful assistant, or properly supported in a position in which it may be levelled, remove the lower guard tube (the wide brass tube hanging down the centre) after screwing out the small pin in its neck. It will be observed that the upper and narrower guard tube consists of two semi-cylindrical parts united. The part in front may now be removed by taking out the two screws which fasten it at the top, and the platinum wire which carries the needle may be examined. If it has got bent it must be straightened; if not, it may be bent carefully just above the needle, so as to raise that end of the needle which was observed to hang lowest. If the cover be supported so that it may be levelled, the needle may be set free, and the operator may observe whether he has succeeded in making it hang parallel to the surfaces above and below it. The needle must not, however, be allowed to hang by the fibres, while bending the platinum wire, or while removing or replacing the guard tubes.

The works being replaced, the process of observing the deflections at different levels and adjusting the tension of the fibres should be repeated, with the view of getting minimum sensibility in the level position.

The two unoccupied holes bored through the cover and flange of the jar are intended to receive the square-pointed keys, when not in use.

Grades of Sensitiveness.

372. There are several ways of making the connections to the terminals of the quadrants, frame, and induction plate, so as to get various degrees of sensitiveness for measuring potentials of various strengths.

1st *Grade.*

The following is the most sensitive arrangement, such as would be used for measuring the potential of a Daniell cell:—

One pole of the battery would be connected, through the medium of a reversing key (Fig. 105, page 317), to one quadrant terminal, and the other to the frame of the instrument and to

the second quadrant terminal. This, by reversing the key, would give about 50 divisions on either side of the 360, equal to 100 in all.

2nd Grade.

Leaving one pole of the battery to the frame, the next degree of sensitiveness is obtained by disconnecting the pair of quadrants that are connected to the frame, the electrode being raised for the purpose; the other connections must be the same as in the last case. By this arrangement the needle is acted upon by one pair of quadrants only.

373. By using the induction plate we may still further diminish the sensitiveness of the instrument. For instance, when we connect the pole of the battery to a pair of quadrants, those quadrants take the potential that it has; but if we connect it to the induction plate, then the charge in the quadrant below is only an induced one, and, since there is an interval between the plate and the quadrant, this induced charge will be small, and the effect on the needle proportionally small. Again, if we disconnect one pair of quadrants, and connect the wire from the battery to the induction plate and to the corresponding quadrants, then the charge will be partially *bound*. The effect on the needle will therefore be less still. The actual number of grades of sensitiveness with the induction plate are as follows:—

3rd Grade.

One pair of quadrants connected to one pole of battery. Induction plate and second pole of battery connected to frame. Second pair of quadrants disconnected by raising electrode.

4th Grade.

One pair of quadrants connected to one pole of battery, and also to induction plate. Second pole of battery connected to frame. Second pair of quadrants disconnected by raising electrode.

5th Grade.

Induction plate connected to pole of battery. One pair of quadrants and second pole of battery connected to frame. Second pair of quadrants disconnected by raising electrode.

6th Grade.

Induction plate connected to pole of battery. Second pole of battery connected to frame. Both pairs of quadrants disconnected by raising the electrodes.

374. We can in each of these cases interchange the terminals of the quadrants, that is to say, we can use the left terminal where we used the right, and *vice versâ*.

375. There is one more point to mention in connection with the instrument, and that is, that it may be found, on raising one of the electrodes to disconnect it from the quadrants, that the act of doing so causes the image on the scale to deviate a few degrees from zero in consequence of a current being induced thereby.

In the most recent form of instrument there is a small milled vulcanite head provided, by turning which the quadrants are connected to the frame, and the charge being thereby dissipated, the image returns to zero. When this is done the milled head must be turned back before commencing to test again.

The Use of the Electrometer.

376. The electrometer can be used in every test where a condenser is usually employed.

In using the condenser we have to charge it, and then note its discharge on the galvanometer, which gives the potential. With the electrometer we have simply to connect to its terminals the wires which would be connected to the condenser, and the permanent deflection on the scale gives us the potential, which can be observed at leisure.

Thus, in measuring the resistance of a battery by the method given on page 258 (§ 294), we should first connect the battery wires to the electrometer (through the medium of the reversing key is best), note the deflection, then insert the shunt, again note the deflection, and calculate from the formula.

The great value of the electrometer, however, lies in the fact of its enabling us to notice the continuous fall of charge in a cable, and not, like the condenser method, merely to determine what the potential has fallen to after a certain time. We can see with unfailing accuracy when the charge has fallen to one-half or any other proportion we please.

We see, in fact, exactly what is going on in the cable at any moment.

The connections for such a test could not well be simpler. We charge the cable, connect it to the electrometer, the frame being to earth, and then notice the deflection as it gradually falls down the scale. We do not even require a battery, as we can charge the cable with a few sparks from an electrophorus.

The degree of sensitiveness necessary for any particular cable we can, of course, only tell by experience.

Measurements from an Inferred Zero.

377. When very high resistances, such, for instance, as short lengths of highly insulated cable, are measured by the ordinary fall of charge method, the fall, even in a considerable time, would be so small that the test would be an unsatisfactory one, for the difference between the deflection at the beginning of the test, and that after the interval of time, could only be a small fraction of the whole length of the scale; and if the deflections are not accurately noted, still less can we be satisfied of the correctness of our result when worked out from a formula.

By means of a plan suggested by Professor Fleeming Jenkin, however, such high resistances can be measured by the fall of charge method with considerable precision.

Professor Jenkin's improvement consists in virtually prolonging the scale and counting the divisions from an *inferred zero.*

An explanation of the method of making the test will best show what an inferred zero is.

One pole of the battery being to earth, the other pole is connected to one pair of quadrants and to the framework of the instrument.

The second pair of quadrants is connected to the cable.

By joining for an instant the two pairs of quadrants together, the cable and quadrants take the same potential; therefore, at the moment of disconnecting them, the needle will be at zero.

The potential, however, of the cable, and the quadrants connected to it, will fall, and the needle be deflected.

Suppose, now, one cell connected to the electrometer gave 100 divisions deflection, and suppose the battery which charged the cable was 100 cells, then if the cable lost 1 per cent. of its charge, the charge remaining would be 99, and as the other quadrant, being permanently connected to the 100 cells, has the potential of 100, the difference between the two is $100 - 99 = 1$ cell, which, as we have said, gives 100 divisions. The 2 per cent. loss would give 200 divisions, and so on, whereas by the method mentioned on the last page, if we get 300 say, at first, then 1 per cent. loss would only move the image down to 297, and 2 per cent. would move it down to 294.

When all the charge is lost, the deflection would evidently be $100 \times 100 = 10{,}000$, which is the inferred zero. To obtain this zero for any particular battery, we should have to get the deflection from 1 cell and then determine, by the method given on pages 250 and 262 (§ 299), what the electromotive force of the testing battery is in terms of the 1 cell. Then by multiplying the 1 cell deflection by this value we get what we require.

The numbers representing the potentials we must evidently get by subtracting the deflections on the scale from the inferred zero.

To obtain the full range of the scale we should, at starting, get the image on the actual marked zero, which is, as we have before said, at the end, and not at the middle of the scale.

CHAPTER XV.

MEASUREMENT OF HIGH RESISTANCES.

378. The highest resistance which it is possible to measure by means of the Wheatstone bridge described at the commencement of Chapter VIII., is 1,000,000 ohms. It is true that some bridges have another set of resistances in the top row, which will enable the ratio 10 to 10,000 to be used, and consequently a resistance of

$$\frac{10{,}000 \times 10{,}000}{10} = 10{,}000{,}000 \text{ ohms}$$

to be measured; but this is not often the case, and the values of resistances much greater than this frequently require to be determined.

For this purpose a modification of the deflection method given in Chapter I., page 5 (§ 9), must be adopted.

379. Provide a single, and also about 100 constant cells. Find their respective electromotive forces by the discharge method given on pages 250 and 262 (§ 299). Thus, suppose the discharge taken from the 1 cell, which, as we have explained, should be taken first, gave a deflection of 300, the galvanometer shunt (S_2) being adjusted for this purpose to 560 ohms. Suppose also that the discharge from the 100 cells in the place of the 1 cell, gave a deflection of 302, with a shunt (S_1) of 6 ohms; then by multiplying the 302 by

$$\frac{G + S_1}{S_1}$$

we get the deflection we should have had if no shunt had been used; this will represent the electromotive force of the 100 cells. In like manner, by multiplying the 300 by

$$\frac{G + S_2}{S_2}$$

we get a number representing the electromotive force of the 1 cell. Taking the resistance of the galvanometer (G) to be 5000 ohms, and giving the other numerical values to the

quantities, the ratio of the electromotive force of the 1 cell to the electromotive force of the 100 cells would be

$$\frac{5000 + 560}{560} \times 300 : \frac{5000 + 6}{6} \times 302\,,$$

or as

$$2980 : 252{,}000\,.$$

If now we divide the greater number by the less, we get the value of the 100 cells in terms of the 1 cell. This value is 84·6, that is to say, the 100 cells are 84·6 times stronger than the 1 cell, and not 100 times. This might arise from some of the cells being defective, or imperfectly insulated. This does not matter, however, so long as we determine, as we have done, *how* much more powerful the 100 cells are than the 1 cell.

Calculation may be saved in the foregoing measurement if we adjust the galvanometer, by means of the directing magnet, so that a convenient discharge deflection is obtained with the 1 cell when there is *no* shunt between the terminals of the instrument. The exact value of this deflection being noted, the discharge deflection from the 100 cells is next taken with the $\frac{1}{99}$ shunt (page 43, § 48); then the latter deflection multiplied by 100 and divided by the first deflection, obviously at once gives the value of the 100 cells.

380. Having found the value of the 100 cells in terms of the single cell, we next proceed to join up the galvanometer, with a shunt, &c., between its terminals, in circuit with a resistance coil and the single cell, as shown by Fig. 106.

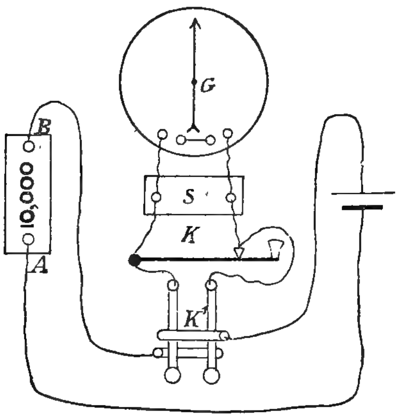
Put a resistance of 10,000 ohms in A B (a resistance of 10,000 ohms in a separate box is often used for this measurement), and having first inserted all the plugs in S, press down the short-circuit key, and proceed to remove some of the plugs, until a deflection of, say, 300 is obtained, then raise the key and see if the spot of light comes back to zero properly: if it does not, then by disconnecting one of the wires, see that the cause is not from the short-circuit key not making proper contact. If this has not the required effect, the adjusting magnet of the galvanometer must be slightly shifted, and, if necessary, put a little lower down, so as to make the needle a little sensitive. After a few trials this will be satisfactorily done, and the spot of light will always come back to the zero point when no current is passing through the galvanometer.

Let the deflection be $301\frac{1}{2}$, the shunt being 7 ohms.

Multiply 301·5 by $\frac{5000 + 7}{7}$, which gives 215,700.

This is the deflection we should get through 10,000 ohms, with no shunt to the galvanometer. There is really in the circuit, besides the 10,000 ohms, the resistance of the 1 cell, and also the resistance of the galvanometer and shunt combined

FIG. 106.



(which will be practically 7 ohms), but this will be so small as to be of no consequence; it may, however, be added on to the 10,000 when working out the results, if preferred.

Now, if we had used the 100 cells instead of the 1 cell, our deflection would have been 84·6 times as great as it was with the 1 cell. If, then, we multiply 215,700 by 84·6 we shall get the deflection obtainable with the 100 cells through a resistance of 10,000. This value will be found to be 18,248,000. Multiplying this number by 10,000 we get the *constant;* this constant is obviously the theoretical resistance which would give a deflection of 1 division with the 100 cells.

If it is required to use, say, 200 cells instead of 100 only, then in cases where galvanometer shunts of a fixed value ($\frac{1}{9}$th, $\frac{1}{99}$th, $\frac{1}{999}$th), only, are available, it would be advisable to employ 2 cells instead of the 1 cell, for making the test, so as to cause the deflections to be of an approximately equal value (page 58, § 61); this would not of course alter the foregoing process of calculation in any way, it would only result in the numerical value of the "constant" being different The actual number of cells used, it may be pointed out, has nothing to do

with the calculations; in fact, it is usual to speak of the 100, or 200, cells as the "battery" simply. A *one*-cell battery is used for producing the permanent deflection through 10,000 ohms, because 100 cells would deflect the spot of light off the scale with the lowest shunt that could be used; one cell happens to be a convenient electromotive force to employ, but, as pointed out, it might be preferable to use two, or even more, in certain cases.

It may be pointed out that the constant deflection with 1 cell through 10,000 ohms may usually be taken with the $\frac{1}{999}$ shunt in the place of a shunt of a particular numerical value (as in the foregoing example); this simplifies calculation, as we have then simply to multiply the constant deflection by 1000 instead of by $\frac{G+S}{S}$.

381. The foregoing process is simplified by using a resistance of 1,000,000 in the place of 10,000. The constant can then be found with the "battery" at once.

382. Having measured and worked out the constant (which is best done by the help of logarithmic tables*), we insert the resistance which is to be measured, in the place of A B, using the 100 cells in the place of the 1 cell. Having adjusted S till a deflection of 300, or near to 300, is obtained, note S and also the deflection. Let S be 2500, and deflection 298. Then the deflection without the shunt would be

$$298 \times \frac{5000 + 2500}{2500} = 894.$$

Dividing the "constant" by this number, we get

$$\frac{182{,}480{,}000{,}000}{894} = 204{,}100{,}000 \text{ ohms},$$

which is the value of the resistance.

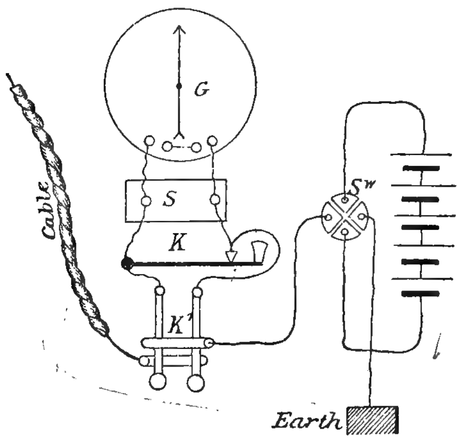
Practically, we may say the value of the resistance is 204,000,000 ohms, or 204 megohms, for inasmuch as we can only be certain of the values of the observed deflections to 3 places of figures, so we can only be certain of the worked out values to 3 places of figures. A great deal of time is often wasted in working out results to 5 or 6 places of figures when, in the observations necessary to obtain these results, it is impossible to be certain of their value beyond 3 places of figures.

* 'Chambers' Mathematical Tables' are those generally used.

MEASUREMENT OF THE INSULATION RESISTANCE OF A CABLE.

383. In measuring the insulation resistance of a cable, the constant having been taken in the foregoing manner, we should join up the galvanometer, shunt, short-circuit key, reversing key, battery switch, battery, and cable, as shown by Fig. 107.*

FIG. 107.



By having both a galvanometer reversing key and a battery switch the trouble of reversing the wires on the galvanometer, when the battery current is reversed, is avoided, as it can be done more readily by means of the key. The object of reversing the galvanometer connections when the battery is reversed is to obtain the deflection always on the same side of the scale.

384. Both ends of the core of the cable must be trimmed by means of a sharp and clean knife, care being taken that the outer surface of the gutta-percha, which has been exposed and oxidised by the air, is completely cut away; the clean surface thus exposed should not be touched with the fingers. It is a good plan to paint the trimmed ends with hot paraffin *wax* (not *oil*).

The ends being thus carefully insulated, and the further end left hanging free, so as not to touch anything, the nearer end of the cable must be connected, through the medium of the lead wire, to the terminal screw of the reversing key, as shown in

* See also 'Rymer Jones' Combination Key for Cable Testing,' Appendix.

Fig. 107, care being taken not to touch the trimmed end in doing so. The switch plugs being inserted, the reversing key which puts the zinc pole to the cable must be clamped down, and (the short-circuit key being depressed) sufficient resistance inserted in the shunt to obtain a deflection of about 300.

At the end of a minute from the time the *reversing* key was clamped down, the exact deflection should be noted.

385. The deflection obtained, it will be found, is not a permanent one, but will gradually decrease as the current is kept on, falling rapidly at first, and then more slowly, until at length it becomes practically stationary; the continued action of the current, in fact, increases the resistance of the dielectric. This phenomenon is known as *electrification*, and its cause is not well understood; it seems to be due to some kind of polarisation.

The following shows the decrease in the deflection observed with a piece of cable core insulated with gutta-percha:—

Minutes' electrification.	Deflection.
½	344
1	304
2	280
3	264
4	256
5	252
10	236
15	222
20	216
25	212
30	208
35	204
40	204
50	200

386. Increase of resistance by electrification is much more marked at a low than at a high temperature; thus in an actual experiment it was found that with a piece of core (insulated with gutta-percha) at a temperature of 0° C. the deflection fell from 240 to 75 in 90 minutes; whereas with the same piece of core at a temperature of 24° C. the deflection fell from 240 to 173 only, in the same time.

The rate at which the resistance increases, also depends upon the nature of the insulating material; it is quicker in some kinds of gutta-percha than in others. If the latter material is of good quality, the rate of fall between the 1st and 2nd minute would not be less than 5 per cent. In indiarubber the increase is very rapid, being as much as 50 per cent. between the 1st and 5th minute.

387. If the cable or insulated wire under test is quite sound, the electrification should take place perfectly regularly, that is to say, the deflection on the galvanometer scale should decrease steadily. An unsteady electrification, as a rule, is a sign that the insulation is defective. It sometimes happens, however, that the unsteadiness is due to the testing battery being in a bad condition, or not properly insulated; if, therefore, the electrification is such as to raise a suspicion that the insulation of the cable or insulated wire under test is not perfect, the battery should be looked to to see whether it is in proper order. An unsteady electrification may also be caused by the ends of the cable or of the lead wire not being properly trimmed, or from their becoming damp. Before concluding, therefore, that the cable is faulty these points should be attended to.

A third cause of unsteady electrification occasionally exists in factories; this is due to induced currents set up by the movement of the machinery in the proximity of the tanks in which the cable is coiled. When a cable is being tested on board ship the rolling of the latter induces comparatively strong currents in the cable, and causes the galvanometer deflections to be very erratic. The effects of these currents, in both cases, may be completely got rid of by the simple device of making the insulation test with *both* ends of the cable connected to the testing apparatus, instead of with one end only.

388. It is found with a good cable, that if the battery be taken off after electrification has proceeded for some time, and the cable be put to earth through a galvanometer, a continually decreasing current will flow through the latter from the cable; and if the deflections be noted after intervals of time equal to those during which deflections were noted whilst the battery was kept on the cable, it will be found that the relative values of the two sets of deflections will correspond. This will only be the case, however, if the cable be sound, and therefore the correspondence of the two sets of deflections may be taken as a guarantee of the good condition of the cable. It is rarely, however, that a test of this kind is made.

389. Although the deflection after the first and second minute with a zinc current is usually all that is required when testing each of the lengths of core (2 knots) of which a cable is composed, yet, when the cable is complete, the deflections for several successive minutes (usually fifteen) with both currents should be observed and noted. In this case the deflections having been observed with the cable connected to one pole of the battery, the latter should be taken off, and the cable put to earth to

discharge it by shifting the plugs in the battery switch, the one reversing key of the galvanometer being still left clamped down. The cable does not get rid of its charge immediately, as a considerable amount remains *absorbed;* hence it is necessary to keep it connected to earth for some time. If the length of the cable does not exceed 10 or 15 miles, a quarter of an hour will usually be sufficient to render it neutral, but greater lengths require a proportionally longer time. It can easily be seen when the absorbed charge is got rid of, for if the cable is neutral no deflection will be observed on depressing the short-circuit key, but if a charge is still retained a slight permanent deflection will be produced.

390. When the cable is discharged to earth great care must be taken that the short-circuit key K is first raised, otherwise the whole discharge will pass through the galvanometer coils and the needles be demagnetised, or, at least, their magnetic power will be altered.

391. As soon as the cable is found to be neutral, the second reversing key of the galvanometer should be clamped down, and the first one released, so as to reverse the galvanometer; the plugs of the battery switch should then be inserted so that the battery sends its current to the cable in the reverse direction to that it did at first; this being done, the deflections on the galvanometer should be noted at intervals of a minute, as before, until the same number of readings are obtained.

392. It is usual to take readings, first with the zinc pole and then with the copper pole, of the battery, connected to the cable.

If the cable is in good condition the readings with the zinc and copper currents will be the same.

If there is not time to take readings both with the zinc and copper currents the zinc should be the one employed, as in case of a fault it renders the latter very apparent; the copper current has the effect, to a certain extent, of sealing up a defect.

The fall of the deflection in both cases will be quick at first, and will afterwards slowly decrease.

The measurements being made, the resistance at the end of each minute may be worked out from the different deflections obtained.

393. When the cable is connected to the testing instruments by a long leading wire, then at the commencement of the test the end of the lead should be disconnected from the cable, and insulated; if any deflection is observable on the galvanometer when the battery current is put on, this deflection must be subtracted from the deflection obtained when the cable is attached to the lead. In making this correction care must be taken that

the same shunt (if any) is connected to the galvanometer as will be employed when the cable is connected to the lead, or if no shunt is used with the lead the necessary allowance for this must not be forgotten to be made.

The ends of the lead must be trimmed in the same manner as the ends of the cable.

The practical way of noting down and working out these tests will be found in Chapter XXVI.

394. When a large number of cables have to be tested daily at a factory any contrivances or methods for shortening calculations are of great value. Now the use of shunts of different values for obtaining readable deflections on the galvanometer scale with different cables is continual, and the working out of the multiplying power of these shunts is a somewhat tedious operation when a large number have to be calculated. If the resistance of the galvanometer used for making the tests were constant, a small table could easily be calculated which would show the multiplying power of any particular shunt at a glance; but the resistance of a galvanometer varies considerably with change of temperature, and therefore under ordinary conditions a table of the kind cannot be employed.

A very simple method of getting over this difficulty, due, it is believed, to Mr. Herbert Taylor, has been adopted in the testing rooms of the Telegraph Construction and Maintenance Company. The method is to have a small set of resistance coils directly in circuit with the galvanometer, so that the resistance of the latter can practically be always preserved the same.

The resistance of the ordinary reflecting galvanometer usually averages between 5000 and 6000 ohms; by having the galvanometer wound, therefore, so that in the hottest weather the latter value is never exceeded, and by having a set of resistance coils adjustable from 1 up to about 1000 ohms, the resistance in the circuit can always be kept up to 6000 under all conditions, and therefore a table giving the multiplying power of shunts for a galvanometer of 6000 ohms resistance can always be made use of. Tables of this description will be found at the end of the book. The tables also give the combined resistance of the galvanometer and shunt, which is sometimes required to be taken into account.

CHAPTER XVI.

MEASUREMENT OF RESISTANCES BY POTENTIALS.

395. There are two distinct ways of measuring resistances by potentials:—

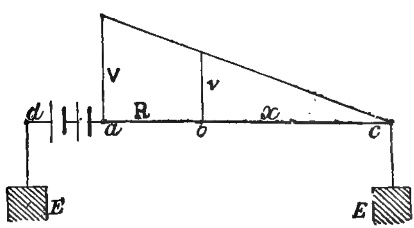
1st. By noting the fall of potential along a known resistance with which the unknown resistance is in connection.

2nd. By noting the rate at which a condenser, of a known capacity, loses its potential when it discharges itself through the unknown resistance.

Fall of Potential Method.

396. If we connect a battery to a resistance $R + x$, as shown by Fig. 108, the potential of the battery may be regarded as* falling regularly along the resistance, being full at a and zero

Fig. 108.



at c. The same would be the case if c and d were connected together instead of being put to earth. By similar triangles we have

$$V : v :: R + x : x,$$

therefore

$$V\,x = v\,R + v\,x,$$

or

$$x\,(V - v) = R\,v,$$

* See Chapter XI., page 248, § 284.

from which

$$x = \mathrm{R}\frac{v}{\mathrm{V} - v}; \qquad [A]$$

V being the potential at a, and v the potential at b. So that, if R is a known resistance, we can—by observing the values of V and v—determine the value of x.

For example.

If R = 1000 ohms, V = 300, and v = 200, then

$$x = 1000\frac{200}{300 - 200} = 2000 \text{ ohms.}$$

397. The relative values of the potentials can be measured by means of a condenser. To do this we should join up our condenser and galvanometer, as shown by Fig. 77, page 240, the only difference being that the terminals which are there represented as being in connection with a battery would, in the present case, be connected to the points a and d (or c) for determining V, and to b and d (or c) for determining v. The condenser discharges in the two cases give V and v.

Another, and for most cases a preferable, method of measuring the potentials, is to insert a galvanometer between the point at which the potential is to be measured and the earth, there being in the circuit a resistance several thousand times greater than the resistance of the conductor of the cable. The permanent deflections in this case indicate the potentials (§ 286, page 249).

398. Instead of measuring the potential V, we can, if we please, at once determine the value of V − v by connecting the wires from the condenser, &c. (or from the galvanometer and high resistance), to the points a and b; the deflection in this case at once gives us V − v. So that if we call v' this difference of potential, we get

$$x = \mathrm{R}\frac{v}{v'}. \qquad [B]$$

399. The conditions for making the test by formula [A] in the best possible manner are precisely similar to those in the case of the "Divided Charge Method" of measuring the electrostatic capacity of a cable or condenser (page 303); for equation [A] in this latter test is similar to equation [A] (§ 396) of the test under consideration. We must, in fact, adjust R until we make v approximately equal to $\frac{\mathrm{V}}{3}$, that is to say, we must make R about half as large as x.

In the case of equation [B] the conditions are slightly different, for here the quantity v' replaces $(V - v)$, and although v' and $(V - v)$ are equal, yet inasmuch as v' is the result of a single observation only, there can be but one error in it; consequently, to determine the best conditions for making the test, we must take equation [A], and assume an error δ to exist in v only.

Let λ be the error in x caused by an error δ in v, then

$$x + \lambda = R \frac{v + \delta}{V - (v + \delta)};$$

but since

$$x = R \frac{v}{V - v}, \quad \text{or,} \quad R = x \frac{V - v}{v},$$

therefore

$$\lambda = x \left\{ \frac{V - v}{v} \times \frac{v + \delta}{V - (v + \delta)} - 1 \right\} = x \frac{V \delta}{v\{V - (v + \delta)\}};$$

or, since δ is a very small quantity, we may say

$$\lambda = x \frac{V \delta}{v (V - v)}.$$

Now we have to make λ as *small* as possible; this we shall do, since x, V, and δ are constant quantities, by making $v (V - v)$ as *large* as possible.

But

$$v (V - v) = \frac{V^2}{4} - \left(\frac{V}{2} - v\right)^2;$$

and to make this expression as large as possible we must make $\frac{V}{2} - v$ as small as possible; that is, since v must be positive, we must make it equal to 0, or

$$\frac{V}{2} - v = 0,$$

therefore

$$V = 2 v.$$

But

$$v' = V - v,$$

therefore

$$v' = 2 v - v = v.$$

In which case we get

$$x = R;$$

that is to say, in order to make the test as accurately as possible, we must make R approximately equal to x.

400. If, instead of introducing the unknown resistance x, and the known resistance R, between the points a and c, we join the pole a of the battery direct on to b, we can determine the value of x by simply noting V, and then inserting an adjustable resistance in the place of x, and altering it until we make the potential at b to be V, as at first, when of course $x = \mathrm{R}$.

Best Conditions for making the Test.

401. In the case of formula

$$x = \mathrm{R}\frac{v}{\mathrm{V} - v}, \qquad [\mathrm{A}]$$

make R approximately equal to $\frac{x}{2}$.

In the case of formula

$$x = \mathrm{R}\frac{v}{v'}, \qquad [\mathrm{B}]$$

make R approximately equal to x.

Possible Degree of Accuracy attainable.

In the case of formula [A],

$$\text{Percentage of accuracy} = \frac{\delta\,(\mathrm{V} + v)\,100}{v\,(\mathrm{V} - v)}.$$

In the case of formula [B],

$$\text{Percentage of accuracy} = \frac{\delta\,(v + v')\,100}{v\,v'}; \qquad [\mathrm{C}]$$

where δ is the fraction of a division to which each of the deflections V, v, and v' can be read.

Loss of Potential Method.

402. In Chapter XIII., page 292, an equation

$$\mathrm{F} = \frac{\mathrm{T}}{2{\cdot}303\,\mathrm{R}\log\frac{\mathrm{V}}{v}}$$

was obtained, where F was the electrostatic capacity, in microfarads, of a condenser, or cable, the potential of whose charge

fell from V to v when it was discharged during T seconds through a resistance of R megohms.

Now if F is the known and R the unknown quantity, then

$$R = \frac{T}{2 \cdot 303 \text{ F} \log \frac{V}{v}};$$

so that we can determine the value of a resistance by a capacity and loss of charge measurement.

403. The connections for making such a test would be precisely similar to those given for determining electrostatic capacities by loss of charge (§ 329, page 295).

If we were measuriing the resistance of a short cable by this method, the discharge deflection V, compared with the discharge deflection obtained with the same battery from a standard condenser, would give us the value of F. For long cables, however, as we have before explained, this does not give correct results, so the capacity must be determined by other methods, Thomson's for example (page 298).

404. From (§ 334, page 293) it is obvious that we must have

Best Conditions for making the Test.

Make v as nearly as possible equal to $\frac{V}{3 \cdot 5}$.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = R \frac{\cdot 200\ \delta}{2 \cdot 303\ v \log \frac{V}{v}}$$

where δ is the fraction of a division to which each of the deflections V and v can be read.

405. When the insulation resistance of a cable is measured by the foregoing method, the result obtained is a mean of the resistances which the cable has at the commencement and at the end of the test, as *electrification* (§ 385, page 332) goes on the whole time the charge is falling.

406. Experimental results show that in the case of a cable whose core is insulated with gutta-percha, if the cable be charged ten seconds before taking the discharge V, and again ten seconds before insulating it preparatory to observing the discharge v, then the value of R after one minute, obtained from the formula, agrees with that obtained by the constant deflection method given in the last chapter (§ 383, page 331).

407. If we know the potential which the cable has when fully charged, and also its potential after a certain time, we can determine the potential it will have after any other time, in the following manner:—

A charged cable loses equal percentages of its charge in equal times, that is to say—if, for example, 5 per cent. of its charge were lost during the first second, then 5 per cent. of *what remained* would be lost in the second second.

Let V	be the potential	at first ;		
v	„	„	after 1 sec. ;	
v_1	„	„	„	t_1 secs. ;
v_2	„	„	„	t_2 „

and let us suppose the charge loses $\frac{1}{n}$th of its potential during the first second ; then the potential at the end of first second will be

$$V - \frac{V}{n} = v = V\frac{v}{V};$$

and the potential at end of second second will be

$$v - \frac{v}{n}; \qquad [A]$$

but from last equation

$$n = \frac{V}{V - v};$$

therefore, substituting this value in [A] the latter becomes

$$\frac{v^2}{V}, \quad \text{which equals} \quad V\left(\frac{v}{V}\right)^2,$$

and consequently the potential at the end of t_1 seconds will be

$$V\left(\frac{v}{V}\right)^{t_1} = v_1.$$

Also we must have

$$V\left(\frac{v}{V}\right)^{t_2} = v_2;$$

therefore

$$t_1 = \frac{\log \frac{v_1}{V}}{\log \frac{v}{V}},$$

and

$$t_2 = \frac{\log \frac{v_2}{V}}{\log \frac{v}{V}}$$

that is,

$$t_2 = \frac{\log \frac{v_2}{V}}{\log \frac{v_1}{V}} \cdot t_1 = \frac{\log \frac{V}{v_2}}{\log \frac{V}{v_1}} \cdot t_1 .$$

For example.

The potential at first was 300 (V), and after 20 seconds (t_1) it fell to 200 (v_1). After what time (t_2) would it fall to 100 (v_2)?

$$t_2 = \frac{\log \frac{300}{100}}{\log \frac{300}{200}} \times 20 = \frac{\overline{2 \cdot 4771213} - \overline{2 \cdot 0000000}}{\overline{2 \cdot 4771213} - \overline{2 \cdot 3010300}} \times 20 = 54 \text{ secs.}$$

408. It being usually required to know the time the charge in a cable will take to fall to half charge, the formula becomes

$$t_2 = \frac{\cdot 30103}{\log \frac{V}{v_1}} \cdot t_1 .$$

409. The formulæ we have given are capable of various modifications, which, however, are more of a fanciful than of an actual and practical value.

Thus the formulæ

$$R = \frac{T}{F \log_e \frac{V}{v}} \quad \text{and} \quad F = \frac{T}{R \log_e \frac{V}{v}}$$

may be simplified if we make $v = \frac{V}{2}$, for in this case

$$\log_e \frac{V}{v} = \log_e 2 = \cdot 693 ;$$

therefore

$$R = \frac{T}{\cdot 693\, F} = 1\cdot 433 \frac{T}{F}.$$

To obtain experimentally the time occupied in falling to half charge, repeated trials would be necessary, and the time taken in doing this would hardly compensate for the advantage of using a simpler formula.

The object of obtaining the time of fall to half charge is to get a convenient unit for comparison with other cables, and this time of fall is easily calculated from the formula before given, in which the potential after any time may be used, this being obtained by one observation only.

410. A useful formula is that suggested by Mr. W. H. Preece, which is obtained in the following manner:—

In the equation

$$t_2 = \frac{\cdot 30103}{\log \frac{V}{v}} \cdot t_1$$

let n = percentage of loss in time t_1, then

$$n = \frac{(V - v_1)\, 100}{V};$$

therefore

$$v_1 = V \frac{100 - n}{100}.$$

Substituting this value of v_1 in the above equation, we get

$$t_2 = \frac{\cdot 30103}{\log \frac{100}{100 - n}} \cdot t_1 = \frac{\cdot 30103}{2\cdot 000 - \log (100 - n)} \cdot t_1 .$$

For example.

If a cable lost 20 per cent. of its charge in 5 minutes; in how many minutes would it fall to half charge?

$$t_2 = \frac{\cdot 30103}{2\cdot 000 - \log (100 - 20)} \times 5 = 15'\, 32''.$$

411. From the equations

$$V \left(\frac{v}{V}\right)^{t_1} = v_1,$$

$$V \left(\frac{v}{V}\right)^{t_2} = v_2,$$

we can find what would be the potential, v_2, after a certain interval of time, t_2, the potential at first, and the potential v_1, after a time, t_1, being given.

Thus we have from these two equations

$$\left(\frac{v_1}{V}\right)^{t_2} = \left(\frac{v_2}{V}\right)^{t_1};$$

therefore

$$v_2 = V\left(\frac{v_1}{V}\right)^{\frac{t_2}{t_1}}.$$

This formula we should have to work out by the aid of logarithmic tables.

For example.

The potential of the charge in a cable when full was 300 (V). After 20 minutes (t_1) the potential fell to 200 (v_1). What would be the potential v_2 at the end of 30 minutes (t_2)?

$$v_2 = 300\left(\frac{200}{300}\right)^{\frac{30}{20}} = 300\left(\frac{2}{3}\right)^{\frac{3}{2}}$$

$$\begin{array}{rr}
\log 2 = & \cdot 3010300 \\
\log 3 = & \cdot 4771213 \\ \hline
 & \bar{1}\cdot 8239087 \\
 & 3 \\ \hline
2) & \bar{1}\cdot 4717261 \\ \hline
 & \bar{1}\cdot 7358631 \\
\log 300 = & 2\cdot 4771213 \\ \hline
 & 2\cdot 2129844 = \log \text{ of } \underline{163\cdot 3}.
\end{array}$$

CHAPTER XVII.

LOCALISATION OF FAULTS BY FALL OF POTENTIALS.

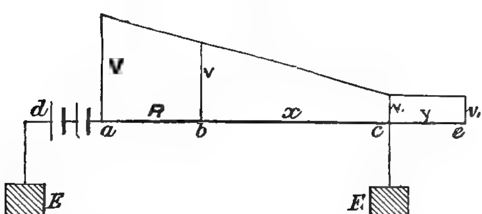
Clark's Method.

412. In Fig. 108 in the last chapter, if $b\,c$ were a portion of a cable making *full* earth at c, then by the method described for determining $b\,c$ we should find the position of the break.

Supposing, however, a cable had a fault which did not make full earth, then the potential would not fall to zero at that point, but would have a value depending upon the resistance of the fault. The potential, however, would be the same as the potential at the further end of the cable, provided that end were insulated.

If we can determine the value of this potential we can readily localise the position of the fault.

Fig. 109.



In Fig. 109 let $b\,e$ be the cable which has a fault at c, the end of the cable at e being insulated; and let R be a resistance between the battery and the end of the cable b, then

$$\mathrm{V} - v_1 : v - v_1 :: \mathrm{R} + x : x\,;$$

therefore

$$x(\mathrm{V} - v_1) = (v - v_1)\,\mathrm{R} + (v - v_1)\,x\,,$$

or

$$x\,[(\mathrm{V} - v_1) - (v - v_1)] = \mathrm{R}\,(v = v_1)\,;$$

therefore

$$x = \mathrm{R}\,\frac{v - v_1}{(\mathrm{V} - v_1) - (v - v_1)},$$

that is,
$$x = R\frac{v - v_1}{V - v}. \qquad [A]$$

413. If, as explained in the last test (§ 397, page 337), we at once determine the value of $V - v$, by connecting the wires from the condenser, &c. (or from the galvanometer and high resistance), to the points a and b, then if we call v' this difference of potential, we get
$$x = R\frac{v - v_1}{v'}. \qquad [B]$$

414. In order to determine the relative values of the potentials at the two ends of the cable, their values with reference to some standard of potential or electromotive force must be obtained. For this purpose any of the standard cells mentioned in Chapter VII. (page 118) may be used.

The way in which such standard cells would be employed for making the test we have been considering, would be as follows:—

The electrician at a charges a condenser from one of the standard cells, and notes the discharge deflection on his galvanometer. This deflection, then, represents the potential of the cell.

The wires from the standard cell are now disconnected, and one wire is connected to earth, and the other to a, and again a discharge reading is taken; then this reading, divided by the reading obtained with the standard cell, gives the value of V in terms of the standard cell. The wire at a is then disconnected and joined to b, and another discharge measured, which result divided by the standard discharge gives the value of v in terms of the standard cell.

The electrician at the other end, e, of the cable makes a similar test, and thus determines the value of v_1.

Since the standard cells at the two stations are exactly equal in electromotive force, the relative values of V, v, and v_1 will be obtained exactly.

The capacities of the condensers at the two stations, it may be observed, need not be alike.

For example.

The discharge deflection obtained from a condenser at station e with a standard cell, was 180 divisions; and the potential v_1, measured from the same condenser, gave a discharge deflection of 360 divisions; therefore
$$v_1 = \frac{360}{180} = 2 \cdot 0.$$

At the other end of the cable, from a standard cell of the same electromotive force as the one employed at station e, a discharge deflection of 150 divisions was obtained from a condenser. The potentials V and v, measured with the same condenser, gave deflections equivalent to 2550 and 1050 divisions respectively; therefore

$$V = \frac{2550}{150} = 17 \cdot 0.$$

$$v = \frac{1050}{150} = 7 \cdot 0.$$

R was equal to 1000 ohms. What was the value of x?

$$x = 1000 \frac{7 - 2}{17 - 7} = 500 \text{ ohms},$$

showing that the fault was 500 ohms distant from the end b of the cable. If the length of the cable were, say, 80 knots, and its total conductivity resistance 800 ohms, or 10 ohms per knot, then the distance of the fault from b would be $\frac{500}{10}$, or 50, knots.

The value of v_1 when obtained at e would be telegraphed to b; this could be done since the cable would not be entirely broken down.

If the potentials are measured by observing the permanent deflections obtained through a high resistance (§ 286, page 249), the observations with the standard cells must be made in the same manner.

415. In making the test we are are liable to make errors in V, v, and v_1, and these errors will produce the greatest total error in x, when the errors in V and v_1 are minus, and the error in v is plus; let each of the errors be δ, and let λ be the total error produced in x, we then have

$$x + \lambda = R \frac{(v + \delta) - (v_1 - \delta)}{(V - \delta) - (v + \delta)} = R \frac{v - v_1 + 2\,\delta}{V - v - 2\,\delta},$$

or

$$\lambda = R \frac{v - v_1 + 2\,\delta}{V - v - 2\,\delta} - x;$$

but

$$x = R \frac{v - v_1}{V - v}, \quad \text{or,} \quad R = x \frac{V - v}{v - v_1},$$

therefore

$$\lambda = x\left[\frac{V - v}{v - v_1} \times \frac{v - v_1 + 2\delta}{V - v - 2\delta} - 1\right] = x\frac{2\,\delta\,(V - v_1)}{(v - v_1)\,(V - v - 2\,\delta)};$$

but, since δ is small, we may say

$$\lambda = x\frac{2\,\delta\,(V - v_1)}{(v - v_1)\,(V - v)},$$

or

$$\lambda = x\frac{2\,\delta\,(V - v_1)}{(v - v_1)\,[\,(V - v_1) - (v - v_1)\,]}.$$

Now if we regard $(V - v_1)$ as a constant quantity, then in order to make λ as small as possible we must make the denominator of the fraction as small as possible; from (§ 398, page 337) we can see that in order that this may be the case we must make

$$(v - v_1) = \frac{(V - v_1)}{2},$$

that is to say, we must make R approximately equal to x.

In the case of formula [B] (page 346) the conditions for making the test in the most satisfactory manner are slightly different from the foregoing; for since $(V - v)$ in this case is obtained by a single measurement, v', there can be but one error, δ, in it. We have, in fact,

$$\lambda = x\left[\frac{V - v}{v - v_1} \times \frac{v - v_1 + 2\,\delta}{V - v - \delta} - 1\right] = x\,\frac{\delta\,(2\,V - v - v_1)}{(v - v_1)\,(V - v - \delta)};$$

but, since δ is very small, we may say

$$\lambda = x\,\frac{\delta\,(2\,V - v - v_1)}{(v - v_1)\,(V - v)},$$

or

$$\lambda = x\,\frac{\delta\,[2\,(V - v_1) - (v - v_1)]}{(v - v_1)\,[(V - v_1) - (v - v_1)]}.$$

Now this equation is of the same form as equation [F] (page 85), consequently the investigation there given may be applied to the present case. In the latter, the coefficients of $(V - v_1)$ and $(v - v_1)$ are 2 and -1 respectively; if therefore, in equation [G] (page 86) we substitute $-\frac{1}{2}$ for k, and also if we

substitute x and R, for C_2 and C_1, respectively, we shall obtain the conditions we require; we have then

$$v - v_1 = \frac{V - v_1}{\sqrt{-\frac{1}{2} + 1} + 1}, \text{ or, } V - v_1 = (v - v_1)\left(\frac{1}{\sqrt{2}} + 1\right)$$

$$= (v - v_1)\, 1 \cdot 7071;$$

that is to say, we must have

$$R = 1 \cdot 7071\, x.$$

Practically we may say, make

$$R = 2\, x$$

approximately.

We have therefore

Best Conditions for making the Test.

416. In the case of formula

$$x = R\,\frac{v - v_1}{V - v} \qquad [A]$$

make R approximately equal to x.

In the case of formula

$$x = R\,\frac{v - v_1}{v'} \qquad [B]$$

make R approximately equal to $2\,x$.

Possible Degree of Accuracy attainable.

In the case of formula [A]

$$\text{Percentage of accuracy} = \frac{\delta\,(V - v_1)\,200}{(v - v_1)\,(V - v)}.$$

In the case of formula [B]

$$\text{Percentage of accuracy} = \frac{\delta\,(2\,v' + v - v_1)\,100}{(v - v_1)\,v'}$$

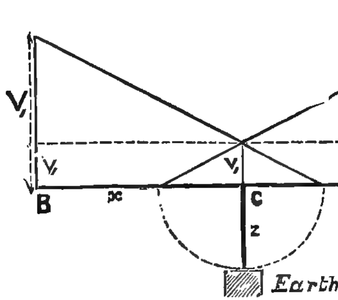
where δ is the fraction of a division to which each of the deflections can be read.

SIEMENS' EQUAL POTENTIAL METHOD.

417. In Fig. 110 let B E be the cable which has a fault at c, x and y being the distances on either side of the fault, and z the

equivalent length of the latter. Suppose that one pole of a battery is connected at B, the other pole being to earth, then if the end of the cable at E is insulated we shall have, as in the last test, the potential at E to be the same as the potential at

FIG. 110.



the fault. Next suppose that the battery at B is removed, and that that end of the cable is insulated; then, if a battery is connected to E, of such a strength that the potential at the fault, and therefore at B, is the same as was the potential at E in the first case, then V_2 will be the new potential at E.

Now,

$$V_1 - v_1 : V_2 - v_1 :: x : y,$$

therefore

$$\frac{x}{y} = \frac{V_1 - v_1}{V_2 - v_1}.$$

If l be the length of the cable, then

$$l = x + y, \quad \text{or,} \quad y = l - x;$$

therefore

$$\frac{x}{l - x} = \frac{V_1 - v_1}{V_2 - v_1};$$

that is,

$$x (V_2 - v_1) = l (V_1 - v_1) - x (V_1 - v_1),$$

or

$$x = l \frac{V_1 - v_1}{(V_2 - v_1) + (V_1 - v_1)}.$$

For example.

In a faulty cable 500 knots (l) long, after adjusting the potentials according to the foregoing method, the values of the same were found to be

$$V_1 = 200,$$
$$V_2 = 300,$$
$$v_1 = 40.$$

What was the distance (x) of the fault from B?

$$x = 500 \frac{200 - 40}{(300 - 40) + (200 - 40)} = 190 \cdot 5 \text{ knots.}$$

418. In making the test practically, the following course would be pursued:—

Station B first connects one pole of a battery direct on to the cable, the other pole being to earth, whilst E insulates his end of the cable. This being done, B notes the potential V_1, and E the potential v_1. When B thinks that sufficient time has elapsed for E to have taken his observation, he removes the battery and insulates his end of the cable. E noting that his potential has fallen to zero, connects up his speaking apparatus, and B having done the same, E communicates to B the result he has obtained.

Station E now connects up his battery to the cable, taking care that the pole connected to the latter is similar to that employed by B in the first instance. The latter observes the potential at his end of the cable, and if it is not the same as that previously obtained at E, he informs the latter, by means of signals agreed upon, that such is the case, whereupon E increases or decreases his battery power, and regulates it by varying a resistance in its circuit until the potential at B is made the same as it was at E on the first occasion. The potential V_2 is then noted by E, and the result being reduced to terms of a standard cell,* is communicated to B. The latter station, having also reduced his results to terms of a standard cell, then works out the formula, and thus determines the position of the fault.

419. For localising faults in long cables this method is more accurate than the previous one, as it is not so much influenced by the *resultant* fault † produced by the conductive power of the insulating sheathing, more especially if the fault is near the middle of the cable.

It must be understood that both tests are only accurate in cases where the total insulation resistance of the cable is very high compared with the resistance of the fault, for in such cases the fall of potential is practically represented by a straight line, and the formulæ are constructed on this assumption.

* See page 346.

† See page 230, § 261.

When, however, the cable is very long and the total insulation resistance consequently comparatively low, then the potential cannot be regarded as falling regularly from end to end, but must be graphically represented by a curve, and the potential at the fault is less than that indicated in the straight line diagram, and the potential at the extreme end is lower than this still. The exact formulæ for these tests are considered in Chapter XXII.

420. From the nature of the test it must be evident that there are no particular conditions which enable a maximum degree of accuracy to be obtained, except in so far that the battery power employed should be sufficient to enable high deflections to be produced.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{200\,\delta}{V - v_1},$$

where δ is the fraction of a division to which each of the deflections can be read.

Siemens' Equilibrium Method.

421. If two batteries have their opposite poles connected to the ends of a perfect cable, their other poles being to earth, then the fall of potential along the cable is continuous and cuts the latter at a certain point. The position of this point can be varied by altering the relative electromotive forces of the batteries, or by adding in resistances between the batteries and the ends of the cable. In the case of a faulty cable, if the fault be at this point, then no current passes from the batteries to earth, consequently any alteration in the resistance of the fault does not affect the values of the potentials at the different points along the line of fall.

By observing what arrangement of resistances and electromotive forces is necessary to bring the zero point to the fault, the position of the latter can be accurately determined.

In Fig. 111 let x and y be the portions of the cable on either side of the fault, and let r, r be equal resistances connected to either end of the cable, also let R_1 and R_2 be resistances whose values can be varied at pleasure.

Now, in making the test we have to adjust R_1 and R_2 so that the potential at the fault shall be zero, and consequently that A B shall be a straight line. To obtain this result we must have

$$v_1 : v_2 :: x : y,$$

or

$$\frac{v_1}{x} = \frac{v_2}{y};$$

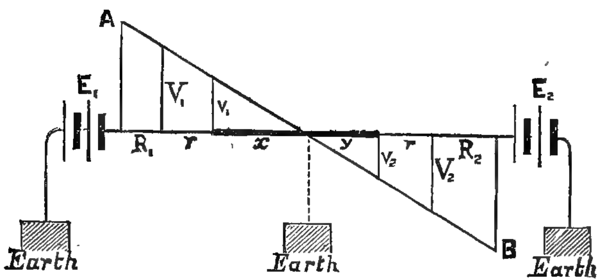
and also

$$V_1 : v_1 :: r + x : x,$$

or

$$V_1 - v_1 = \frac{v_1 r}{x};$$

FIG. 111.



and again

$$V_2 : v_2 :: r + y : y,$$

or

$$V_2 - v_2 = \frac{v_2 r}{y} = \frac{v_1 r}{x};$$

from which we get

$$V_1 - v_1 = V_2 - v_2;$$

that is to say—in order that A B may be a straight line the differences of the potentials on either side of r, at both ends of the cable, must be the same.

422. To obtain this result in practice only one of the resistances R_1 and R_2 need be adjusted. The best way of making the test would then be as follows:—

The two stations should first adjust their galvanometers by means of the movable magnets so that they both give precisely the same deflections when a current from a standard cell through a standard resistance is sent through them. This being done, batteries E_1 and E_2 are connected by the two stations on to the ends of the cable, and then the adjusted galvanometers are severally connected on each side of the respective resistances r and r at the two stations, there being in the circuit of each

galvanometer very high, but equal, resistances. Station A, say, now adjusts R_1 and watches the effect on his galvanometer; B also watches the effect on his own galvanometer, and from time to time signals to A the deflection he obtains; this signalling is easily done by having the front contact of a well-insulated key connected to the end of the cable, and the back contact connected to earth, whilst the lever of the key is connected to one terminal of a small condenser whose second terminal is to earth. By pressing down this key a small quantity of the charge in the cable will rush into the condenser, and a momentary movement of the galvanometer needle at station A will be produced; by arranging then that so many movements shall represent a particular deflection, B can easily communicate his results to A.

When exact adjustment is obtained, that is to say, when $(V_1 - v_1)$ and $(V_2 - v_2)$ are equal, the galvanometers are disconnected from either side of r and r, and the potential v_1 is measured; x is then obtained from the formula

$$x = r\,\frac{v_1}{v'}$$

where v' equals $(V_1 - v_1)$, as in the "Fall of Potential Method," of measuring a resistance, page 336.

423. To make the foregoing test as accurately as possible it is advisable, for the reason explained in § 398, page 337 (after the value of x has been obtained by a rough test), to adjust r and r so that they shall each be approximately equal to x.

With regard to the "Possible degree of accuracy attainable," we are liable to make an error in obtaining the value of $V_1 - v_1$, but inasmuch as $V_1 - v_1$ may itself contain an error due to $V_2 - v_2$ being incorrectly measured, the actual total error which may exist in x must be twice that given by formula [C] (page 339); consequently we have

Best Conditions for making the Test.

Make r, r, each approximately equal to x.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{\delta\,(v_1 + v')\,200}{v_1\,v'},$$

where δ is the fraction of a division to which each of the deflections can be read.

CHAPTER XVIII.

TESTS DURING THE LAYING OF A CABLE.

424. The immediate detection of a fault which may occur in a cable during its submersion is a point of great importance. To enable this to be done, a good system of testing is requisite.

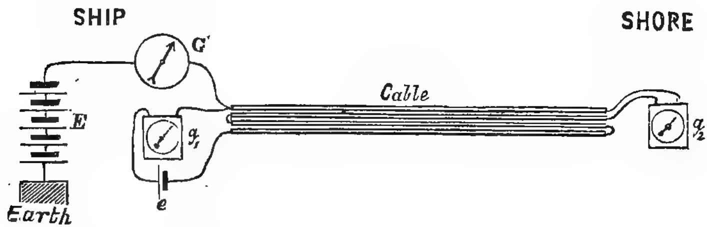
Whatever the system be, it should be a continuous one, that is to say, the cable should be continuously and visibly under test, so that the moment a fault occurs it may be detected by the ship and traced.

SYSTEM FOR COMPOUND CABLES.

425. For laying cables which are not more than 200 miles or so in length, and which have several wires, the method shown by Fig. 112 may be employed.

In this system the wires are all connected up in one continuous length as shown. Should there be an odd number of wires, the odd one would have to be coupled on to one of the others in "multiple arc."

FIG. 112.



In Fig. 112, g_1 and g_2 are two ordinary "detector" galvanometers *well* insulated. The battery e, of one or two cells (also well insulated), keeps a continuous current circulating through these galvanometers and the conducting wires of the cable; this serves as a "continuity" test, for if any of the wires should break within their insulating sheathing, the circuit becomes

interrupted, and consequently the needles of both galvanometers will fall back to zero. In the case of a cable with an odd number of wires, should the conductor of either of the two which are coupled together become broken, then the needles will only fall back a little way and not back to zero; this, however, will be quite sufficient to indicate that the conductor is fractured.

The galvanometer G is of the marine description, shown on page 46, and is connected to one of the wires. The battery E, of about 200 cells, keeps a continuous current flowing through the galvanometer and through the insulating covering of the wires. If a fault occurs in the insulation, the current by escaping direct to earth causes an immediate and very large increase in the deflection of the needle of G.

In order to keep up communication with the shore, the current from battery e is reversed after certain equal intervals of time. If the shore perceives that the reversal has not taken place, or that the needle of g_2 is not steadily deflected, he knows that something has gone wrong, or that the ship wishes to communicate with him, and he joins up his speaking apparatus and tries to communicate with the ship. The galvanometers g_1 and g_2 could be used for this purpose by having *well*-insulated keys inserted in their circuit at the ship and shore, these keys being so arranged that their depression breaks the circuit; the movements of the needles could then be worked according to the ordinary Morse code, and communication be kept up without interrupting the insulation test.

System for Single Wire Cables.

426. The method just described is only applicable to a cable which has more than one wire, for although with the latter the insulation test would be kept up, there would be no means of communicating with the shore. In such cases the following plan may be adopted:—

The end of the cable on board the ship is well insulated and connected to a battery and Thomson galvanometer as in the previous test and as shown by Fig. 113. On shore (Fig. 114) a condenser is provided, one terminal of which is connected to a brass lever which plays between two insulated contacts; one of these contacts is connected to the second terminal of the condenser, which latter terminal is also connected, through a Thomson galvanometer, to earth; the other contact is connected to the conductor of the cable. The battery connected to the cable on board the ship charges the former to a certain potential,

and the value of this potential will be the same throughout the whole length, provided no fault exists. If the lever on shore be moved against the contact connected to the cable, a portion of the charge in the latter will rush into the condenser and will charge up the set of plates, to which it is connected, to the same

FIG. 113.

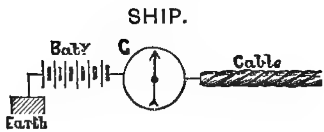
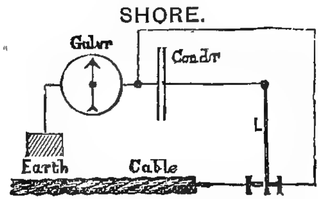


FIG. 114.



potential as the cable; the second set of plates will become charged to the opposite potential by a charge rushing in from earth through the galvanometer; this in-rush will produce a *throw*, or momentary deflection of the needle, the amount of which will represent the potential of the charge in the condenser, that is, the potential at the end of the cable. If now the lever be moved from the cable contact to the contact connected to the condenser, the latter will be short circuited and discharged. The rush of the charge into the condenser when the latter is connected to the cable contact, produces a simultaneous rush into the cable from the battery on the ship, and as this takes place through the galvanometer on board the ship a sudden throw is produced on the needle. Now if a fault occurs during the laying, the *steady* deflection on the ship's galvanometer, which is due to the flow of current through the dielectric of the cable, and which is distinct from the *throw* which takes place when the condenser becomes connected to the cable at the shore end, becomes greatly increased and renders the presence of the fault evident immediately. On the shore the effect of the fault is to reduce the potential at that end of the cable, and consequently the charge which the condenser takes becomes correspondingly reduced; when then the condenser becomes charged through the galvanometer a reduced throw is produced, which thus shows the shore the existence of the fault.

The lever on shore which charges and discharges the condenser is moved by clockwork which causes it to act every five minutes, so that every hour twelve throws are observed on each

galvanometer. At the end of every hour the ship reverses the battery so that the direction of the throws is changed.

In order to enable the ship to communicate with the shore, instructions are given that if at the end of the hour the throws do not become reversed, or if they become reversed before the expiration of the hour, it is a sign that the ship wishes to communicate with the shore; in this case, then, the shore disconnects the cable from the clock lever and connects it with the speaking apparatus, and as the ship does the same, the necessary communications can be carried on. If, on the other hand, the shore wishes to call the attention of the ship, he can do so by moving a lever, corresponding to the clock lever, two or three times quickly by hand; the ship then observing that the throws on her galvanometer take place quickly, instead of at intervals of five minutes, immediately joins up her speaking apparatus, and thus communicates with the shore.

The movement of the lever L in the foregoing system of testing is effected, as has been pointed out, by means of a clock, but L may be a hand-worked key, and this is sometimes preferred, as although a clock ensures the discharges being obtained after regular intervals of time, yet the hand method ensures the necessary watchfulness of the electrician on shore, which is a point of importance.

Willoughby Smith's System.

427. For long single-wire cables a refinement of the foregoing method, devised by Mr. Willoughby Smith, has been adopted. This system is shown by Figs. 115 and 116.

On shore, the cable is connected to a key K, galvanometer G_2, and condenser C_1 as in the last method of testing. To the cable there is also connected a resistance in circuit with a galvanometer G. This resistance is very much greater than the total insulation resistance of the cable, and consequently it does not appreciably affect the potential measured by the key K, whilst it allows sufficient current to pass through the galvanometer G to produce a sensible deflection of its needle.

The high resistance is made of selenium, and it must be carefully excluded from light, and kept at as uniform a temperature as possible, otherwise it will vary considerably.

On the ship the cable is connected to a slide resistance Wheatstone bridge similar to that described in Chapter VIII., page 188.

The working of the apparatus is then as follows:—

On the ship, plugs are inserted at p_1 and p_2 and balance is

kept on the galvanometer G_4 by adjusting the slides of the slide resistances, the resistance R being preserved constant. This gives the insulation resistance of the cable.

Galvanometer G_5 is kept short circuited under ordinary conditions, it being only used occasionally for the purpose of ascertaining whether the batteries are in good condition.

FIG. 115.

SHORE.

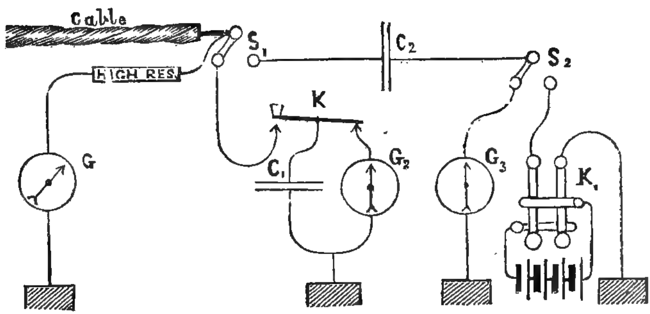
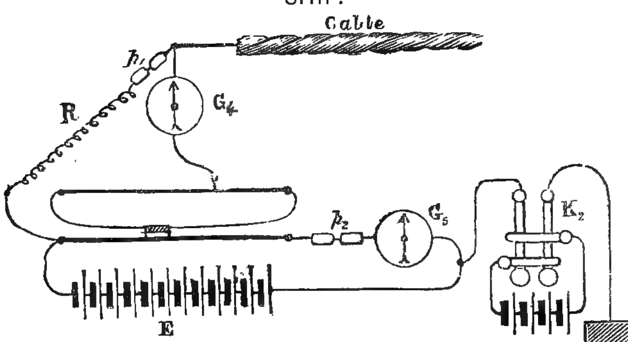


FIG. 116.

SHIP.



Should it be thought advisable, as a check, to take an ordinary deflection insulation test,* this can be done by removing the plugs p_1 and p_2; the current then passes direct from the battery through the galvanometer G_4 into the cable.

* Page 331.

On shore the potential at the end of the cable is observed on G_2 by depressing the key K every five minutes. The deflections obtained are carefully noted and recorded.

The battery E is reversed every fifteen minutes by the ship, and this is observed on the galvanometer G and shows that the conductor of the cable is entire. If the ship requires to communicate with the shore it reverses the battery several times after short intervals; this is acknowledged by the shore by means of his key K; when this is done, the shore moves over his switch S_1 and receives signals from the ship on galvanometer G_3 through the medium of the condenser C_2. The insulation test is not interrupted by this signalling, as the cable remains insulated the whole time. The effect of working the signalling key K_2 is only to add or subtract a little from the charge in the cable through the medium of the condenser and thereby to produce momentary deflections on the galvanometer G_3. The same in the case when the shore signals to the ship, the switch S_2 being moved over to key K_1 for that purpose.

Various slight modifications have been, and are, employed in practically using this method, but the general arrangement is that which has been indicated.

CHAPTER XIX.

JOINT-TESTING.

428. Joints are the weak points in a cable, and it is therefore essential that they should be not only carefully made but carefully tested.

A joint, being a very short length of the core, offers, or should offer, a very high resistance; it would consequently be impossible to test it by a direct deflection method, that is, a method similar to that by which the insulation resistance of a cable is taken (page 331). Even with a very powerful battery, the galvanometer deflection, provided the joint were good, would be quite inappreciable. One or other of the following methods must therefore be adopted.

A condenser can be charged through the medium of the joint and after a noted time the discharge taken, which gives the amount which has leaked through the joint. This is known as *Clark's accumulation method.*

Or a charged condenser may be allowed to discharge itself through the joint, and the amount lost after a certain time noted.

In both these methods the discharge deflections are compared with the results obtained with a few feet of perfect core.

Clark's Accumulation Method.

429. A gutta-percha or ebonite trough is provided, which is suspended by long ebonite rods from any convenient hook.

The good insulation of the trough is a point of great importance, and consequently the suspending rods should be quite dry and clean. The most effectual way of obtaining this result is to well scour the surface of the ebonite with a glass or emery paper; this is a much better method than covering the surface with hot paraffin wax as is sometimes done.

430. We may here remark that *surface* leakage is almost the only medium of loss to be feared in electrical apparatus, and this should always be seen to by keeping all surfaces over which leakage is likely to occur, in a proper condition. The peculiar formation of ebonite causes minute quantities of sulphuric acid

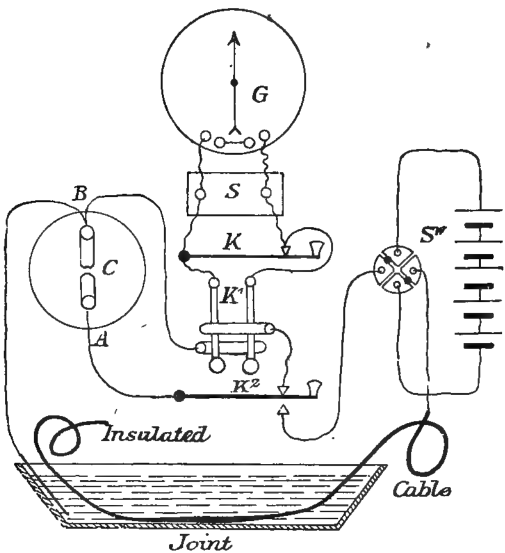
to form on its surface, so that the latter should be often rubbed over with a dry cloth. Hot paraffin wax painted over the dry surfaces is very advantageous, but, where appearance is immaterial, nothing is so effectual as a surface well scoured with glass or emery paper.

431. The trough is filled with water, and the joint to be tested is immersed and held down in it by two hooks placed at the bottom.

The portion of the core on either side of the joint should be carefully dried (not paraffined), for the same reason that the suspending rods were so treated.

The connections for the test, shown by Fig. 117, are very similar to those shown by Fig. 77, page 240; the only difference

FIG. 117.



being that the pole of the battery, which in that figure was connected directly to the condenser, is, in the joint test, connected to it through the medium of the joint. The battery used should be as large as possible; 200 cells is the number very commonly employed.

432. After the joint is placed in the trough for testing, it is necessary to see that the latter is sufficiently well insulated.

To do this the pole of the battery, which for the regular test would be connected to the core, must be connected to the wire attached to the plate in the trough, and the discharge key pressed down; this charges the condenser; the battery being then disconnected from the plate, an interval of time (usually one minute) equal to that which would be occupied by the test of the joint, is allowed to elapse, and then the "Discharge" trigger is pressed and the discharge noted; this should be equal, or very nearly so, to the instantaneous discharge.

433. The good insulation of the trough being satisfactorily obtained, and the connections being made as shown by Fig. 117, the short-circuit plug of the condenser must be inserted in its place, the discharge key pressed down, and then the short-circuit plug removed; the battery then charges the condenser through the joint.

After a certain time, usually one minute, the discharge deflection must be noted. A similar measurement must also be made, using a length of perfect core in the place of the joint. If, in the latter case, the discharge deflection after the same interval of time is much less than that obtained from the joint, the latter is defective and must be remade.

434. It is a very important point in making the test to insert the short-circuit plug in the condenser previous to depressing the discharge key; if this is not done an induced charge is thrown into the condenser by the sudden rush of the battery current into the core when the discharge key is depressed. This induced charge will give a considerable deflection when the condenser is discharged, which deflection is in no way due to *leakage* through the joint, though it might be mistaken for such. By keeping the condenser short circuited this induced charge is dissipated.

435. If the joint is good, the discharge deflection seldom exceeds two or three divisions. Indeed, the fact that it does not do so is usually a quite sufficient proof of the soundness of the joint, and it is not often the case that a comparison with a piece of perfect core is necessary.

Discharge Method.

436. This is a reversal of the foregoing and consists in charging the condenser full and letting it discharge itself through the joint.

The connections for making this test would be similar to those employed in measuring high resistances by the fall of charge method given in Chapter XVI., page 339 (§ 401), the

one end of the core taking the place of one end of the resistance, and the plate in the trough the place of the other end.

The system of charging the condenser through the joint cannot of course be carried out unless one end of the core is at hand to which to attach one pole of the battery.

When a joint is made in a cable core at sea neither end can be got at. The joint, however, could be tested by making the connections as for the last method of testing, only instead of joining the core to the condenser terminal, the latter, and also the cable end, would be put to earth. To carry out the test in this manner, arrangements would have to be made with the shore, previous to the manufacture of the joint, that at a certain time the end of the cable shall be put to earth.

The first method could also be adopted for testing at sea by using an earth in the foregoing manner.

As a matter of fact, joints made at sea are never tested, though there seems no reason why they should not be so.

437. We may if we please, in both the foregoing tests, place the galvanometer between the back terminal of the key and the condenser, and join the two terminals from which it was removed by a piece of wire. We should then get a charge as well as a discharge deflection, and there is this advantage, that if the joint is very bad or the trough not well insulated, we should get a permanent deflection after the charge deflection has taken place.

438. The connections should always be so made that the zinc pole of the battery is connected to the core and the copper pole to the plate.

439. It is very advisable to employ a special condenser for making these tests, for if one is used which has been charged at any time with a high battery power, it will often be found that a portion of this charge will have become absorbed, and when the condenser is left to itself, this portion will become free and give a discharge which may be mistaken for an accumulation through the joint.

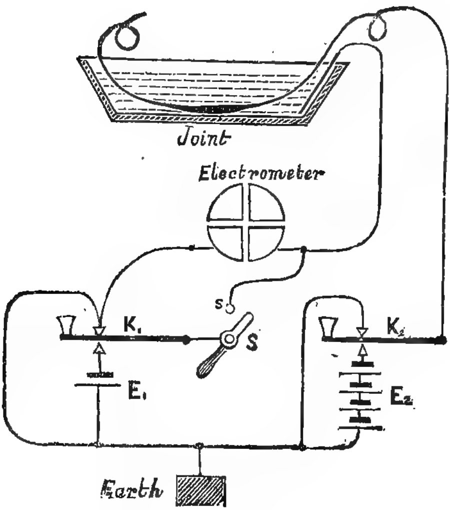
Electrometer Method.

440. Although the preceding methods of testing are often the only ones which can be adopted, yet when possible it is best to make joint tests by means of an electrometer, as the results are always more trustworthy than those obtained by the condenser method, since they are free from the source of error mentioned at the end of the last paragraph.

Fig. 118 shows the connections for making this test, which is executed in the following manner:—

After the insertion of the joint in the trough, the insulation of the latter must be tested; this is done by pressing down key K_1 and moving the switch S over to its *well*-insulated contact stop *s*; this puts the ten-cell battery E_1 in connection with the quadrants of the electrometer, and thereby charges them and causes a steady deflection of the needle. Key K_1 being kept

FIG. 118.



depressed, switch S is now opened and the deflection watched for two minutes to see whether there is any sensible fall due to the charge on the quadrants leaking to earth through the medium of the trough; if this loss is only equal to two or three divisions the insulation of the trough may be considered to be good.

Key K_1 is now released and switch S closed so as to discharge the electrometer. Switch S is now again opened and key K_2 depressed; this puts the 200-cell battery E_2 in connection with the core of the cable, and the momentary rush of current into the latter causes an induced charge to rush out of the trough and produce a sudden deflection of the electrometer needle; it is usual to record this deflection, although it is of no value, except to show that the various connections have been properly made, and that the joint has been placed in the trough.

Key K_2 being kept depressed, switch S is now moved over to *s* (so as to discharge the electrometer), and then again opened. The scale of the electrometer is then watched as the current leaking through the joint into the trough accumulates and causes a gradually increasing deflection of the needle; the amount of this deflection should be noted at the end of one and two minutes after the opening of the switch.

After the observations with the joint have been made, a piece of perfect core must be inserted in the trough and a similar test executed, the results of which should not differ much from those obtained with the joint. It always happens that a joint gives a greater accumulation than an equal length of perfect core, unless indeed the joint has been made several days before being tested, which is seldom, if ever, the case.

CHAPTER XX.

SPECIFIC MEASUREMENTS.

441. In order to compare the relative or *specific* "Conductivity," "Insulation," and "Inductive capacity" of the materials used in the construction of the core of submarine cables, it is necessary that they should each of them be referred to some standard unit with which the comparison can be made.

Specific Conductivity.

442. For the *specific conductivity* of a wire, the conductivity of the pure metal is taken as the standard.

Experiments by Dr. Matthiessen have proved that 1 foot of pure copper wire weighing 1 grain has a resistance of ·2262 ohms at a temperature of 24° Cent., which is equivalent to a resistance of ·2261 ohms at 75° Fahr.; consequently, l feet of such wire at the latter temperature will have a resistance of $l \times \cdot 2261$ ohms. But l feet of the wire will weigh, not 1 grain, but l grains, and therefore the resistance of l feet weighing 1 grain must be $l \times \cdot 2261 \times l$, or, $l^2 \times \cdot 2261$ ohms; and, further, if the l feet weighed w grains then the resistance would be

$$\frac{l^2 \times \cdot 2261}{w} \text{ ohms.}$$

But, again, the resistance of the wire will vary with the temperature, consequently to obtain the resistance at any particular temperature we must correct the same by means of a coefficient k; we have then

$$\left.\begin{array}{l}\text{Resistance, R, of } l \text{ feet of pure copper} \\ \quad \text{wire weighing } w \text{ grains}\end{array}\right\} = \frac{l^2 \times \cdot 2261}{w\,k}.$$

The numerical value of k for various temperatures is given in Table IV.*

Having thus obtained a simple formula which expresses the

* The general question of corrections for temperature is considered in Chapter XXI.

relation between the length, &c., and the resistance, of a pure copper wire, we are in a position to determine the specific conductivity of any other wire; for having measured off a definite length of the latter and ascertained its weight, temperature, and resistance, then the latter compared with the resistance of a pure copper wire of the same length, temperature and weight, gives us, by direct proportion, what we require.

For example.

Suppose the length of our sample of wire was 20 feet, its weight 500 grains, its resistance ·200 ohms, and its temperature 60° Fahr. From Table IV. we get: $k = 1 \cdot 0323$, consequently we have

$$\mathrm{R} = \frac{20 \times 20 \times \cdot 2261}{500 \times 1 \cdot 0323} = \cdot 1752 \text{ ohms.}$$

Then to get the specific conductivity (x) of the wire sample, we have the inverse proportion

$$\cdot 200 : \cdot 1752 :: 100 : x,$$

or

$$x = \frac{\cdot 1752 \times 100}{\cdot 200} = 87 \cdot 6;$$

that is to say, the conductivity of the wire sample is 87·6 per cent. of that of pure copper.

443. In the case of a cable where the weight per knot of the conductor is always known, the calculations are much simpler, as they can be made by reference to Table II. (given at the end of the book), which gives the resistances corresponding to various percentages of conductivity of a conductor 1 knot long weighing 1 lb., and at a temperature of 75° Fahr. The way in which this table would be used is as follows:—

Supposing we had a cable whose conductor weighed 107 lbs. per knot (this is a very usual weight for the conductor of a cable), and whose resistance per knot at 75° Fahr. was found by experiment to be 11·56 ohms, then by multiplying 11·56 by 107 we get the resistance of a knot-pound of copper of a corresponding conductivity. $11 \cdot 56 \times 107 = 1236 \cdot 92$ and this resistance in the table corresponds to a conductivity of 96·8, which is therefore the percentage of conductivity of the conductor of the cable.

444. In calculating out Table II., the determination of Dr. Matthiessen before referred to, given in the British Association

report on electrical standards, has been taken as the basis: this determination makes the resistance of a foot-grain of pure copper at 24° C. (75·2° F.) to be ·2262 ohms; the latter value appears to be the most trustworthy one yet obtained.

445. It is sometimes required to determine the specific conductivity of a wire whose length and *diameter* (d) are known; in this case, the determination of Dr. Matthiessen—viz., the resistance of 1 foot of pure copper wire whose diameter is 1 mil ($\frac{1}{1000}$th of an inch) is 10·323 ohms at 60° F. (or 10·656 ohms at 75° F.)—may be taken as the standard.

Since the resistance of a wire varies inversely as its sectional area, that is, inversely as the square of its diameter (d), we must have:—

$$\left.\begin{array}{l}\text{Resistance of } l \text{ feet of pure copper}\\ \quad\text{wire } d \text{ mils in diameter}\end{array}\right\} = \frac{l \times 10 \cdot 656}{d^2\, k} \text{ ohms.}$$

For example.

The resistance of 50 feet of copper wire, 14 mils in diameter, equals 2·746 ohms, at a temperature of 65° F.; what is the specific conductivity of the wire?

For 65°, $k = 1 \cdot 0214$ (Table IV.), therefore

$$\left.\begin{array}{l}\text{Resistance of 50 feet of pure}\\ \quad\text{copper wire 14 mils in}\\ \quad\text{diameter, at 65° F.}\end{array}\right\} = \frac{50 \times 10 \cdot 656}{14 \times 14 \times 1 \cdot 0214} = 2 \cdot 661 \text{ ohms};$$

then by inverse proportion we have

$$2 \cdot 746 : 2 \cdot 661 :: 100 : x$$

or

$$x = \frac{2 \cdot 661 \times 100}{2 \cdot 746} = 96 \cdot 9;$$

that is to say, the conductivity of the wire sample is 96·9 per cent. of that of pure copper.

446. In the case of small wires where it is difficult to measure the diameter with great accuracy, it is preferable to test for specific conductivity by weight rather than by gauge, for by taking a sufficient length of wire, we can determine the value of the weight as accurately as we please.

447. Table III.,* at the end of the book, shows the resistances &c., of various gauges of pure copper wire at 60° F.

* This Table was compiled by Messrs. W. T. Glover and Co., electrical wire makers, of Manchester, and is inserted by permission.

Specific Insulation.

448. To obtain the *specific insulation* resistance of any material is not an easy matter, for we have no pure standard material with which to compare it, and even if we had, the resistance would be so enormously high that we could not, as in the case of the wire, get a piece of a certain length and compare it by measurement with another. We must therefore look for some other method.

Now, the form in which gutta-percha is used for submarine cables is that of a cylinder, in which the conducting wire is concentrically placed; and to compare the relative resistances of different cores we must first ascertain the law of the insulation resistance of cores whose sheathings have various thicknesses. As this is an interesting problem, we will give it at length.

Looking at a transverse section, let us suppose the sheathing to be divided into a number of concentric circles, such that the resistance of the piece between any two circles equals ρ. For this to be the case, it is evident that the circles nearer the circumference must be of a greater thickness than those near the centre, since their circumferences are greater.

Let there be n of these circles, so that $n\ \rho = W$ (ρ here corresponds to the little interval of time t in the fall of charge problem, page 290, § 333).

Now, if $\left.\begin{matrix} r_d \\ r_{d+1} \end{matrix}\right\}$ be the internal and external radii or diameters of any one cylinder, and if the difference $r_{d+1} - r_d$ is very small, the resistance of the cylinder will be

$$\frac{(r_{d+1} - r_d)\,s}{2\,\pi\,l\,r_d}$$

when l is the length of the cable, and s the specific resistance of the insulating material.

Now, the smaller we make $r_{d+1} - r_d$, the nearer will this be true. But in order to do this, we must make ρ small and n large. Now

$$\frac{(r_{d+1} - r_d)\,s}{2\,\pi\,l\,r_d} = \rho$$

since ρ equals the resistance of each cylinder; therefore

$$r_{d+1} = r_d\left(1 + \frac{2\,\pi\,l\,\rho}{s}\right).$$

Then, as in the problem we have referred to,

$$r_n = r_d\left(1 + \frac{2\,\pi\,l\,\rho}{s}\right)^n$$

where r_n, or R, is the external, and r_d, or r, the internal radius of the sheathing; that is,

$$R = r\left(1 + \frac{2\,\pi\,l\,\rho}{s}\right)^n = r\left(1 + \frac{2\,\pi\,l\,W}{s\,n}\right)^n,$$

and the larger n is the nearer is this true; therefore make $\rho = 0$ and $n = \infty$ so that $n\,\rho$ still equals W; we then get a perfectly accurate result. Let

$$\frac{2\,\pi\,l\,W}{s\,n} = \frac{1}{x},$$

so that $x = \infty$ when $n = \infty$. Then

$$R = r\left[\left(1 + \frac{1}{x}\right)^x\right]^{\frac{2\,\pi\,l}{s}W}$$

when $x = \infty$, but when this is the case the expression within the square brackets is known to be equal to e,* thus

$$\frac{R}{r} = e^{\frac{2\,\pi\,l}{s}W};$$

therefore

$$W = \frac{s\log_e\frac{R}{r}}{2\,\pi\,l} = \frac{s\,\log\frac{R}{r}}{2\cdot728\,l};$$

therefore

$$s = W\frac{2\cdot728\,l}{\log\frac{R}{r}}.$$

Now, if we take for our standard a cable-core which has a length of one knot, an insulation resistance of 1 megohm, and which has such external and internal radii or diameters (that is, the radii or diameters of the insulating sheathing and the conductor), that $\log\frac{R}{r} = 2\cdot728$, then $s = 1$.

* See Todhunter's Algebra, Fifth Edition, Chapter XXXIX.

If, then, we measure the insulation resistance per knot, and also the external and internal diameters of our sample core, and insert the values in the formula, we get its specific insulation resistance.

Since we take the data of 1 knot to insert in the formula, we may write the latter

$$s = W \frac{2 \cdot 728}{\log \frac{R}{r}}.$$

For example.

The core of a cable had an insulation resistance of 300 megohms per knot. Its external diameter was $\frac{4}{10}$th inch, and internal diameter $\frac{1}{10}$th inch. What was its specific insulation resistance?

$$s = 300 \frac{2 \cdot 728}{\log \frac{4}{1}} = 1360.$$

449. It may be remarked that the foregoing standard of specific insulation resistance is really that of a *cube knot* of the material, that is to say, of a cube whose dimensions are one knot each way, and whose resistance is assumed to be 1; this standard was introduced by Messrs. Clark and Sabine.

Specific Inductive Capacity.

450. From what was said on page 339, § 402, it will be evident that the formula for giving the specific inductive capacity (K) of a cable-core will be

$$K = F \frac{\log \frac{R}{r}}{2 \cdot 728},$$

where F is the capacity per knot of the core in microfarads.

CHAPTER XXI.

CORRECTIONS FOR TEMPERATURE.

451. In order to make tests for Conductivity, Insulation Resistance, or Electrostatic Capacity strictly comparative, it is either necessary that they be made at the same temperature, or, when this cannot be done, the temperatures at which they are taken should be noted, and a correction made.

Various methods have been suggested to enable this to be done, but the following seems to be a satisfactory and simple one.

It is found, when the temperatures are not very widely different, that for every degree of increase in temperature, an equal percentage of increase in resistance takes place; that is to say, if the resistance increased at a certain rate per cent. by a rise of one degree of temperature, it will be increased by the next degree of rise at the same rate per cent. calculated on the new resistance.

This being so, it will be evident, on consideration, that the percentage of increase for a certain number of degrees will be the same at whatever part of the scale these degrees are taken. Thus, if a resistance increased 25 per cent. between 30° and 40°, it would increase 25 per cent. between 65° and 75°.

Corrections for Conductor Resistance.

452. If we take a wire of any metal, and determine how much its resistance is increased by any number of degrees of temperature, we can determine how much the resistance of any other wire of the same metal and quality would be increased.

The law we have stated is exactly the same as the law for the fall of potential in an insulated cable. We have simply, in fact, to substitute resistances for potentials, and degrees of temperature for intervals of time, in any of the formulæ we had for the above case, and we get our formulæ for change of resistance by change of temperature.

At the end of Chapter XVI. (page 344) we obtained a formula

$$v_2 = V\left(\frac{V}{v}\right)^{\frac{t_2}{t_1}},$$

If we suppose a resistance to have increased from r to R, by an increase of temperature of n°, and to R_1 by an increase of n_1°, then by substitution in the foregoing formula, we get the equation

$$R_1 = r\left(\frac{R}{r}\right)^{\frac{n_1^\circ}{n^\circ}}$$

as representing the connection between these quantities.

If we determine R and r experimentally, we can find what R_1 will be for an increase in the temperature of 1°, 2°, 3°, &c.; and by embodying the results in a table, we can determine, from an experimental measurement made at any temperature, what the resistance of a wire, of the same kind as that from which the table was constructed, would be at any temperature we may require.

An example will render this clearer:—Suppose we had a wire of pure copper, whose resistance r at 32° F. was found to be 2064 ohms, and whose resistance R at 75·2° F. was 2262 ohms.* Then this wire had, by an increase of $75 \cdot 2^\circ - 32^\circ = 43 \cdot 2\ (n)$ increased its resistance $\frac{2262}{2064}$, or 1·096 times. We therefore know that any other wire of similar quality will increase its resistance by that amount, by an increase of 43·2° of temperature. This result, then, is the *coefficient* of *increase* for 43·2°, by which we must multiply our observed resistance to obtain its value at the required temperature.

The result we have obtained will enable us to determine the increase of resistance for any other number of degrees of temperature, and also the coefficients, for our formula becomes

$$R_1 = 2064\left(\frac{2262}{2064}\right)^{\frac{n_1^\circ}{43 \cdot 2^\circ}}.$$

Thus, if we want to find the coefficient for 30° (n_1°) of increase, we have

$$R_1 = 2064\left(\frac{2262}{2064}\right)^{\frac{30^\circ}{43 \cdot 2^\circ}} = 2199 \cdot 564;$$

showing that an increase of 30° has increased 2064 (r) to 2199·564, or $\frac{2199 \cdot 564}{2064} = 1 \cdot 06568$ times, which gives the coefficient for 30° of increase in temperature.

* These are the exact figures from experiments made by Dr. Matthiessen on pure copper.

Since we have to divide R_1 by r to obtain the coefficient, we have

$$\text{Coefficient} = \left(\frac{R}{r}\right)^{\frac{n_1^\circ}{n^\circ}};$$

or for the quality of wire in question,

$$\text{Coefficient} = \left(\frac{2262}{2064}\right)^{\frac{n_1^\circ}{43 \cdot 2^\circ}}.$$

As the coefficients have to be worked out by logarithmic tables, we may say

$$\log \text{coefficient} = (\log 2262 - \log 2064)\frac{n_1^\circ}{43 \cdot 2^\circ};$$

that is,

$$\log \text{coefficient} = (\cdot 0009209)\, n_1^\circ.$$

When all the coefficients are worked out by this formula and embodied in a table (Table IV.), if we require to find the resistance of a wire at the higher temperature, that at the lower being given, we must *multiply* the latter by the coefficient corresponding to the number of degrees of difference between the two temperatures.

If, on the contrary, it is required to reduce or correct from a higher to a lower temperature, we must *divide* by the coefficient corresponding to the number of degrees of difference between the two temperatures.

Influence of Conducting Power upon Variation of Resistance by Change of Temperature.

453. The influence of temperature upon the resistance of metals varies according to the conducting power of the metal. According to Matthiessen,* "the percentage of decrement in the conducting power of an impure metal, between 0° C. and 100° C., is to that of the pure one, between 0° C. and 100° C., as the conducting power of the impure metal at 100° C. is to that of the pure one at 100° C." A numerical example will best explain this law:—

Supposing we have two wires of the same metal, one of which is pure and the other impure, and we take such a length of each that they both have a resistance of 300 ohms at 0° C.; and suppose that the relative specific conductivities of the two kinds of metal are as 100 to 90. Now if we found that the pure

* Phil. Trans., 1864, p. 167.

sample increased its resistance from 300 ohms to 420 ohms, or 40 per cent., when the temperature was increased to 100° C.; then we should find that the impure sample when raised to 100° C. would have increased its resistance to 408 ohms, or 36 per cent., for

$$100 : 90 :: 40 : 36.$$

From this it is evident that the correction coefficients require to be varied according to the purity of the metal, but if we know what the coefficients are for the pure metal, and also the specific conductivity of the metal, we can correct the coefficients accordingly. Let R be the resistance of both the pure and impure metals at a temperature t, and R_1 the resistance of the *pure* metal at a temperature t_1, and let κ be the coefficient required to correct R to the latter temperature, that is, let

$$R_1 = R\,\kappa. \qquad [1]$$

Let R_2 be the resistance of the *im*pure metal at the temperature t_1, and let κ_1 be the coefficient required to correct R to this temperature, that is, let

$$R_2 = R\,\kappa_1. \qquad [2]$$

Also let C and C_1 be the specific conductivities of the pure and impure metals.

Lastly, let p and p_1 be the percentages of increase in resistance of the two samples respectively, between the temperatures t and t_1.

We then have the following equations:—

$$p = \frac{R - R_1}{R} 100 \qquad [3]$$

$$p_1 = \frac{R - R_2}{R} 100, \qquad [4]$$

and the proportion

$$p : p_1 :: C : C_1$$

or

$$\frac{p}{p_1} = \frac{C}{C_1};$$

but from equations [3] and [4] we get

$$\frac{p}{p_1} = \frac{\frac{R - R_1}{R} 100}{\frac{R - R_2}{R} 100} = \frac{R - R_1}{R - R_2};$$

therefore

$$\frac{C}{C_1} = \frac{R - R_1}{R - R_2},$$

or, substituting the values of R_1 and R_2, obtained from equations [1] and [2], we get

$$\frac{C}{C_1} = \frac{R - R\kappa}{R - R\kappa_1} = \frac{1 - \kappa}{1 - \kappa_1};$$

therefore

$$1 - \kappa_1 = \frac{C_1}{C}(1 - \kappa),$$

that is

$$\kappa_1 = 1 - \frac{C_1}{C}(1 - \kappa) = 1 + \frac{C_1}{C}(\kappa - 1).$$

As the specific conductivity of the pure metal is always taken as 100, the formula becomes

$$\kappa_1 = 1 + \frac{C_1}{100}(\kappa - 1). \qquad [A]$$

For example.

From Table IV., the correction coefficient for correcting from 45° to 75° (equal to 30° of difference of temperature) is 1·0657, for pure copper. What is the coefficient for copper whose conductivity is 96 per cent. of that of the pure metal?

$$\kappa_1 = 1 + \frac{96}{100}(1 \cdot 0657 - 1) = 1 \cdot 0631.$$

Corrections for Insulation Resistance.

454. The law of change of resistance by change of temperature for *Insulators* is the reverse of that for *Conductors*, that is to say, *increase* of temperature *diminishes* their resistance, and *vice versâ*. If, therefore, we obtain our coefficients from the formula

$$\text{Coefficient} = \left(\frac{R}{r}\right)^{\frac{n_1^\circ}{n^\circ}},$$

where R is the observed higher resistance at the lower temperature, r the observed lower resistance at the higher temperature, n° the number of degrees of difference between the two temperatures, and n_1° the number of degrees of difference for which

the coefficient is required, then we must *divide* by the coefficient when we require to find the resistance at the higher temperature, that at the lower being given, and *vice versâ*.

455. The influence of temperature is very much greater on Insulators than on Conductors, thus whereas a wire containing 96 per cent. of pure copper only increased its resistance from 1000 ohms to 1030 ohms by an *increase* of 15° of temperature, a particular gutta-percha core increased its resistance from 1000 ohms to 9080 ohms by the same amount of *decrease* of temperature.

The amount of the change of resistance by change of temperature which takes place in the case of insulating materials is dependent upon the quality of the latter, and, therefore, the correction coefficients for the same can only be regarded as approximately correct.

456. The range of temperatures required in practice is not large. If we calculate coefficients for a difference of from 0° to 45° it will usually be sufficient.

457. Tables of coefficients for copper and two kinds of gutta-percha, calculated on the foregoing principles, will be found at the end of the book. As it is usual in practice to correct all measurements to 75° Fahr., the coefficient corresponding to the number of degrees of difference between any particular temperature and 75°, is placed opposite that particular temperature.

458. It was pointed out on page 375 that if all the coefficients are worked out by the formula "log coefficient $= (\cdot 0009209) n_1°$," then in order to correct from a lower to a higher temperature it is necessary to *multiply* by the coefficient corresponding to the number of degrees of difference between the two temperatures, but to correct from a higher to a lower temperature we must *divide*; now, if in the latter case we employ the reciprocal of the coefficient, then we must *multiply* as in the first case. By taking advantage of this fact, in Table IV. the coefficients are so calculated that whether we have to correct from 100° down to 75°, or from 32·5° up to 75°, in all cases we have to *multiply*; in Tables V. and VI., in all cases we have to *divide*.

If it should be required to correct up to any temperature other than 75°, then we must first ascertain the number of degrees of difference between the two temperatures, and then the coefficient opposite to the temperature corresponding to 75° minus the number of degrees of difference, will be the coefficient required. Thus, if we wanted the coefficient for correcting the resistance of a pure copper wire from 45° up to 60°, then $60° - 40° = 20°$, and $75° - 20° = 55°$, and the coefficient corresponding to this temperature in Table IV. is 1·0434, which is

the required coefficient. Should it be necessary to correct from 60° down to 45°, then in this case the coefficient will be that corresponding to 75° + 20°, or 95°, the value of which is ·9586.

459. The exact effect of temperature on Electrostatic Capacity has not, it is believed, been yet determined or published; it is, however, very slight.

Determination of the Temperature of a Wire by Change of Resistance.

460. By a reverse process to the foregoing we can tell what the temperature of a wire is if we know what is its resistance at one temperature, and also its resistance at the unknown temperature. For all we have to do is to divide one resistance by the other, and note with what number of degrees of temperature the coefficient so obtained corresponds, then this result shows the number of degrees the wire has above or below the temperature at which the wire was measured.

For example.

To take the case on page 374, we found that the wire had a resistance of 2064 ohms at 32°, and at the temperature which we will suppose to be unknown, a resistance of 2262, then the coefficient is $\frac{2262}{2064} = 1 \cdot 096$, which corresponds to an increase of $43 \cdot 2°$; the temperature of the wire is therefore $32° + 43 \cdot 2° = 75 \cdot 2°$.

In this way, if we ascertained the resistance of a cable at a noted temperature before it was laid, and then measured its resistance after it was laid, we could tell the mean temperature of the sea by referring to the Tables.

Or if we ascertained the resistance of the cable at two different temperatures before it was laid, then we could determine its temperature after it was laid without the use of Tables, thus from the formula

$$R_1 = r\left(\frac{R}{r}\right)^{\frac{n_1°}{n°}}$$

we get

$$\left(\frac{R_1}{r}\right) = \left(\frac{R}{r}\right)^{\frac{n_1°}{n°}};$$

therefore

$$\frac{n_1°}{n°} = \frac{\log \frac{R_1}{r}}{\log \frac{R}{r}};$$

therefore

$$n^\circ = \frac{\log \frac{R}{r}}{\log \frac{R_1}{r}} \cdot n_1^\circ.$$

For example.

The conductor resistance (R_1) of a cable at 60° Fahr. when lying in the tanks at the factory, was 2000 ohms, and at a temperature of 45° Fahr. the resistance (r) was 1941·6 ohms. When the cable was laid the resistance (R) was found to be 1961 ohms. What was the temperature of the sea?

$$n_1^\circ = 60^\circ - 45^\circ = 15^\circ,$$

$$n^\circ = \frac{\log \frac{1961}{1941\cdot 6}}{\log \frac{2000}{1941\cdot 6}} \cdot 15^\circ = 5^\circ.$$

The temperature of the sea will therefore be 45° + 5° = 50° F.

461. It is very often the case in cable factories that two sections of the cable are in different tanks at different temperatures, as, for instance, when several miles of core are added on to the made-up cable in a colder tank.

As the whole length must be tested in one section, it is necessary to know what correction must be applied to the measured resistance of the whole length of cable to correct it to the value it would have at one uniform temperature.

Corrections for Conductor Resistance when Two Sections of a Cable are at Different Temperatures.

462. Let l_1 and l_2 be the two lengths of the cable in the different tanks, also let r_1 and r_2 be the respective conductor resistances of the two sections at the temperatures of the tanks, and let P_c be the resistance of the two together. Also let k_1 and k_2 be the coefficients by which r_1 and r_2 must be *multiplied* respectively in order to reduce them to the values they would have at one uniform temperature, and let R_c be the total resistance of the cable at this uniform temperature; we then have the following equations:

$$P_c = r_1 + r_2,$$

$$R_c = r_1 k_1 + r_2 k_2, \qquad [1]$$

$$\frac{l_1}{l_2} = \frac{r_1 k_1}{r_2 k_2};$$

therefore

$$\frac{R_c}{P_c} = \frac{r_1 k_1 + r_2 k_2}{r_1 + r_2}, \quad \text{or,} \quad R_c = P_c \frac{r_1 k_1 + r_2 k_2}{r_1 + r_2};$$

and also we may say

$$\sigma l_1 = r_1 k_1, \quad \text{or,} \quad r_1 = \frac{\sigma l_1}{k_1},$$

$$\sigma l_2 = r_2 k_2, \quad \text{or,} \quad r_2 = \frac{\sigma l_2}{k_2},$$

where σ is a constant; therefore

$$R_c = P_c \frac{\sigma (l_1 + l_2)}{\sigma \left(\frac{l_1}{k_1} + \frac{l_2}{k_2}\right)} = P_c \frac{l_1 + l_2}{\frac{l_1}{k_1} + \frac{l_2}{k_2}}.$$

If l_1 and l_2 are the lengths of the portions of the cable in knots, then the corrected resistance per knot (r_c) will be

$$r_c = P_c \frac{l_1 + l_2}{\frac{l_1}{k_1} + \frac{l_2}{k_2}} \div (l_1 + l_2) = \frac{P_c}{\frac{l_1}{k_1} + \frac{l_2}{k_2}}. \qquad [2]$$

For example.

At a cable factory there were 15 knots (l_1) of manufactured cable lying in a tank whose temperature was 50° Fahr. Connected to this cable were 5 knots (l_2) of core in a tank whose temperature was 55° Fahr. The total observed conductor resistance of the 20 knots was 215 ohms (P_c). What would be the conductor resistance per knot (r_c) of the cable and core at 75° Fahr.? The conductor was of copper of 96 per cent. conductivity.

From Table IV. we have (after applying the correction [A], page 377)

$$k_1 = 1 \cdot 0523, \quad k_2 = 1 \cdot 0417;$$

therefore

$$r_c = \frac{215}{\frac{15}{1 \cdot 0523} + \frac{5}{1 \cdot 0417}} = 11 \cdot 283 \text{ ohms.}$$

Corrections for Insulation Resistance when Two Sections of a Cable are at Different Temperatures.

463. Let l_1 and l_2 be the lengths of the two sections, r_1 and r_2 their respective insulation resistances at the temperature of the tanks, P_i the combined resistance of the two sections, k_1 and k_2 the coefficients by which r_1 and r_2 must be *divided* in order to reduce them to the values they would have at one uniform temperature, also let R_i be the combined resistance of the two sections at this uniform temperature; then we have the following equations:

$$P_i = \frac{r_1 r_2}{r_1 + r_2},$$

$$R_i = \frac{\dfrac{r_1}{k_1} \cdot \dfrac{2}{k_2}}{\dfrac{r_1}{k_1} + \dfrac{r_2}{k_2}} = \frac{r_1 r_2}{r_1 k_2 + r_2 k_1}, \qquad [3]$$

$$\frac{l_1}{l_2} = \frac{\dfrac{r_2}{k_2}}{\dfrac{r_1}{k_1}} = \frac{r_2 k_1}{r_1 k_2};$$

therefore

$$\frac{R_i}{P_i} = \frac{r_1 + r_2}{r_1 k_2 + r_2 k_1}, \quad \text{or,} \quad R_i = P_i \frac{r_1 + r_2}{r_1 k_2 + r_2 k_1};$$

and also we may say

$$\sigma l_1 = r_2 k_1, \quad \text{or,} \quad r_2 = \frac{\sigma l_1}{k_1},$$

$$\sigma l_2 = r_1 k_2, \quad \text{or,} \quad r_1 = \frac{\sigma l_2}{k_1}$$

where σ is a constant; therefore

$$R_i = P_i \frac{\sigma \left(\dfrac{l_1}{k_1} + \dfrac{l_2}{k_2}\right)}{\sigma (l_1 + l_2)} = P_i \frac{\dfrac{l_1}{k_1} + \dfrac{l_2}{k_2}}{l_1 + l_2}.$$

If l_1 and l_2 are the lengths of the sections in knots, then the corrected resistance per knot (r_i) will be

$$r_i = R_i (l_1 + l_2) = P_i \left(\frac{l_1}{k_1} + \frac{l_2}{k_2}\right).$$

For example.

Taking the same lengths and temperatures as in the previous example, let us suppose the total observed insulation resistance of the 20 knots was 160 megohms (P_i), what would be the insulation resistance per knot (r_i) at 75° Fahr. the insulator being Willoughby Smith's gutta-percha?

From Table VI. (by reducing the logarithms to natural numbers) we find

$$k_1 = 6 \cdot 928, \quad k_2 = 4 \cdot 704,$$

therefore

$$r_i = 160 \left(\frac{15}{6 \cdot 928} + \frac{5}{4 \cdot 704} \right) = 516 \cdot 5 \text{ megohms.}$$

CHAPTER XXII.

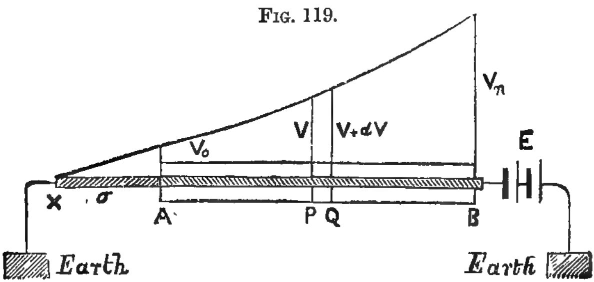
LOCALISATION OF FAULTS OF HIGH RESISTANCE.

Faults in Cables.

464. In all the tests for localising faults hitherto described, with the exception of the loop test (page 224), it has been assumed that the insulation resistances of the portions of cable on either side of the fault are infinitely great compared with the resistances of the conductor. Such an assumption practically holds good in cases where the cable under test is short, and also if the resistance of the fault is small, but when we come to deal with long cables having faults of high resistance, the formulæ we have obtained are no longer correct. The following investigation* is made for the purpose of obtaining a test which shall be correct for cables of any length and having faults of any resistance:—

Let A B (Fig. 119), be a cable of any length, connected to a battery as shown, and having its further end to earth through

Fig. 119.



a resistance σ. By putting $\sigma = 0$ the end of the cable will be direct to earth, and by putting $\sigma = \infty$, it will be insulated.

$$\text{Let the length } A\,B = n$$
$$\text{"} \quad \text{"} \quad A\,P = x$$
$$\text{"} \quad \text{"} \quad P\,Q = d\,x.$$

* See 'On the Leakage of Submarine Cables,' by A. B. Kempe, B.A., 'Journal of Society of Telegraph Engineers,' Vol. IV., page 90.

Let the potential at A $= V_0$
„ „ „ B $= V_n$
„ „ „ P $= V$
„ „ „ Q $= V + dV$.

Let the current strength at A $= C_0$
„ „ „ B $= C_n$
„ „ „ P $= C$
„ „ „ Q $= C + dC$.

Let the resistance of X A P $= R$
„ „ „ X A Q $= R + dR$
„ „ „ X A B $= R_n$
„ „ „ X A $= R_0 = \sigma$.

Also let resistance of unit length of conductor $= r$
And „ „ „ sheathing $= i$.

Calling E the electromotive force of the battery, then since the flow of electricity from any point to any other point close to it is from the point of higher to that of lower potential, and is equal to the difference of potential divided by the resistance separating the two points, therefore the current along A B at P is

$$\frac{(V + dV) - V}{r\,dx} = \frac{dV}{r\,dx} = C.$$

The resistance of the wire P Q is evidently $r\,dx$, because it varies *directly* as the length of the wire, but the resistance of the insulating sheath P Q is $\frac{i}{dx}$, because it varies *inversely* as the length. Hence the "leakage" or the current from the surface of the conductor between the points P and Q to the earth where the potential is zero, is

$$\frac{V - 0}{\frac{i}{dx}} = \frac{V\,dx}{i} = dC.$$

Hence

$$\frac{dC}{dx} = \frac{V}{i};$$

but

$$C = \frac{d\,V}{r\,dx},$$

and, therefore,

$$\frac{d\,C}{dx} = \frac{1}{r} \cdot \frac{d^2\,V}{dx^2};$$

therefore

$$\frac{d^2\,V}{dx^2} = \frac{r\,V}{i} m^2\,V,$$

where

$$m^2 = \frac{r}{i}, \text{ i.e. } m = \sqrt{\frac{r}{i}}.$$

The solution of this differential equation, obtained by the well-known method,* is

$$V = A\,e^{mx} + B\,e^{-mx}, \qquad [1]$$

and

$$C = \frac{1}{r}\;\frac{d\,V}{dx} = \frac{m}{r}\Big[A\,e^{mx} - B\,e^{-mx}\Big]. \qquad [2]$$

Now when $x = n$

$$V = V_n = E, \text{ and } C = C_n,$$

therefore, since resistance $= \dfrac{\text{potential}}{\text{current strength}}$,

$$R_n = \frac{V_n}{C_n} = \frac{E}{C_n};$$

and similarly when $x = 0$

$$V = V_0, \text{ and } C = C_0,$$

and

$$R_0 = \sigma = \frac{V_0}{C_0}.$$

Taking, therefore, $x = n$, we get

$$E = V_n = A\,e^{mn} + B\,e^{-mn} \text{ by [1]}$$

$$C_n = \frac{m}{r}\Big[A\,e^{mn} - B\,e^{-mn}\Big] \text{ by [2]},$$

* See Boole's 'Differential Equations,' Second Edition, Chapter IX., p. 194.

therefore

$$R_n = \frac{E}{C_n} = \frac{r}{m}\left[\frac{A\,e^{mn} + B\,e^{-mn}}{A\,e^{mn} - B\,e^{-mn}}\right]. \qquad [3]$$

Again, taking $x = 0$, we have

$$\sigma = \frac{V_0}{C_0} = \frac{A + B}{\frac{m}{r}(A - B)};$$

therefore

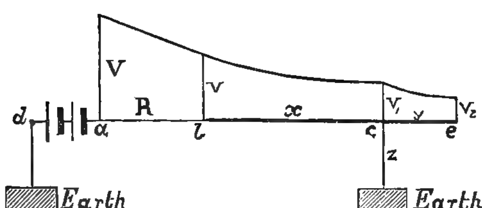
$$\frac{A}{B} = \left(\frac{\sigma\frac{m}{r} + 1}{\sigma\frac{m}{r} - 1}\right), \qquad [4]$$

and

$$R_n = \frac{r}{m}\left[\frac{e^{mn}\left(\sigma\frac{m}{r} + 1\right) + e^{-mn}\left(\sigma\frac{m}{r} - 1\right)}{e^{mn}\left(\sigma\frac{m}{r} + 1\right) - e^{-mn}\left(\sigma\frac{m}{r} - 1\right)}\right]. \qquad [5]$$

465. Let us now see how we can apply this investigation so as to obtain a test which shall be strictly accurate for a cable of any length and resistance. The following, which accomplishes this, is in reality the fall of potential test given on page 345, with the formula corrected.

FIG. 120.



Let $b\,c$ (Fig. 120) be the cable having a fault z at c, x and y being the *lengths* on either side of the fault, and let R be a resistance.

Now from equation [1] we have

$$v = A\,e^{mx} + B\,e^{-mx} \qquad [6]$$

and at c, where $x = 0$,

$$v_1 = A + B; \text{ therefore } A = v_1 - B;$$

therefore

$$v = v_1 e^{mx} - B e^{mx} + B e^{-mx},$$

or

$$B = \frac{v_1 e^{mx} - v}{e^{mx} - e^{-mx}}.$$

Now from [6]

$$v - 2 B e^{-mx} = A e^{mx} - B e^{-mx}.$$

therefore

$$v - 2 e^{-mx} \frac{v_1 e^{mx} - v}{e^{mx} - e^{-mx}} = \frac{v(e^{mx} + e^{-mx}) - 2 v_1}{e^{mx} - e^{-mx}} = A e^{mx} - B e^{-mx}. \quad [7]$$

Now, calling R_n the resistance beyond b, we have

$$\frac{V}{v} = \frac{R + R_n}{R_n};$$

therefore

$$\frac{R}{V - v} = \frac{R_n}{v} = \frac{R^n}{A e^{mx} + B e^{-mx}} \qquad \text{from [6];}$$

but from [3]

$$R_n = \frac{r}{m}\left[\frac{A e^{mx} + B e^{-mx}}{A e^{mx} - B e^{-mx}}\right]$$

therefore

$$\frac{R}{V - v} = \frac{r}{m}\left[\frac{1}{A e^{mx} - B e^{-mx}}\right],$$

therefore from [7]

$$\frac{R}{V - v} = \frac{r}{m}\left[\frac{e^{mx} - e^{-mx}}{v(e^{mx} + e^{-mx}) - 2 v_1}\right]. \qquad [8]$$

Again, considering the portion of the cable y, we have

$$\frac{v_1}{v_2} = \frac{A_1 e^{my} + B_1 e^{-my}}{A_1 + B_1},$$

and from [4]

$$\frac{A_1}{B_1} = \frac{\sigma \frac{m}{r} + 1}{\sigma \frac{m}{r} - 1},$$

in which, since the end of the cable is insulated, $\sigma = \infty$; therefore

$$\frac{A_1}{B_1} = 1,$$

from which

$$\frac{v_1}{v_2} = \frac{e^{my} + e^{-my}}{2};$$

therefore

$$v_1 = v_2 \frac{e^{my} + e^{-my}}{2}.$$

Inserting this value in [8], we get

$$\frac{R}{V - v} = \frac{r}{m}\left[\frac{e^{mx} - e^{-mx}}{v\,(e^{mx} + e^{-mx}) - v_2\,(e^{my} + e^{-my})}\right].$$

Now if l be the length of the cable, then $y = l - x$, therefore

$$\frac{R\,\frac{m}{r}}{V - v} = \frac{e^{mx} - e^{-mx}}{v\,(e^{mx} + e^{-mx}) - v_2\,(e^{m(l-x)} + e^{-m(l-x)})}.$$

Multiplying the top and bottom of the equation by e^{mx} we get

$$\frac{R\,\frac{m}{r}}{V - v} = \frac{e^{2mx} - 1}{v\,(e^{2mx} + 1) - v_2\,(e^{ml} + e^{-ml}e^{2mx})}$$

$$= \frac{e^{2mx} - 1}{e^{2mx}\,(v - v_2\,e^{-ml}) + v - v_2\,e^{ml}};$$

therefore

$$e^{2mx} = \frac{(V - v) + (v - v_2\,e^{ml})\,R\,\frac{m}{r}}{(V - v) - (v - v_2\,e^{-ml})\,R\,\frac{m}{r}},$$

from which

$$x = \frac{1}{2\,m}\log_e \frac{(V - v) + (v - v_2\,e^{ml})\,R\,\frac{m}{r}}{(V - v) - (v - v_2\,e^{-ml})\,R\,\frac{m}{r}}. \qquad [9]$$

In this form the formula cannot be practically used, as we require to know r and m, that is $\sqrt{\frac{r}{i}}$, r being the resistance per unit length of the conductor, and i the resistance per unit length of the insulating sheathing. These we cannot determine individually, for the measurement made when the end of the cable is to earth is not that of the conductor alone but that of the conductor diminished by the insulation resistance; and similarly, when the end is insulated the measurement made is not that of the insulating sheathing alone but of the latter combined with the resistance of the conductor. If, however, we know what these measured values are we can substitute them in the formula in the place of m and r.

Let the measured resistance of the cable when to earth at the further end be R_e, and when insulated R_i; then

$$\mathrm{R}_e = \frac{r}{m}\left[\frac{e^{ml} - e^{-ml}}{e^{ml} + e^{-ml}}\right],$$

$$\mathrm{R}_i = \frac{r}{m}\left[\frac{e^{ml} + e^{-ml}}{e^{ml} - e^{-ml}}\right].$$

This value of R_e we obtain from equation [5] by putting $\sigma = 0$, and of R_i by putting $\sigma = \infty$.

By multiplying one equation by the other we get

$$\mathrm{R}_e \mathrm{R}_i = \frac{r^2}{m^2};$$

therefore

$$\frac{m}{r} = \frac{1}{\sqrt{\mathrm{R}_e \mathrm{R}_i}}. \qquad [10]$$

Also, we have

$$\frac{\mathrm{R}_e}{\mathrm{R}_i} = \left(\frac{e^{ml} - e^{-ml}}{e^{ml} + e^{-ml}}\right)^2;$$

therefore

$$e^{ml}\left(\sqrt{\frac{\mathrm{R}_e}{\mathrm{R}_i}} - 1\right) = - e^{-ml}\left(\sqrt{\frac{\mathrm{R}_e}{\mathrm{R}_i}} + 1\right);$$

therefore

$$e^{2ml} = \frac{1 + \sqrt{\frac{\mathrm{R}_e}{\mathrm{R}_i}}}{1 - \sqrt{\frac{\mathrm{R}_e}{\mathrm{R}_i}}}; \qquad [11]$$

that is

$$\frac{1}{2m} = \frac{l}{\log_e \left(\dfrac{1 + \sqrt{\dfrac{R_e}{R_i}}}{1 - \sqrt{\dfrac{R_e}{R_i}}} \right)};$$

and also from [11] we have

$$e^{ml} = \sqrt{\frac{1 + \sqrt{\dfrac{R_e}{R_i}}}{1 - \sqrt{\dfrac{R_e}{R_i}}}}, \quad \text{and} \quad e^{-ml} = \sqrt{\frac{1 - \sqrt{\dfrac{R_e}{R_i}}}{1 + \sqrt{\dfrac{R_e}{R_i}}}}. \qquad [12]$$

We have thus determined $\frac{1}{2m}$, $\frac{m}{r}$, e^{ml}, and e^{-ml}, and can consequently insert their values in any equations we may require.

466. Instead of employing the resistance R, we may make the test by connecting the battery direct on to b through a galvanometer, so that the resistance R_n of the cable can be measured by the ordinary deflection method (§ 9, page 5). Then, since

$$V : v :: R + R_n : R_n,$$

therefore

$$V - v = v \frac{R}{R_n}.$$

If we substitute this value of $V - v$ in equation [9] we get

$$x = \frac{1}{2m} \log_e \frac{\dfrac{v}{R_n} + (v - v_2 e^{ml}) \dfrac{m}{r}}{\dfrac{v}{R_n} - (v - v_2 e^{-ml}) \dfrac{m}{r}}$$

$$= \frac{1}{2m} \log_e \frac{v + (v - v_2 e^{ml}) R_n \dfrac{m}{r}}{v - (v - v_2 e^{-ml}) R_n \dfrac{m}{r}}.$$

For example.

A cable 1000 knots (l) long had a very small fault in it which was required to be localised. When the cable was good its resistance with the further end insulated, after five minutes' electrification, was 700,000 ohms (R_i), and its resistance with the further end put to earth, 5000 ohms (R_e). When the cable was faulty its resistance with the end insulated, after five minutes' electrification was 100,000 ohms (R_n). The potentials at the ends of the cable, after five minutes' electrification, were 300 (v) and 292 (v_2) respectively. What was the distance (x) of the fault from the nearer end of the cable?

$$\frac{m}{r} = \frac{1}{\sqrt{5000 \times 700{,}000}} = \frac{1}{59{,}161},$$

$$\frac{1}{2m} = \frac{1000}{\log \dfrac{1 + \sqrt{\dfrac{5000}{700{,}000}}}{1 - \sqrt{\dfrac{5000}{700{,}000}}} \times 2 \cdot 3026} \doteq 5902 \cdot 1,$$

$$e^{ml} = \sqrt{\frac{1 + \sqrt{\dfrac{5000}{700{,}000}}}{1 - \sqrt{\dfrac{5000}{700{,}000}}}} = 1 \cdot 0884,$$

$$e^{-ml} = \sqrt{\frac{1 - \sqrt{\dfrac{5000}{700{,}000}}}{1 + \sqrt{\dfrac{5000}{700{,}000}}}} = \cdot 9188.$$

Inserting these values in the equation we get

$$x = 5902 \cdot 1 \times \log \frac{300 + [300 - (292 \times 1 \cdot 0884)]\, 100{,}000 \times \dfrac{1}{59{,}161}}{300 - [300 - (292 \times \cdot 9188)]\, 100{,}000 \times \dfrac{1}{59{,}161}}$$
$$\times 2 \cdot 3026 = 538 \text{ knots.}$$

467. Since in the case of a small fault the difference between the potentials at the two ends is comparatively small it is essential that they should be measured with great accuracy, otherwise a small error made in determining their value will

make a considerable error in the value of x. The readings on the scale of the galvanometer or electrometer must therefore be made as high as possible; it is even advisable to extend the length of the scale so that this may be done more effectually.

468. The relative values of the potentials at the two ends of the cable must be determined in the manner described in Chapter XVII (§ 414, page 346).

Localisation of Faults in Insulated Wires.

Warren's Method.*

469. This method is adapted for localising faults of high resistance in lengths of cable core which have not been covered with the iron sheathing which forms the complete cable.

The length of wire to be operated on is immaterial, provided that the whole or a portion of it can be coiled on an insulated drum, and that between the parts coiled the surface of the core for a length of 6 or 8 inches can be cleaned and dried so as to prevent conduction.

In the first case (when the whole can be coiled on a drum), one-half is coiled off on a second drum, the two drums being carefully insulated. The surface of the core between the drums is well cleaned and dried. The conductor is attached to one set of quadrants of an electrometer, the other set being to earth, and the two drums are connected to earth by an attendant at each drum; one pole of the battery (whose second pole is to earth) is then connected to the conductor, so that the whole becomes charged; the battery is then disconnected from the electrometer, and the earth-wires simultaneously taken off the drums. It is best to leave the battery on until the earth-wires are removed from the drums.

The insulation of the drums and electrometer should be such that no loss can be perceived after a few minutes, when, if the earth-wire be applied first to one drum and then to the other, the fault will be found on that drum which causes the greatest fall in the electrometer. The wire is coiled from the faulty side to the other, and the test repeated as often as is required. A mile of core with a small fault in it can by a little practice be put right in an hour or two, involving no more waste than a portion of the insulator which can be held between the fingers, and without even cutting the conductor. The position of the fault, when it is obtained between the two drums, can be found more closely by cleaning and drying the surfaces on each side of it, and then touching the place where the fault appears to be,

* 'Philosophical Magazine,' No. 314, June 1879.

with the earth-wire, and seeing whether there is a fall in the electrometer.

In the second case, where the bulk would prevent the whole from being insulated, we should continue to coil the core upon an insulated drum until the fault disappeared—that is, until it was coiled on the drum. This is a useful method when dealing with "served core," that is core with its hemp covering only, at a cable factory.

470. By the foregoing method a *joint* may be tested with great ease by immersing it in an insulated trough of water, and putting the latter to earth, or even by simply touching the moist joint with the earth-wire.

471. The tests can be made with a galvanometer instead of an electrometer, although it is not such a sensitive arrangement. In this case the battery would be connected through the galvanometer to the conductor, as in an ordinary insulation test, and then the drums be connected to earth alternately, when the deflection of the needle shows on which drum the fault exists; but as the lengths on each drum may be very unequal, and consequently one drum may show a greater deflection simply in virtue of its having a greater length of core on it, the rush of current alone is not sufficient to enable the drum on which the fault is, to be found; but by carefully watching the electrification, and seeing whether the fall is regular or not, no difficulty will be found in fixing upon the drum containing the fault. The battery-power required will vary with the magnitude of the fault and the sensitiveness of the instrument, and can only be determined by experience and experiment.

CHAPTER XXIII.

LOCALISATION OF A DISCONNECTION FAULT IN A CABLE.

Localisation of a Total Disconnection.

472. The localisation of a total disconnection in a cable is a very easy matter. The conductor being broken inside the insulating sheathing, a battery connected to the end of the cable will charge the latter up as far as the fault only, consequently if we measure the discharge and compare it with the discharge from a condenser of a known capacity charged from the same battery, we shall obtain the capacity of the portion up to the fault. Also since the capacity per knot of the cable is always known, the observed capacity of the length in question, divided by the capacity per knot, will give at once the distance of the fault.

Localisation of a Partial Disconnection.

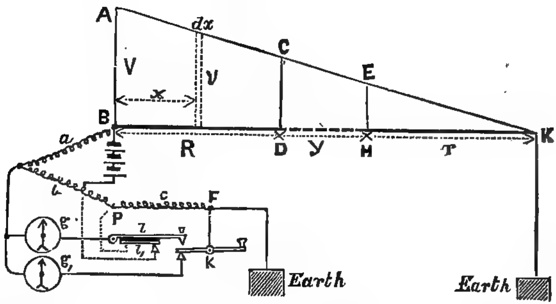
473. Partial disconnection faults, although they are seldom met with in cables with gutta-percha cores, frequently occur in those whose insulating material is indiarubber. This arises from the elasticity of the substance; for when any undue strain is put on the core the conductor breaks, but the indiarubber only stretches, and an earth fault is not made. When the strain is taken off, the two ends of the conductor come together and make contact more or less perfectly. If the break is noticed at the moment the cable is being laid from the ship, its position is of course known. But in some cases a fault of this nature does not develop itself until some time after the submersion; its locality can then only be found by testing.

Such faults are difficult to localise, as none of the ordinary tests are applicable to them. The following method, however, devised by the author, is susceptible of considerable accuracy if carefully made.

In Fig. 121, B K represents the cable with its further end to earth; R and r are the resistances of the portions of the cable on either side of the disconnection, and y is the resistance of the latter: a. b. and c are the three sides of a Wheatstone bridge,

of which the cable forms the fourth side; g and g_1 are two galvanometers, the former being of the ordinary Thomson form, and the latter also a Thomson, but provided with heavy needles, so that its movements are very sluggish.

FIG. 121.



Connected to the battery, and also to the galvanometers, is a key of a peculiar description; it is formed in two parts. The ordinary lever k of the key has its back-stop connected through the galvanometer g_1 to the junction of the resistances a, b; thus when the key is in its normal position the galvanometer g_1 is connected to earth. The second portion of the key consists of a lever l, to the underneath part of which is fixed the metal piece l_1, which is insulated from l. Normally, as shown in the figure, l_1 rests on a stop connected to one pole of the battery, the other pole of the latter being connected to B. The point P is connected permanently with l_1, whilst the lever l is itself permanently connected to the galvanometer g.

Now, the result of this arrangement is, that normally the battery is connected between the points B and P, and the galvanometer g_1 is connected between the junctions of a and b and with earth, that is with the end B of the cable; the whole arrangement, in fact, forms an ordinary Wheatstone bridge.

Now, if a, b, and c are adjusted that balance is produced, then the needle of the galvanometer g_1 will stand at zero; if, when this is the case, the key k be depressed, g_1 will be disconnected, and when the lever of k touches the end of l, g will be put in circuit in the place of g_1; but immediately this takes place l_1 will be lifted off its contact and the battery will be cut off; exactly at this moment then the charges in the cable will discharge and divide themselves, portions flowing out at the further end and the other portions flowing out through g, a, and

b, and thence through c to earth. A *throw* of the needle of the galvanometer will thus be produced.

Supposing the key k to be in its normal position, so that the battery causes a current to flow through the cable, whilst the resistances a, b, and c are so adjusted that the galvanometer g_1 is unaffected, then let V be the potential at the beginning, and v be the potential at any other point of the portion B D.

If now the key be depressed, the charges in the cable represented by the areas A B D C and E H K, will flow out at the two ends of the cable in proportions dependent upon the values of the resistances R, y, and r, and the combined resistances of a, b, g, and c.

Let $v\,dx$ be a differential part of the charge A B D C, then this portion will split, and the portions flowing out at the two ends of the cable will be inversely proportional to the resistances on either side of $v\,dx$; thus the portion flowing out at B will be

$$d\,Q' = v\,\frac{R + y + r - x}{R_1 + R + y + r}\,dx\,,$$

where R_1 is the combined resistance of a, b, g, and c.

Now

$$V : v :: R + y + r : R + y + r - x\,,$$

therefore

$$v = V\,\frac{R + y + r - x}{R + y + r}\,,$$

that is

$$d\,Q' = V\,\frac{(R + y + r - x)^2}{(R_1 + R + y + r)\,(R + y + r)}\,,$$

and the integral of this between the limits

$$x = R \text{ and } x = 0$$

will give the quantity Q′ flowing through the galvanometer, that is

$$Q' = \int_0^R V\,\frac{(R + y + r - x)^2}{(R_1 + R + y + r)\,(R + y + r)}\,dx$$

$$= \frac{V}{(R_1 + R + y + r)\,(R + y + r)}\int_0^R (R + y + r - x)^2\,dx$$

$$= \frac{V}{(R_1 + R + y + r)\,(R + y + r)}\left[\frac{(R + y + r)^3 - (y + r)^3}{3}\right]$$

$$= \frac{V}{3}\cdot\frac{(R + y + r)^3 - (y + r)^3}{(R_1 + R + y + r)\,(R + y + r)}\,.$$

Similarly we should find that the quantity Q'' flowing out from the portion r of the cable would be

$$Q'' = \frac{V}{3} \cdot \frac{r^3}{(R_1 + R + y + r)(R + y + r)},$$

and therefore the total quantity Q flowing through the galvanometer will be

$$Q' + Q'' = \frac{V}{3} \cdot \frac{(R + y + r)^3 - (y + r)^3 + r^3}{(R_1 + R + y + r)(R + y + r)} = Q \, . [1]$$

Now the total quantity Q_1 which the cable would take if its further end were insulated and the end B maintained at the potential V, would be

$$Q_1 = V(R + r).$$

Again, if f be the capacity in microfarads of such a length of the cable as would have a conductor resistance of 1 ohm, then $(R + r) f$ will be the actual total capacity of the cable, and if Q_2 be the charge held by a condenser of F microfarads capacity, also charged to the potential V, then

$$Q_1 : Q_2 :: (R + r) f : F;$$

therefore

$$Q_1 = \frac{Q_2 (R + r) f}{F} = V(R + r),$$

or

$$V = \frac{Q^2 f}{F}.$$

Substituting then this value of V in equation [1] we get

$$Q = \frac{Q_2 f}{3\,F} \cdot \frac{(R + y + r)^3 - (y + r)^3 + r^3}{(R_1 + R + y + r)(R + y + r)}.$$

Let

$$R + y + r = L, \text{ therefore } y + r = L - R;$$
$$R + r = L_1, \text{ therefore } r = L_1 - R.$$

Substituting these values in the last equation we get

$$Q = \frac{Q_2 f}{3\,F} \cdot \frac{L^3 - (L - R)^3 + (L_1 - R)^3}{(R_1 + L)\,L}$$

$$= \frac{Q_2 f}{3\,F} \cdot \frac{L^3 - L^3 - 3\,L\,R^2 + 3\,L^2\,R + R^3 + L_1^3 + 3\,L_1\,R^2 - 3\,L_1^2\,R - R^3}{(R_1 + L)\,L}$$

$$= \frac{Q_2 f}{3\,F} \cdot \frac{L_1^3 + 3\,R\,(L^2 - L_1^2) - 3\,R^2\,(L - L_1)}{(R_1 + L)\,L};$$

therefore

$$\frac{3\,Q\,F\,(R_1 + L)\,L}{Q_2 f} - L_1^3 = 3\,R\,(L + L_1)\,(L - L_1) - 3\,R^2\,(L - L_1);$$

therefore

$$R^2 - R\,(L + L_1) = -\frac{3\,Q\,F\,(R_1 + L)\,L - Q_2 f L_1^3}{3\,Q_2 f\,(L - L_1)};$$

therefore

$$R^2 - R\,(L + L_1) + \left(\frac{L + L_1}{2}\right)^2 = \left(\frac{L + L_1}{2}\right)^2 - \frac{3\,Q\,F\,(R_1 + L)\,L - Q_2 f L_1^3}{3\,Q_2 f\,(L - L_1)},$$

that is

$$R = \frac{L + L_1}{2} - \sqrt{\frac{(L + L_1)^2}{4} - \frac{3\,Q\,F\,(R_1 + L)\,L - Q_2 f L_1^3}{3\,Q_2 f\,(L - L_1)}}. \quad [2]$$

Now the quantity Q discharged at B will split between the resistances g, and $a + b$, the quantity Q_3 passing through the galvanometer being

$$Q_3 = Q\,\frac{b + c}{b + c + g},$$

from which

$$Q = Q_3\,\frac{b + c + g}{b + c}.$$

The value of R_1, the combined resistance of a, b, g, and c, will be

$$R_1 = a + \frac{(b + c)\,g}{b + c + g},$$

and since balance is produced in the bridge

$$L = \frac{a\,c}{b},$$

therefore

$$R_1 + L = a + \frac{(b + c)\,g}{b + c + g} + \frac{a\,c}{b}$$

$$= (b + c)\,\frac{g}{b + c + g} + (b + c)\,\frac{a}{b} = \frac{(b + c)}{(b + c + g)\,b}\,[b\,g + a\,(b + c + g)] = \frac{b + c}{(b + c + g)\,b}\,[g\,(a + b) + a\,(b + c)];$$

and therefore

$$Q(R_1 + L) = \frac{Q_3}{b}[g(a+b) + a(b+c)].$$

Substituting this value in equation [2] we get

$$R = \frac{L + L_1}{2} - \sqrt{\frac{(L+L_1)^2}{4} - \frac{\frac{3 Q_3 F}{b}[g(a+b) + a(b+c)] L - Q_2 f L_1^3}{3 Q_2 f (L - L_1)}}.$$

In which, as we have before stated,

$$L = \frac{a\,c}{b}.$$

Should it be necessary to employ a shunt for the galvanometer g, of the $\frac{1}{n}$th value say, then the observed deflection will have to be multiplied by n in order to give the value of Q_3, and also the value of g in the formula will be $\frac{1}{n}$th of the actual resistance of the galvanometer.

For example.

In localising a partial disconnection in a cable by the foregoing test, the branches a and b of the bridge were made 100 ohms each, and balance was obtained on g when c was adjusted to 5000 ohms; consequently since a and b are equal

$$L = C = 5000.$$

The conductor resistance L_1 when the cable was perfect was 2000 ohms.

The resistance of the galvanometer was 5000 ohms, but when the discharge was noted it was necessary to employ the $\frac{1}{10}$th shunt, so that in the formula we must put

$$g = \frac{5000}{10} = 500.$$

The discharge deflection observed on depressing the key was 248 divisions, therefore

$$Q_3 = 248 \times 10 = 2480.$$

The discharge deflection Q_2 observed from a condenser of 1 microfarad capacity (F) when charged to the potential V was 202 divisions with no shunt, therefore

$$Q_2 = 202.$$

The cable having a conductor resistance of 10 ohms per mile, and an inductive capacity of ·3 microfarad per knot, the capacity in microfarads of such a length of the cable as would have a conductor resistance of 1 ohm, would be $\frac{\cdot 3}{10} = \cdot 03$ microfarad, that is

$$f = \cdot 03\,;$$

then

$$R = \frac{5000 + 2000}{2} - \sqrt{\frac{(5000 + 2000)^2}{4} - \frac{\frac{3 \times 2480 \times 1}{100}\left[500\,(100 + 100) + 100\,(100 + 5000)\right]5000 - 202 \times \cdot 03 \times 2000^3}{3 \times 202 \times \cdot 03\,(5000 - 2000)}}$$

$$= 3500 - 2996 = 504 \text{ ohms.}$$

474. In making this test practically, after c and Q_3 have been obtained the cable must be disconnected from the bridge, and a resistance equal to L be connected between B and F, the potential at the point B will then still be V, and further the galvanometer can be removed without altering this potential; the condenser and galvanometer must then be joined up in the manner shown by Fig. 77, page 240, the wires, however, which in that figure are shown as connected to the battery, being connected in the present case to the points B and F, Fig. 121; then the discharge obtained, multiplied by the shunt (if one is employed), gives Q_2.

475. It will sometimes be found that the cable is traversed by an earth current. The effects of this may best be neutralised in the manner indicated on page 224, Chapter IX., the compensating battery being connected between the cable and the point B, and adjustment effected with the lever $l\,l_1$ raised so as to cut the testing battery off; when the galvanometer g_1 is unaffected the adjustment is correct, the lever $l\,l_1$ is then let down, and the test made as if no earth current existed.

476. As it would be a matter of considerable difficulty, if not of impossibility, to adjust the bridge balance with an ordinary Thomson galvanometer (g_1) in consequence of the latter being greatly affected by slight changes in the earth current, a galvanometer with a heavy needle whose movements are very sluggish, and which is consequently unaffected by slight and

sudden changes of current, is necessary for the purpose. For measuring the discharge, however, a highly sensitive instrument (g) is necessary, which must be brought into use only at the exact moment required, since it is necessary that its needle be steady at zero at that time. By the arrangement of key shown in Fig. 121, this object is completely effected, as the galvanometer g is only brought into use at the moment when the battery is cut off, and the cable discharged.

CHAPTER XXIV.

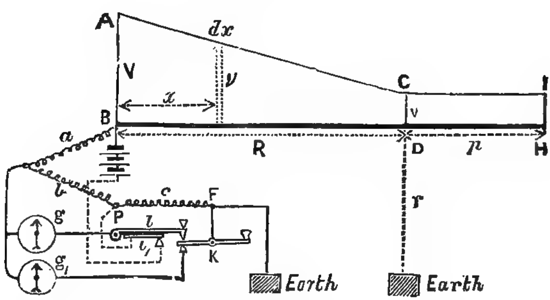
A METHOD OF LOCALISING EARTH FAULTS IN CABLES.

LOCALISATION OF FAULT WHEN CABLE IS NOT BROKEN.

477. This test is of the same nature as the forgoing, and possesses the advantage of having all the necessary observations taken simultaneously, and from one end of the cable only.

In Fig. 122, R and p represent the resistances of the portions of the conductor of the cable on either side of the fault, and r represents the resistance of the fault itself. As in the previous

FIG. 122.



test, a, b, and c are the three sides of a Wheatstone bridge, of which the cable forms the fourth side, and g and g_1 are two galvanometers. $k\ l\ l_1$ is a key, the construction and working of which were fully described in the previous test, and which it is unnecessary to consider again here.

Supposing the key to be in its normal position, then let V represent the potential at the beginning of the cable, v the potential at the fault and also at the further end H, and v the potential at any point between B and D.

If now the key k is depressed, the charge in the cable, which is represented by the area A B C D, will flow out at B and at D in proportions dependent upon the values of the resistances r, R, and the combined resistances a, b, g, and c.

Let $v\,dx$ be a differential part of the charge. Then the portion of this which will flow out at B will be

$$d\,q_1 = v\,\frac{R + r - x}{R_1 + R + r}\,dx\,,$$

where R_1 is the combined resistance of a, b, g, and c.

Now

$$V : v :: R + r : R + r - x\,;$$

therefore

$$v = V\,\frac{R + r - x}{R + r}\,;$$

therefore

$$d\,q_1 = V\,\frac{(R + r - x)^2}{(R_1 + R + r)\,(R + r)}\,dx\,;$$

and the integral of this between the limits $x = R$ and $x = 0$ will give the total quantity q_1, due to the charge A B D C, flowing out at B, that is

$$q_1 = \int_0^R V\,\frac{(R + r - x)^2}{(R_1 + R + r)\,(R + r)}\,dx$$

$$= \frac{V}{(R_1 + R + r)\,(R + r)}\int_0^R (R + r - x)^2\,dx$$

$$= \frac{V}{(R_1 + R + r)\,(R + r)}\left[\frac{(R + r)^3 - r^3}{3}\right]$$

$$= \frac{V}{3}\cdot\frac{(R + r)^3 - r^3}{(R_1 + R + r)\,(R + r)}.$$

Now besides the quantity q_1 there will be a quantity q_2 flowing out at B, due to the charge represented by the area C D H I. Let this charge be q', then

$$q' = \mathrm{v}\,p\,;$$

but

$$V : \mathrm{v} :: R + r : r\,,$$

therefore

$$\mathrm{v} = V\,\frac{r}{R + r}\,;$$

therefore

$$q' = V\frac{r\,p}{R+r}.$$

This quantity q' in flowing out at D will divide, the portion q_2 flowing along R and out at B, being

$$q_2 = q'\frac{r}{R_1+R+r} = V\frac{r^2\,p}{(R_1+R+r)\,(R+r)}.$$

Consequently the *total* quantity flowing out at B will be

$$q_1+q_2 = \frac{V}{3}\cdot\frac{(R+r)^3 - r^3}{(R_1+R+r)\,(R+r)} + V\frac{r^2\,p}{(R_1+R+r)\,(R+r)}$$

$$= \frac{V}{3}\cdot\frac{(R+r)^3 - r^3 + 3\,r^2\,p}{(R_1+R+r)\,(R+r)} = Q. \qquad [1]$$

Now if the cable had no fault in it, and its further end were insulated, and if it had been charged to the potential V, then the quantity Q_1, which the length B D would contain, would be represented by the equation

$$Q_1 = V\,R.$$

Again, if f be the capacity in microfarads of such a length of the cable as would have a conductor resistance of 1 unit, then $R\,f$ will be the actual total capacity of the length B D; and if Q_2 be the charge held by a condenser of F microfarads capacity also charged to the potential V, then

$$Q_1 : Q_2 :: R\,f : F;$$

therefore

$$Q_1 = \frac{Q_2\,R\,f}{F} = V\,R,$$

or

$$V = \frac{Q_2\,f}{F}.$$

Substituting the value of V in equation [1] we get

$$Q = \frac{Q_2\,f}{3\,F}\cdot\frac{(R+r)^3 - r^3 + 3\,r^2\,p}{(R_1+R+r)\,(R+r)}, \qquad [A]$$

or

$$(R + r)^3 - r^3 + 3\, r^2\, p = \frac{3\, Q\, F\, (R_1 + R + r)\, (R + r)}{Q_2 f}.$$

Let

$$R + r = L$$

and

$$R + p = L_1;$$

therefore

$$r = L - R$$

and

$$p - r = L_1 - L, \quad \text{or,} \quad p = L_1 - L + r;$$

therefore

$$\begin{aligned}(R + r)^3 - r^3 + 3\, r^2\, p &= L^3 - r^3 + 3\, r^2\, (L_1 - L) + 3\, r^3 \\ &= L^3 + 3\, r^2\, (L_1 - L) + 2\, r^3 \\ &= L^3 + 3\, (L - R)^2\, (L_1 - L) + 2\, (L - R)^3;\end{aligned}$$

therefore

$$\begin{aligned}(L - R)^3 + \frac{3\, (L_1 - L)}{2}\, (L - R)^2 &= \frac{3\, Q\, F\, (R_1 + L)\, L}{2\, Q_2 f} - \frac{L^3}{2} \\ &= \frac{C}{2}, \text{ say.} \qquad [1]\end{aligned}$$

Also if Q_3 equals the quantity discharged through the galvanometer, then by substituting this quantity and the combined values of a, b, c, and g, to which R_1 is equal, in the manner shown on page 399, in the last chapter, we shall have

$$C = \frac{3\, Q_3\, F\, [g\, (a + b) + a\, (b + c)]\, L}{b\, Q_2 f} - L^3.$$

If in making the test it is found necessary to employ a shunt with the galvanometer when taking the discharge, then if the value of this shunt be $\frac{1}{n}$th, we must multiply the observed deflection by n in order to obtain Q_3, and also the value of g in the above equation will be $\frac{1}{n}$th of the actual resistance of the galvanometer.

From the *cubic* equation [1] R has now to be determined; this can be done in the following manner:—

Dividing each side by $(L_1 - L)^3$, we get

$$\left(\frac{L - R}{L_1 - L}\right)^3 + \frac{3}{2}\left(\frac{L - R}{L_1 - L}\right)^2 - \frac{C}{2\,(L_1 - L)^3} = 0\,;$$

therefore

$$\left(\frac{L - R}{L_1 - L} + \frac{1}{2}\right)^3 - \frac{3}{4}\left(\frac{L - R}{L_1 - L}\right) - \frac{1}{8} - \frac{C}{2\,(L_1 - L)^3} = 0\,;$$

therefore

$$\left(\frac{L - R}{L_1 - L} + \frac{1}{2}\right)^3 - \frac{3}{4}\left(\frac{L - R}{L_1 - L} + \frac{1}{2}\right) + \frac{1}{4} - \frac{C}{2\,(L_1 - L)^3} = 0\,;$$

that is,

$$4\left(\frac{L - R}{L_1 - L} + \frac{1}{2}\right)^3 - 3\left(\frac{L - R}{L_1 - L} + \frac{1}{2}\right) + 1 - \frac{2\,C}{(L_1 - L)^3} = 0\,. \quad [2]$$

Now this equation is of the same form as the identity

$$4\cos^3\alpha - 3\cos\alpha - \cos 3\,\alpha = 0\,.$$

If then we put

$$\frac{2\,C}{(L_1 - L)^3} - 1 = \cos 3\,\alpha\,, \quad [3]$$

we shall have

$$\frac{L - R}{L_1 - L} + \frac{1}{2} = \cos\alpha\,,$$

or

$$L - R = (L_1 - L)\,(\cos\alpha - \tfrac{1}{2})\,;$$

that is,

$$R = L - (L_1 - L)\,(\cos\alpha - \tfrac{1}{2})\,. \quad [4]$$

So that, having worked out the numerical value of $\frac{2\,C}{(L_1 - L)^3} - 1$, and ascertained in a table of cosines to what angle this corresponds, then the cosine of $\frac{1}{3}$rd of this angle gives $\cos\alpha$, which value inserted in equation [4] enables the value of R to be obtained.

For example.

In localising a fault by the foregoing test, the two arms a and b of the bridge were made 100 ohms each, and balance was obtained on g when c was adjusted to 700 ohms; therefore $L = 700$ ohms.

The resistance of the galvanometer was 5000 ohms, but when the discharge was noted on it the $\frac{1}{10}$th shunt was inserted, so that $g = \frac{5000}{10} = 500$ ohms.

The discharge deflection observed on depressing the key was 350 divisions; therefore $Q_3 = 338 \times 10 = 3380$. The discharge deflection Q_2 obtained from a condenser of 1 microfarad capacity (F) charged to the potential V was 106 divisions with the $\frac{1}{10}$th shunt; therefore $Q_2 = 106 \times 10 = 1060$. The capacity f of such a length of the cable as would have a conductor resistance of 1 ohm was ·03 microfarad; and lastly, the total conductor resistance L_1 of the cable when sound was 1100 ohms. Thus we have

$$
\begin{aligned}
a &= 100\\
b &= 100\\
g &= 500\\
c &= 700\\
L &= 700\\
L_1 &= 1100\\
Q_2 &= 1060\\
Q_3 &= 3380\\
F &= 1\\
f &= \cdot 03
\end{aligned}
$$

we then get

$$C = \frac{3 \times 3380 \times 1\,[500\,(100 + 100) + 100\,(100 + 700)]\,700}{100 \times 1060 \times \cdot 03} - 700^3 = 401{,}770{,}000 - 343{,}000{,}000 = 58{,}770{,}000;$$

therefore

$$\frac{2\,C}{(L_1 - L)^3} - 1 = \frac{2 \times 58{,}770{,}000}{(1100 - 700)^3} - 1 = \cdot 8366 = \cos 3\,\alpha$$
$$= \cos \text{ of } 33^\circ\, 13';$$

therefore

$$\alpha = \frac{33^\circ\ 13'}{3} = 11^\circ\ 4',$$

the cosine of which is ·9814; therefore

$$R = 700 - (1100 - 700)\,(\cdot 9814 - \tfrac{1}{2}) = 507 \text{ ohms},$$

which gives the distance of the fault.

478. It may be remarked that the foregoing test is an excellent example of one of those rare cases in which the solution of an equation of the third degree is practically required, and in which the application of trigonometrical formulæ for the purpose is useful.*

479. Now the cosine of an angle can never exceed 1, and it will sometimes be found, on working out the value of $\frac{2\,C}{(L_1 - L)^3} - 1$, that its value will exceed unity; consequently in such a case R cannot be determined by the help of a cosine table, but some other method must be adopted. Let us determine this method.

In equation [2] (page 407) let

$$\frac{L - R}{L_1 - L} + \frac{1}{2} = y + \frac{1}{4\,y};$$

we then have

$$4\,y^3 + 3\,y + \frac{3}{4\,y} + \frac{1}{16\,y^3} - 3\,y - \frac{3}{4\,y} + 1 - \frac{2\,C}{(L_1 - L)^3} = 0\,,$$

or

$$y^3 + \frac{1}{64\,y^3} + \frac{1}{4}\left(1 - \frac{2\,C}{(L_1 - L)^3}\right) = 0\,.$$

Let

$$\frac{2\,C}{(L_1 - L)^3} - 1 = K;$$

therefore

$$y^6 - \frac{K}{4}\,y^3 + \frac{1}{64} = 0\,,$$

a *quadratic* equation, from which y^3 can be determined in the ordinary manner. Thus

$$y^6 - \frac{K}{4}\,y^3 + \left(\frac{K}{8}\right)^2 = \frac{K^2}{64} - \frac{1}{64};$$

* See Todhunter's Trigonometry, Third Edition, Chapter XVII., page 202.

therefore

$$y^3 - \frac{K}{8} = \pm \tfrac{1}{8}\sqrt{K^2 - 1},$$

or

$$y = \tfrac{1}{2}[K \pm \sqrt{K^2 - 1}]^{\frac{1}{3}},$$

and

$$y + \frac{1}{4\,y} = \tfrac{1}{2}\{[K + \sqrt{K^2 - 1}]^{\frac{1}{3}} + [K + \sqrt{K^2 - 1}]^{-\frac{1}{3}}\}$$

$$= \tfrac{1}{2}\{[K + \sqrt{K^2 - 1}]^{\frac{1}{3}} + K - \sqrt{K^2 - 1}]^{\frac{1}{3}}\};$$

so that we get

$$R = L - (L_1 - L)\tfrac{1}{2}\{[K + \sqrt{K^2 - 1}]^{\frac{1}{3}} + [K - \sqrt{K^2 - 1}]^{\frac{1}{3}} - 1\},$$

in which

$$K = \frac{2\,C}{(L_1 - L)^3} - 1$$

and

$$C = \frac{3\,Q_3\,F\,[g\,(a + b) + a\,(b + c)]\,L}{b\,Q_2\,f} - L^3.$$

For example.

In making the test, suppose the following to have been the numerical values of the different quantities:—

$$\begin{aligned} a &= 100 \\ b &= 100 \\ g &= 500 \\ c &= 900 \\ L &= 900 \\ L_1 &= 1100 \\ Q_2 &= 300 \\ Q_3 &= 1230 \\ F &= 1 \\ f &= \cdot 03 \end{aligned}$$

therefore

$$C = \frac{3 \times 1230 \times 1\,[500\,(100 + 100) + 100\,(100 + 900)]\,900}{100 \times 300 \times \cdot 03} - 900^3 = 538{,}000{,}000 - 729{,}000{,}000 = 9{,}000{,}000;$$

therefore

$$K = \frac{2 \times 9{,}000{,}000}{(1100-900)^3} - 1 = 2 \cdot 25 - 1 = 1 \cdot 25\,;$$

therefore

$$\sqrt{K^2 - 1} = \sqrt{1 \cdot 25^2 - 1} = \cdot 75\,;$$

from this we get

$$R = 900 - (1100 - 900)\,\tfrac{1}{2}\,\{2^{\frac{1}{3}} + \cdot 5^{\frac{1}{3}} - 1\}$$

$$= 900 - \frac{200}{2}\,\{1 \cdot 2599 + \cdot 7937 - 1\} = 795 \text{ ohms.}$$

Localisation of Fault when Cable is broken.

480. In this case, referring to page 405, the quantity discharged at B when the key is depressed will be only q_1 instead of $q_1 + q_2$; consequently equation [A], on the same page, will become

$$Q = \frac{Q_2 f}{3\,F} \cdot \frac{R^3 + 3\,R^2 r + 3\,R\,r^2}{(R_1 + R + r)(R + r)}\,,$$

or

$$(R + r)^3 - r^3 = \frac{3\,Q\,F\,(R_1 + R + r)(R + r)}{Q_2 f}\,;$$

and putting

$$R + r = L, \text{ and } r = L - R\,,$$

we get

$$L^3 - (L - R)^3 = \frac{3\,Q\,F\,(R_1 + L)\,L}{Q_2 f}\,;$$

therefore

$$(L - R)^3 = L^3 - \frac{3\,Q\,F\,(R_1 + L)\,L}{Q_2 f}\,;$$

therefore

$$L - R = \sqrt[3]{L^3 - \frac{3\,Q\,F\,(R_1 + L)\,L}{Q_2 f}}\,,$$

or

$$R = L - \sqrt[3]{L^3 - \frac{3\,Q\,F\,(R_1 + L)\,L}{Q_2 f}}\,;$$

and by substituting a, b, c, g, and Q_3, in the manner shown on page 399, we get

$$R = L - \sqrt[3]{L^3 - \frac{3\,Q_3\,F\,[g\,(a + b) + a\,(b + c)]\,L}{b\,Q_2 f}}\,.$$

For example.

In localising a fracture in a submarine cable by the foregoing test, a and b were made 100 ohms each, and balance was obtained on g when c was adjusted to 700 ohms.

The resistance of the galvanometer was 5000 ohms, but when the discharge was noted, the $\frac{1}{10}$th shunt was inserted, therefore

$$g = \frac{5000}{10} = 500 \text{ ohms.}$$

The discharge deflection observed on depressing the key was 186 divisions, therefore $Q_3 = 186 \times 10 = 1860$. The discharge deflection Q_2 obtained from a condenser of 1 microfarad capacity (F) charged to the potential V was 120 divisions with the $\frac{1}{10}$th shunt, therefore $Q_2 = 120 \times 10 = 1200$. The capacity f of such a length of the cable as would have a conductor resistance of 1 ohm was $\cdot 03$ microfarad, then

$$R = 700 - \sqrt[3]{700^3 - \frac{3 \times 1860 \times 1}{100 \times 1200 \times \cdot 03}[500(100+100)+100(100+700)]700}$$

$$= 700 - 529 = 171 \text{ ohms.}$$

481. A great merit in the foregoing methods of testing for faults lies in the fact that the two cable measurements can be made almost simultaneously; thus the moment balance is obtained on g_1 by adjusting c, at that moment the key is depressed, and the discharge deflection Q_3 noted on the galvanometer g. The other measurement, viz. that from the condenser, can be made at leisure. Thus after c and Q_3 are obtained, the cable must be disconnected from the bridge, and a resistance equal to c be connected between B and F, the potential at the point B will then still be V, and further, the galvanometer g can be removed without altering this potential; the condenser and galvanometer must then be joined up in the manner shown by Fig. 77, page 240. The wires, however, which in the latter figure are shown as connected to the battery, must in the present case be connected to the points B and F, Fig. 122; then the discharge obtained, multiplied by the shunt (if one is employed), gives Q_2.

482. Should earth currents be present when the test is about to be made, they may be neutralised in the manner explained on page 224, in Chapter IX., and also at the end of the last chapter (§ 475, page 401).

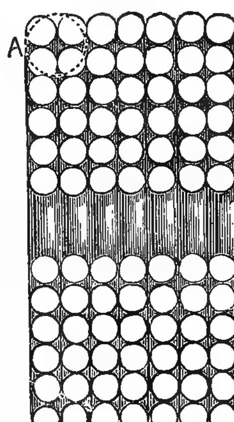
CHAPTER XXV.

GALVANOMETER RESISTANCE.

483. The question of what resistance a galvanometer should have in order that its figure of merit may be high, involves several points, such as "the shape of the coil," "the diameter of the wire," &c. The determination of all these points, however, would be more useful for the purpose of finding what are the most economical conditions under which a galvanometer can be made, than (what is more to the purpose of the practical electrician) for showing how any particular galvanometer can be arranged so as to enable any particular test to be made with accuracy.

The problem we have to solve in the latter case is as follows:—Having given a galvanometer with a coil of a certain size, should thin or thick wire be on it in order that any particular test may be made under the most favourable conditions? Or supposing the coil to be divided into several sections, how should the latter be coupled up?

Fig. 123.



Referring to Fig. 123, which represents a section of a galvanometer coil, let us direct our attention to the 4 turns of wire at A. If these 4 turns be joined up in one continuous length, then calling the resistance of each turn 4, their total resistance will be 4×4, or 4^2. If, however, the 4 turns be coupled up for "quantity," then their joint resistance will be 1.

If we suppose the *total* current flowing to be constant, then in the case where the 4 wires are joined up in one continuous length, the current makes 4 turns round the needle of the galvanometer, its effect will therefore be equal to 4; but in the second case, where the turns of wire are coupled up for "quantity," the same current only makes 1 turn round the needle, hence its effect can only be equal to 1.

If instead of 4 turns we have 9 turns, then the relative values of the resistances when joined up in one continuous length, and when joined up for "quantity," will be as 1 to 9×9, or 9^2, whilst the relative effect of the current on the galvanometer needle will be as 1 to 9.

In the first case then, where the resistance was reduced 4^2 times, the effect on the needle was only reduced 4 times; and in the second case, where the resistance was reduced 9^2 times, the effect was only reduced 9 times; or, in other words, the effect varied directly as the *square root* of the resistance; consequently for the whole of the galvanometer the effect varies directly as the square root of its resistance.

If we replace the 4 wires at A by a solid wire of twice their diameter, then this wire, shown by the dotted lines, will have the same resistance as these 4 wires coupled up for "quantity," and its influence on the magnetic needle will be very nearly the same. As a matter of fact, the effect will be rather less, in consequence of the metal being differently distributed over the area which the 4 wires occupy. But inasmuch as the silk covering with which the wires are insulated is practically of the same thickness for large as for small wires, if the thick wire were wound on the coil the sectional area of that wire would actually be rather larger than the area of the small wires which it takes the place of, consequently we may without any considerable error say that the effect varies directly as $\sqrt{g}$.

484. This fact enables us to determine what should be the resistance of the galvanometer in order that any particular test may be made under the best possible conditions. Let us take the case of the Wheatstone bridge.

On page 173 we obtained an equation which gave the strength of the current flowing through the galvanometer when equilibrium was very nearly produced, viz.:

$$c_6 = \frac{E\,x\,(a\,d_1 - b\,x)}{\{g\,(a+x) + a\,(d+x)\}\;\{r\,(d+x) + d\,(a+x)\}}.$$

This equation may be written

$$c_6 = \frac{1}{\left\{g + \dfrac{a\,(d+x)}{a+x}\right\}} \times \frac{E\,x\,(a\,d_1 - b\,x)}{(a+x)\,\{r\,(d+x) + d\,(a+x)\}}$$

$$= \frac{1}{(g+k)} \times K.$$

We have shown that the effect of the galvanometer coil on the needle varies directly as the square root of the resistance of

the former. Its effect must also vary directly as the current passing through the coils, consequently the total effect M will be

$$M = \frac{\sqrt{g}}{(g+k)} \times K\kappa = \frac{K\kappa}{\sqrt{g} + \dfrac{k}{\sqrt{g}}},$$

where κ is a constant dependent upon the shape of the coil, the magnetic strength of the needle, &c.

We have to find what value of g will make M as *large* as possible, and this we shall do, since $K\kappa$ is constant, by finding what value of g will make $\sqrt{g} + \dfrac{k}{\sqrt{g}}$ as *small* as possible.

Now

$$\sqrt{g} + \frac{k}{\sqrt{g}} = \left(\sqrt[4]{g} - \frac{\sqrt{k}}{\sqrt[4]{g}}\right)^2 + 2\sqrt{k},$$

and this will be made a minimum by making $\sqrt[4]{g} - \dfrac{\sqrt{k}}{\sqrt[4]{g}}$ a minimum, that is, by making

$$\sqrt[4]{g} - \frac{\sqrt{k}}{\sqrt[4]{g}} = 0,$$

therefore

$$\sqrt{g} = \sqrt{k}, \quad \text{or,} \quad g = k;$$

but

$$k = \frac{a(d+x)}{a+x}$$

and $\dfrac{a(d+x)}{a+x}$ is the same as $\dfrac{(a+b)(d+x)}{a+b+d+x}$, when $bx = ad$, and this expression is the joint resistance of the resistances on either side of the galvanometer; theoretically therefore we should make g equal to this quantity if we wish M to be as large as possible.

This rule, however, although it shows what value g should have in order to make M an absolute maximum, is one which cannot well be strictly followed out. We should rather seek to determine to what extent the exact rule may be violated without seriously diminishing M.

Let us suppose g to be n times k, then we have

$$M = \frac{\sqrt{n\,k}}{n\,k + k} \times K\,\kappa = \frac{\sqrt{n}}{n + 1} \times \frac{K\,\kappa}{\sqrt{k}};$$

for an absolute maximum $n = 1$, that is

$$M = \frac{1}{2} \times \frac{K\,\kappa}{\sqrt{k}}.$$

Suppose, now, we make g nine times as large as k, that is, make $n = 9$, then we have

$$M = \frac{\sqrt{9}}{9 + 1} \times \frac{K\,\kappa}{\sqrt{k}} = \frac{1}{3{\cdot}3} \times \frac{K\,\kappa}{\sqrt{k}}.$$

In other words, although g is *nine* times as great as it should be for making M a maximum, yet M has only been reduced from $\frac{1}{2}$ down to $\frac{1}{3{\cdot}3}$. Or, to put it in another way: supposing we were making a bridge test, employing a galvanometer of the exact theoretical value for obtaining a maximum deflection, and supposing that having nearly obtained equilibrium, the deflection of the galvanometer needle was 3·3 divisions, then, if the resistance of the galvanometer had been 9 times the theoretical value, the deflection would only have been reduced down to 2 divisions.

It must therefore be evident that, unless we employ a galvanometer whose resistance *very* much exceeds the theoretical value, this resistance will practically be the one required. If it is necessary to draw a limit, we may say—avoid making the resistance more than 10 times as great (or as small, as can also be shown) as the theoretical value.

485. It will be found that in all tests in which g has a particular best value, an equation of the form

$$M = \frac{\sqrt{g}}{g + k} \times K\,\kappa$$

can be obtained. k in fact is in reality the resistance external to the galvanometer, so that we have simply to find what this resistance is, and then make g as nearly as possible equal to it.

CHAPTER XXVI.

SPECIFICATION FOR MANUFACTURE OF CABLE.—SYSTEM OF TESTING CABLE DURING MANUFACTURE.

486. As soon as the laying of a new cable has been decided upon, and the route which it is to take has been selected, &c., the manufacture has to be commenced. The choice of the types of cable to be adopted, the lengths of the "shore ends," "intermediate," and "deep-sea" sections are purely matters of experience and discretion with the engineers in charge of the work, and no satisfactory rules for general guidance can be laid down.

When the description of cable has been settled upon, a specification has to be drawn up, of which the following is a general specimen.

487. The ______________________ Telegraph Company and

______________________ Telegraph Works.

Contract Specification for the manufacture of the Submarine Telegraph Cable of the ______________________ Telegraph Company, to be laid between the coast of ________, near ________, and the Island of________.

The following lengths of cable will be required:—

Actual distance, 480 knots (each being 2029 yards), or, including 10 per cent. slack,* 528 knots.

A.	Main cable	500 knots.
B.	Intermediate cable	11 "
C.	Shore-end cable	17 "

Core.

The core of the entire length of cable to be as follows:—

Conductor.—To be formed of a strand of seven copper wires of a conductivity of not less than 96 per cent. of pure copper

* The amount of slack required will vary with the length of the cable and with the depth of water it is laid in.

according to Matthiessen's standard,* and weighing one hundred and seven (107) pounds per nautical mile (2029 yards).

Insulator.—Copper conductor to be covered with three coatings of the purest gutta-percha, a coating of Chatterton's compound being placed next the conductor and between each layer of percha. The insulator to weigh one hundred and fifty (150) pounds per nautical mile, making the weight of the conductor, when covered with the insulator, two hundred and fifty-seven (257) pounds per nautical mile.

The insulation resistance of each coil to be not less than 250 megohms per nautical mile after having been kept in water, maintained at a temperature of 75° Fahrenheit, for not less than twenty-four consecutive hours, and after one minute's electrification.

Each coil of insulated wire, before being placed in the temperature tank for testing, to be carefully labelled with the exact length of wire, the exact weight of copper, and the exact weight of insulator it contains.

A margin of 4 pounds over or under the specified total weight (257 lbs.) will be allowed, but the mean weight of the core for the whole cable must not be under the specified weight.

The core during manufacture to be carefully protected from sun and heat, and kept under water.

Joints.—Every joint to be tested by accumulation, and the leakage from any joint during one minute not to be more than double that from an equal length of the perfect core. Notice to be given to the inspecting officer of the company when a joint is about to be made, so that he may test it.

Serving and Sheathing.

Main Cable A.

Serving.—The insulated conductor to be served with the best wet-tanned Russian hemp to receive the sheathing as specified, and to be then kept in tanned water and not allowed to be out of water more than is necessary to feed the closing machine.

Sheathing.—The served core to be sheathed with fifteen galvanised iron wires, each ·120 of an inch in diameter.

The lay to be 10 inches, no loose threads of hemp to be run through the closing machine, and no weld in any one iron wire to be within six feet of a weld in any other wire. The sheathed core to be finally covered with three coatings of Bright and

* See page 368, § 444.

Clark's compound, a serving of tarred yarn made from the best Russian hemp being placed between each layer of compound, each serving of yarn being laid on in contrary directions.*

Intermediate Cable B.

Serving to be similar in every respect to that on the Main Cable A.

Sheathing to be generally similar to that specified for the Main Cable A, but the iron covering to consist of ten galvanised iron wires, each ·180 of an inch in diameter. The lay to be 10 inches.

Shore-End Cable C.

The shore-end cable to consist of Cable A complete, and further well served with the best wet-tanned Russian hemp, and then sheathed with twelve galvanised iron wires, ·300 of an inch in diameter.

The lay to be 17 inches, no loose threads of hemp to be run through the closing machine, and no weld in any one iron wire to be within six feet of a weld in any other wire. The sheathed core to be finally covered with three coatings of Bright and Clark's compound, a serving of tarred yarn made from the best Russian hemp being placed between each layer of compound, each serving of yarn being laid on in contrary directions.

The completed cable as fast as it is made, to be passed into a tank of water and kept covered with water until shipped. A correct indicator to be attached to the closing machine, and the length of cable to be marked as agreed.

Quality of Materials.

The wire used in the Main Cable A to be of the best quality of homogeneous wire, galvanised, and having a tensile strength of 50 tons per square inch area, and 850 lbs. as a minimum breaking strain on a length of 12 inches between the clamps. The wire must elongate not less than $\frac{3}{4}$ per cent. before breaking. It shall bend round itself and unbend without breaking. The joints in the homogeneous wires to be of the form decided upon by the company's and contractor's engineers, and, as far as practicable, no one joint to be within six feet of any other joint.

The iron wire to be used in Cables B and C is to be of the

* In the place of the tarred yarn and Bright and Clark's compound, two layers of tarred *tape*, each layer being wound on in contrary directions, are now frequently employed; this gives an excellent finish to the cable.

quality known as Best Best, free from inequalities, galvanised and annealed, and having a tensile strength of 25 tons per square inch of area. A margin of 5 per cent. will be allowed in weight, provided the average weight is as specified above. The wire for Cables B and C to be capable of being bent round a cylinder four times its own diameter and unbent without breaking. No wire of brittle quality shall be put into the cables, and the engineers or their assistants shall have power to reject any hanks which break frequently in the closing machine, or are of unsatisfactory quality. No weld shall be made in the B and C cables within six feet of any other weld.

The galvanising of the iron to bear four dips of one minute each in a solution of one part by weight of sulphate of copper and five parts of water.

Each intermediate cable to be finished off with suitable tapers to be arranged to the satisfaction of the engineer of the company.

Testing Accommodation.

A proper room and all necessary batteries and leading wires to be provided for testing the cable during the whole manufacture.

Inspection.

The engineer of the company or his agents to have access to the works for inspecting and testing cable and all materials employed, and may reject all materials which are unsatisfactory.

Penalty.

The whole of the cable to be completed on or before the time stated in the tender under a penalty of____________per cent. on the price for each day, or fraction of a day, after the said time, until the day the cable may be actually completed and ready for shipment.

The manufacture may not be carried on at night without the written consent of the engineer of the company or his agent.

The cable ship or ships are not to leave the wharf with cable on board until the cable has been thoroughly tested in all respects by the engineers or their assistants from the shore, and ample time after the shipment of the last mile to be allowed for this purpose.

SYSTEM OF TESTING CABLE DURING MANUFACTURE.

488. The tests made by the cable manufacturers, although systematic, are not as a rule quite so exact or lengthy as those made by the electrician representing the company for whom the cable is being made. The cable, once manufactured, passes out of the hands of the manufacturer, and the latter has no further interest in the matter; whereas the company may require at any time to localise a fault, and the more precise the data they possess the more closely will they be able to determine the position of the defect. Besides, when a large number of cables are being made at once at the factory it would be impossible, without a very large staff, to make an elaborate series of tests for each cable; whereas these can easily be made by the electrician and his assistants when there is only one cable to look after.

The methods of working out the tests, and the forms employed for entering down the same, depend upon the individual opinion of the electrician in charge of the work, but the following will give a general idea of the course to be pursued:—

Tests of the Coils.

489. The core of the cable is usually made in 2-knot lengths approximately, which are coiled upon wooden drums as manufactured, and then placed in tanks of water heated to a temperature of 75° F. to be tested.

After being placed in the tank, the coils should remain there for at least twenty-four hours, so that they may acquire throughout their mass the necessary uniform temperature. At the end of this time the tests may be taken.

Sheets A, B, C, and D are employed for entering all the details of the tests as they are made; the more important of these details are then copied on to sheet E. The working out of the tests of the coils and cable is shown on corresponding pages.

The figures given are such as are often obtained in actual practice. The insulation resistances of the coils are very often considerably higher than those shown, but this entirely depends upon the time which elapses after the manufacture.

(A)

THE ______________________ TELEGRAPH COMPANY.

DETAILS OF COILS FORMING THE CORE OF ______________ CABLE.

Date.	No. of Coil.	Length of Coils.		Total Weight.			Weight per Knot.			Difference from Contract Weight in lbs. per Knot.				Remarks.
										Copper.		Gutta-percha.		
		In yards.	In knots.	Copper.	Gutta-percha.	Total.	Copper.	Gutta-percha.	Total.	+	−	+	−	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	
				lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	
1884. Apr. 6	1	4047	1·9946	213	298	511	106·79	149·40	256·19	..	·21	..	·60	
,,	2	4073	2·0074	214	302	516	106·60	150·44	257·04	..	·40	·44	..	
,,	3	4072	2·0069	215	304	519	107·13	151·48	258·61	·13	..	1·48	..	
,,	4	4056	1·9990	214	299	513	107·05	149·57	256·62	·05	..	..	·43	
,,	5	4056	1·9990	212	296	508	106·05	148·07	254·12	..	·95	..	1·93	

Signature ______________________

Calculations for Sheet (A).

April 6th.

Copper.

No. 1 *Coil.*

log 213 = 2·3283796
log 1·9946 = ·2998558

2·0285238
= log of 106·79

No. 2 *Coil.*

log 214 = 2·3304138
log 2·0074 = ·3026339

2·0277799
= log of 106·60

No. 3 *Coil.*

log 215 = 2·3324385
log 2·0069 = ·3025257

2·0299128
= log of 107·13

No. 4 *Coil.*

log 214 = 2·3304138
log 1·9990 = ·3008128

2·0296010
= log of 107·05

No. 5 *Coil.*

log 212 = 2·3263359
log 1·9990 = ·3008128

2·0255231
= log of 106·05

Gutta-percha.

No. 1 *Coil.*

log 298 = 2·4742163
log 1·9946 = ·2998558

2·1743605
= log of 149·40

No. 2 *Coil.*

log 302 = 2·4800069
log 2·0074 = ·3026339

2·1773730
= log of 150·44

No. 3 *Coil.*

log 304 = 2·4828736
log 2·0069 = ·3025257

2·1803479
= log of 151·48

No. 4 *Coil.*

log 299 = 2·4756712
log 1·9990 = ·3008128

2·1748584
= log of 149·67

No. 5 *Coil.*

log 296 = 2·4712917
log 1·9990 = ·3008128

2·0174789
= log of 148·07

(B)

THE ______________________ TELEGRAPH COMPANY.

CONDUCTOR RESISTANCE TESTS OF COILS AT 75° FAHR.

Date. 1	No. of Coil. 2	Length of Coils. 4	Resistance of Leads. 15	Total Resistance of Conductor and Leads. 16	Total Resistance of Conductor. 17	Resistance per Knot of Conductor. 18	Percentage of Conductivity compared with Pure Copper. 19	Remarks.
1884.		knots.	ohms.	ohms.	ohms.	ohms.		
April 6	1	1·9946	1·46	24·44	22·98	11·52	97·3	
,,	2	2·0074	,,	24·51	23·05	11·48	97·7	
,,	3	2·0069	,,	24·47	23·01	11·47	97·4	
,,	4	1·9990	,,	24·42	22·96	11·49	97·3	
,,	5	1·9990	,,	24·68	23·22	11·62	97·2	

Signature ______________________

Calculations for Sheet (B).

April 6th.

Conductor Resistance.

No. 1 Coil.

log 22·98 = 1·3613500
log 1·9946 = ·2998558
1·0614942 = log of 11·52
log 106·79 = 2·0285238
3·0900180 = log of 1230·3
= 97·3 per cent. pure copper *

No. 2 Coil.

log 23·05 = 1·3626709
log 2·0074 = ·3026339
1·0600370 = log of 11·48
log 106·60 = 2·0277799
3·0878169 = log of 1224·1
= 97·7 per cent. pure copper *

No. 3 Coil.

log 23·01 = 1·3619166
log 2·0069 = ·3025257
1·0593909 = log of 11·47
log 107·13 = 2·0299128
3·0893037 = log of 1228·3
= 97·4 per cent. pure copper *

No. 4 Coil.

log 22·96 = 1·3609719
log 1·9990 = ·3008128
1·0601591 = log of 11·49
log 107·05 = 2·0296010
3·0897601 = log of 1229·6
= 97·3 per cent. pure copper *

No. 5 Coil.

log 23·22 = 1·3658622
log 1·9990 = ·3008128
1·0650494 = log of 11·62
log 106·05 = 2·0255231
3·0905725 = log of 1231·9
= 97·2 per cent. pure copper *

---

* Table III. See also page 368, § 443.

(C)

THE ______________________ TELEGRAPH COMPANY.

INDUCTIVE CAPACITY TESTS OF COILS AT 75° FAHR.

				Condenser.		Coils.										
				Capacity ⅓ Microfarad.			Immediate Discharge after 10 seconds Electrification.			Discharge after 10 seconds Electrification and 60 seconds Insulation.					
No. of Coils.	Length of Coils.	No. of Cells.	Resistance of Galvanometer.	Shunt.	Discharge	Shunt.	Lead.	Coil and Lead.	Coil.	Lead.	Coil and Lead.	Coil.	Percentage of Loss.	Capacity per Knot.	Remarks.
2	4	20	21	22	23	24	25	26	27	28	29	30	31	32	
	knots.		ohms.		divisions.	ohms.	divisions.	divisions.	divisions.	divisions.	divisions.	divisions.		m.farads.	
1	1·9946	10 Dnls.	5460	$\frac{1}{10}$th	172	330	2·5	170	167·5	2·25	78·5	76·25	54·48	·2855	
2	2·0074	,,	,,	,,	,,	,,	,,	172	169·5	,,	79·5	77·25	54·42	·2871	
3	2·0069	,,	,,	,,	,,	,,	,,	172	169·5	,,	78	75·75	55·31	·2872	
4	1·9990	,,	,,	,,	,,	,,	,,	174	171·5	,,	81	78·75	54·08	·2917	
5	1·9990	,,	,,	,,	,,	,	,,	171	168·5	,,	80	77·75	53·86	·2866	

Signature ______________________

Calculations for Sheet (C).

April 6th.

Inductive Capacity.

$$
\begin{array}{rl}
\log\ 3 = & \cdot4771213 \\
\log 1720 = & 3\cdot2355284 \\
\log\ 330 = & 2\cdot5185139 \\
\hline
& 6\cdot2311636 \\
\log 5790 = & 3\cdot7626786 \\
\hline
& 2\cdot4684850
\end{array}
\qquad
\frac{G+S}{S} = \frac{5460+330}{330} = \frac{5790}{330}
$$

No. 1 Coil.

$$
\begin{array}{rl}
\log 167\cdot5 = & 2\cdot2240148 \\
& 2\cdot4684850 \\
\hline
& \bar{1}\cdot7555298 \\
\log\ 1\cdot9946 = & \cdot2998558 \\
\hline
& \bar{1}\cdot4556740 = \log \text{ of } \underline{\cdot2855}
\end{array}
$$

No. 2 Coil.

$$
\begin{array}{rl}
\log 169\cdot5 = & 2\cdot2291697 \\
& 2\cdot4684850 \\
\hline
& \bar{1}\cdot7606847 \\
\log\ 2\cdot0074 = & \cdot3026339 \\
\hline
& \bar{1}\cdot4580508 = \log \text{ of } \underline{\cdot2871}
\end{array}
$$

No. 3 Coil.

$$
\begin{array}{rl}
\log 169\cdot5 = & 2\cdot2291697 \\
& 2\cdot4684850 \\
\hline
& \bar{1}\cdot7606847 \\
\log\ 2\cdot0069 = & \cdot3025257 \\
\hline
& \cdot4581590 = \log \text{ of } \underline{\cdot2872}
\end{array}
$$

No. 4 Coil.

$$
\begin{array}{rl}
\log 171\cdot5 = & 2\cdot2342641 \\
& 2\cdot4684850 \\
\hline
& \bar{1}\cdot7657791 \\
\log\ 1\cdot9990 = & \cdot3008128 \\
\hline
& \bar{1}\cdot4649663 = \log \text{ of } \underline{\cdot2917}
\end{array}
$$

CALCULATIONS FOR SHEET (C)—*continued.*

No. 5 Coil.

$$
\begin{array}{lr}
\log 168\cdot 5 = & 2\cdot 2265999 \\
 & 2\cdot 4684850 \\ \hline
 & \bar{1}\cdot 7581149 \\
\log \quad 1\cdot 9990 = & \cdot 3008128 \\ \hline
 & \bar{1}\cdot 4573021 = \log \text{ of } \underline{\cdot 2866}
\end{array}
$$

Percentage of Loss.

No. 1 Coil.

$$
\begin{array}{lr}
 & 167\cdot 5 \\
 & 76\cdot 25 \\ \hline
\log \ 91\cdot 25 = & 1\cdot 9602329 \\
\log 100 = & 2\cdot \\ \hline
 & 3\cdot 9602329 \\
\log 167\cdot 5 = & 2\cdot 2240148 \\ \hline
 & 1\cdot 7362181 \\
 & = \log \text{ of } \underline{54\cdot 48}
\end{array}
$$

No. 2 Coil.

$$
\begin{array}{lr}
 & 169\cdot 5 \\
 & 77\cdot 25 \\ \hline
\log \ 92\cdot 25 = & 1\cdot 9649664 \\
\log 100 = & 2\cdot \\ \hline
 & 3\cdot 9649664 \\
\log 169\cdot 5 = & 2\cdot 2291697 \\ \hline
 & 1\cdot 7357967 \\
 & = \log \text{ of } \underline{54\cdot 42}
\end{array}
$$

No. 3 Coil.

$$
\begin{array}{lr}
 & 169\cdot 5 \\
 & 75\cdot 75 \\ \hline
\log \ 93\cdot 75 = & 1\cdot 9719713 \\
\log 100 = & 2\cdot \\ \hline
 & 3\cdot 9719713 \\
\log 169\cdot 5 = & 2\cdot 2291697 \\ \hline
 & 1\cdot 7428016 \\
 & = \log \text{ of } \underline{55\cdot 31}
\end{array}
$$

No. 4 Coil.

$$
\begin{array}{lr}
 & 171\cdot 5 \\
 & 78\cdot 75 \\ \hline
\log \ 92\cdot 75 = & 1\cdot 9673139 \\
\log 100 = & 2\cdot \\ \hline
 & 3\cdot 9673139 \\
\log 171\cdot 5 = & 2\cdot 2342641 \\ \hline
 & 1\cdot 7330498 \\
 & = \log \text{ of } \underline{54\cdot 08}
\end{array}
$$

No. 5 Coil.

$$
\begin{array}{lr}
 & 168\cdot 5 \\
 & 77\cdot 75 \\ \hline
\log \ 90\cdot 75 = & 1\cdot 9578466 \\
\log 100 = & 2\cdot \\ \hline
 & 3\cdot 9578466 \\
\log 168\cdot 5 = & 2\cdot 2265999 \\ \hline
 & 1\cdot 7312467 \\
 & = \log \text{ of } \underline{53\cdot 86}
\end{array}
$$

(D)

THE ______________ TELEGRAPH COMPANY.

INSULATION TESTS OF COILS AT 75° FAHR.

Date.	No. of Coil.	Length of Coils.	Number of Cells.	Resistance of Galvanometer.	CONSTANT. Battery Ratio. Discharge from Condenser. 1 Cell with [illegible] Shunt.	CONSTANT. Battery Ratio. Discharge from Condenser. Full Battery with $\frac{1}{100}$ Shunt.	CONSTANT. Battery Ratio. Value of Battery.	CONSTANT. Deflection from 1 Cell through 10,000 ohms, with $\frac{1}{1000}$ Shunt.	CONSTANT. Logarithm of Constant.	Current from Coils before Test. + − No Shunt.	COILS. Shunt.	COILS. Deflection after 1 minute Electrification. Lead.	COILS. Deflection after 1 minute Electrification. Coil and Lead.	COILS. Deflection after 1 minute Electrification. Coil.	COILS. Deflection after 2 minutes Electrification. Lead.	COILS. Deflection after 2 minutes Electrification. Coil and Lead.	COILS. Deflection after 2 minutes Electrification. Coil.	COILS. Percentage of Electrification during second minute.	COILS. Total Resistance after 1 minute Electrification.	COILS. Resistance per Knot.	Remarks.
1	2	4	33	21	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	
		knots.		ohms.	divns.	divns.		divns.		divns.	ohms.	divns.	divns.	divns.	divns.	divns.	divns.		megs.	megs.	
1884. Apr. 3	1	1·9946	100	5460	173	170	98·27	152	11·1117541		780	3·5	151·5	148	3·0	140	137	7·43	126·41	252·1	
,,	2	2·0074	,,	,,	,,	,,	,,	,,	,,		,,	,,	145·5	142	,,	133	130	8·45	131·75	264·5	
,,	3	2·0069	,,	,,	,,	,,	,,	,,	,,		,,	,,	148	144·5	,,	137	134	7·27	129·47	259·8	
,,	4	1·9990	,,	,,	,,	,,	,,	,,	,,		,,	,,	144	140·5	,,	135	132	6·05	133·15	266·2	
,,	5	1·9990	,,	,,	,,	,,	,,	,,	,,		,,	,,	141·5	138	,,	130·5	127·5	7·61	135·56	271·0	

Signature ______________

CALCULATIONS FOR SHEET (D).

April 6th.

Insulation Resistance.

$$
\begin{array}{rl}
\log 17{,}000 = & 4\cdot2304489 \\
\log 173 = & 2\cdot2380461 \\
\hline
 & 1\cdot9924028 = \log \text{ of } 98\cdot27 = \text{value of battery} \\
\log 10{,}020 \times 1000 = & 7\cdot0008677 \\
\log\ 152 = & 2\cdot1818436 \\
\hline
 & 11\cdot1751141 = \log \text{ constant} \\
\log\ 780 = & 2\cdot8920946 \\
\hline
 & 14\cdot0672087 \\
\log 6240 = & 3\cdot7951846 \\
\hline
 & 10\cdot2720241
\end{array}
$$

$$
\begin{array}{lr}
\text{Res. of galv. and shunt} = & 5\cdot46 \\
\text{,, 1 cell} = & 15 \\
\hline
 & \underline{20\cdot46}
\end{array}
$$

$$\frac{5460 + 780}{780} = \frac{6240}{780}$$

No. 1 *Coil.*

$$
\begin{array}{rl}
 & 10\cdot2720241 \\
\log 148 = & 2\cdot1702617 \\
\hline
 & 8\cdot1017624 = \log \text{ of } \underline{126\cdot41} \text{ megs.} \\
\log 1\cdot9946 = & \cdot2998558 \\
\hline
 & 8\cdot4016186 = \log \text{ of } \underline{252\cdot1} \text{ megs.}
\end{array}
$$

No. 2 *Coil.*

$$
\begin{array}{rl}
 & 10\cdot2720241 \\
\log 142 = & 2\cdot1522883 \\
\hline
 & 8\cdot1197358 = \log \text{ of } \underline{131\cdot75} \text{ megs.} \\
\log 2\cdot0074 = & \cdot3026339 \\
\hline
 & 8\cdot4223697 = \log \text{ of } \underline{264\cdot5} \text{ megs.}
\end{array}
$$

No. 3 *Coil.*

$$
\begin{array}{rl}
 & 10\cdot2720241 \\
\log 144\cdot5 = & 2\cdot1598678 \\
\hline
 & 8\cdot1121563 = \log \text{ of } \underline{129\cdot47} \text{ megs.} \\
\log 2\cdot0069 = & \cdot3025257 \\
\hline
 & 8\cdot4146820 = \log \text{ of } \underline{259\cdot8} \text{ megs.}
\end{array}
$$

Calculations for Sheet (D)—*continued.*

No. 4 Coil.

$$\begin{array}{rrl} & 10\cdot2720241 & \\ \log 140\cdot5 = & 2\cdot1476763 & \\ \hline & 8\cdot1243478 & = \log \text{ of } \underline{133\cdot15} \text{ megs.} \\ \log 1\cdot9990 = & \cdot3008128 & \\ \hline & 8\cdot4251596 & = \log \text{ of } \underline{266\cdot2} \text{ megs.} \end{array}$$

No. 5 Coil.

$$\begin{array}{rrl} & 10\cdot2720241 & \\ \log 138 = & 2\cdot1398791 & \\ \hline & 8\cdot1321450 & = \log \text{ of } \underline{135\cdot56} \text{ megs.} \\ \log 1\cdot9990 = & \cdot3008128 & \\ \hline & 8\cdot4329578 & = \log. \text{ of } \underline{271\cdot0} \text{ megs.} \end{array}$$

Percentage of Electrification.

No. 1 Coil.

$$\begin{array}{rr} & 148 \\ & 137 \\ \hline \log\ 11 = & 1\cdot0413927 \\ \log 100 = & 2\cdot \\ \hline & 3\cdot0413927 \\ \log 148 = & 2\cdot1702617 \\ \hline & \cdot8711310 \\ = \log \text{ of } & \underline{7\cdot43} \end{array}$$

No. 2 Coil.

$$\begin{array}{rr} & 142 \\ & 130 \\ \hline \log\ 12 = & 1\cdot0791812 \\ \log 100 = & 2\cdot \\ \hline & 3\cdot0791812 \\ \log 142 = & 2\cdot1522883 \\ \hline & \cdot9268929 \\ = \log \text{ of } & \underline{8\cdot45} \end{array}$$

No. 3 Coil.

$$\begin{array}{rr} & 144\cdot5 \\ & 134 \\ \hline \log\ 10\cdot5 = & 1\cdot0211893 \\ \log 100 = & 2\cdot \\ \hline & 3\cdot0211893 \\ \log 144\cdot5 = & 2\cdot1598678 \\ \hline & \cdot8613215 \\ = \log \text{ of } & \underline{7\cdot27} \end{array}$$

No. 4 Coil.

$$\begin{array}{rr} & 140\cdot5 \\ & 132 \\ \hline \log\ 8\cdot5 = & \cdot9294189 \\ \log 100 = & 2\cdot \\ \hline & 2\cdot9294189 \\ \log 140\cdot5 = & 2\cdot1476763 \\ \hline & \cdot7817426 \\ = \log \text{ of } & \underline{6\cdot05} \end{array}$$

CALCULATIONS FOR SHEET (D)—*continued.*

No. 5 Coil.

$$
\begin{array}{lcl}
 & 138 & \\
 & 127 \cdot 5 & \\
\hline
\log & 10 \cdot 5 & = 1 \cdot 0211893 \\
\log & 100 & = 2 \cdot \\
\hline
 & & \ 3 \cdot 0211893 \\
\log & 138 & = 2 \cdot 1398791 \\
\hline
 & & \ \cdot 8813102 \\
 & & = \log \text{ of } \underline{7 \cdot 61}
\end{array}
$$

(E)

THE ______________ TELEGRAPH COMPANY.

SPECIFICATION.

CONDUCTOR.
Weight per Knot, 107 lbs.
Conductivity compared with Pure Copper, 96 per Cent.

INSULATOR.
Weight per Knot, 150 lbs.
Insulation Resistance at 75° Fahr., 250 megohms.

MANUFACTURE OF ______________ SUBMARINE CABLE

AT ______________ CABLE WORKS.

RECORD SUMMARY OF TESTS OF COILS AT 75° FAHR.

Date.	Details of Coils.													Resistance of Conductor.		Inductive Capacity.			Resistance of Dielectric.			Remarks.
	No. of Coil.	Length of Coils.		Total Weight.			Weight per Knot.			Difference from Contract Weight in lbs. per Knot.				Resistance per Knot.	Percentage of Conductivity compared with Pure Copper.	No. of Cells.	Percentage of loss after 10 secs. Electrification and 60 secs. Insulation.	Capacity per Knot.	No. of Cells.	Percentage of Electrification during second minute.	Resistance per Knot.	
		In Yards.	In Knots.	Copper.	Gutta-percha.	Total.	Copper.	Gutta-percha.	Total.	Copper.		Gutta-percha.										
										+	−	+	−									
1	2	3	4	5	6	7	8	9	10	11	12	13	14	18	19	20	31	32	33	47	49	
				lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	ohms.				m.f.			megs.	
1884. Apr. 6	1	4047	1·9946	213	298	511	106·79	149·40	256·19	..	·21	..	·60	11·52	97·3	10 Dnls.	54·7	·2855	100 Dnls.	7·43	252·1	
„	2	4073	2·0074	214	302	516	106·60	150·44	257·04	..	·40	·44	..	11·48	97·7	„	54·6	·2871	„	8·45	264·5	
„	3	4072	2·0069	215	304	519	107·13	151·48	258·61	·13	..	1·48	..	11·47	97·4	„	55·2	·2872	„	7·27	259·8	
„	4	4056	1·9990	214	299	513	107·05	149·57	256·62	·05	..	..	·43	11·49	97·3		54·0	·2917		6·05	266·2	
	5	4056	1·9990	212	296	508	106·05	148·07	254·12	..	·95	..	1·93	11·62	9·72		53·7	·2866		7·61	271·0	

Signature ______________

Tests of the Cable.

490. As soon as one or more coils have been tested, the manufacture of the cable is commenced, and as each coil is passed through the covering or "closing" machine, another is jointed on, the joint being made at such a time that at least twenty-four hours can elapse between the making and testing of the same. To ensure this necessary time intervening, as soon as one joint is passed through the closing machine the next should be made, so that there is a length of two knots of coil to be sheathed before the new joint is reached.

The system of testing joints has been described in Chapter XIX. A form for entering the results of the tests is shown by Sheet F.

In making a joint it is necessary to cut off a certain length from each coil. The amount of this length varies according to circumstances, but it is seldom more than a few yards.

The order in which the coils are jointed together does not always correspond to the order in which they are tested at 75°, and therefore it is necessary to note down their consecutive order in a column provided on the test sheets for the purpose. In the case of a fault occurring in the cable, this information is of use in enabling an accurate measurement to be made.

Sheets G, H, I, J, and K show the system of entering the tests as they are taken each day. The method of working out and entering the results will be understood from the examples given.

In working out the conductivity test, in order to correct the temperature coefficient, the percentage of purity of the copper is assumed to be the arithmetic mean of the percentages of each 2-knot length at 75°; this, although not strictly correct, is so close an approximation to the truth that no perceptible error is made by considering it to be so.

With reference to columns 49 to 55 on Sheet J, as has been explained on page 206, § 225, the joint insulation resistance of a number of wires is equal to the reciprocal of the sum of the reciprocals of their respective insulation resistances. Column 52 contains, therefore, the reciprocals * of the values in column 51. These reciprocals are added together, and the results noted in column 53, the reciprocals of these numbers (multiplied by 10 million) give the values in column 54.

* These are best obtained from tables (Barlow's are generally used). The numbers are multiplied by 10 million to avoid decimals.

Column 55 is obtained by comparing column 49 with column 54.

491. When the cable is completed it is usual to make special tests for insulation, keeping the current on for half an hour, first with a zinc and then with a copper current, the deflections being noted at the end of each minute. An interval should elapse between the tests with the zinc and copper current, during which time the cable must be put to earth; this enables the cable to become neutral before the copper current is put on. If the cable be connected to a galvanometer immediately after the test with the zinc current, a continuous but decreasing current will be found to flow; as soon as this current ceases, the cable will be neutral, and the test with the copper current can be made. A cable sometimes takes an hour or two to become neutral (page 333, § 389).

492. An electrometer test for fall of charge is also sometimes taken as a check (page 324, § 376).

(F)

THE ______________________ TELEGRAPH COMPANY.

JOINT TESTS OF COILS FOR ______________ CABLE.

Con-secutive Order of Coils.	Joint Made.		Joint Tested.		Time elapsing between Making and Testing Joint.	Leakage from Trough.			Solid Core.			Joint.			Remarks.
	Date.	Time.	Date.	Time.		Full Potential.	Reduced Potential after 1 minute.	Percentage of Loss.	Induced Discharge.	Accumulation by Leakage. 1st min.	Accumulation by Leakage. 2nd min.	Induced Discharge.	Accumulation by Leakage. 1st min.	Accumulation by Leakage. 2nd min.	Number of Cells for Testing Joint, 500 Dnls.
	1884.	P.M.		P.M.	hours.	divisions.	divisions.		divisions.	divisions.		divisions.	divisions.		
1 }	Apr. 6	2.0	Apr. 7	4.0	26	200	194	3·0	150	8	15	145	10	20	
2 }	,, 7	3.30	,, 8	5.0	26½	202	196	3·0	147	9	16	145	13	22	
7 }															
}															
}															
}															

Signature ______________________

Calculations for Sheet (G).

Length Manufactured.

April 9th.

log 1404 = 3·1473671
log 274·25 = 2·4381466

·7092205 = log of 5·1194

5·9926
5·1194

·8732

Calculations for Sheet (H).

Conductor Resistance.

April 9th.

coeff. 56° p. c.* = (1)·0411
97·5

2055
2877
3799

(1)·0410725

coeff. 67° p. c.* = (1)·0171
97·5

855
1197
1539

(1)·0166725

97·3
97·7
97·5
3)292·5
97·5

log 5·1194 = ·7092191
log 1·0411 = ·0174924

·6917267
= log of 4·9173

log ·8732 = $\bar{1}$·9411137
log 1·0167 = ·0071928

$\bar{1}$·9329209
= log of ·85885

4·9173
·85885

5·77615

log 66·34 = 1·8217755
log 5·7762 = ·7616422

1·0601333 = log of 11·49

Estimated Conductor Resistance.

log 11·46 = 1·0591846
log 1·9952 = ·2999864

1·3591710 = log of 22·86

log 68·83 = 1·8377778
log 5·9926 = ·7776153

1·0601625 = log of 11·49

* Pure copper, Table IV. See also page 375, § 453.

Calculations for Sheet (I).

Inductive Capacity.

$$\begin{array}{rl}
\log\ 3 = & \cdot 4771213 \\
\log 1720 = & 3\cdot 2355284 \\
\log\ 100 = & 2\cdot \\
\hline
& 5\cdot 7126497 \\
\log 5560 = & 3\cdot 7450748 \\
\hline
& 1\cdot 9675749
\end{array}
\qquad
\frac{G+S}{S} = \frac{5460+100}{100} = \frac{5560}{100}$$

$$\begin{array}{rl}
\log 165\cdot 5 = & 2\cdot 2187980 \\
& 1\cdot 9675749 \\
\hline
& \cdot 2512231 \\
\log 5\cdot 9926 = & \cdot 7776153 \\
\hline
& \bar{1}\cdot 4736078 = \log \text{ of } \underline{\cdot 2976}
\end{array}$$

Percentage of Loss.

$$\begin{array}{r}
165\cdot 5 \\
90 \\
\hline
\end{array}$$

$$\begin{array}{rl}
\log\ 75\cdot 5 = & 1\cdot 8779470 \\
\log\ 100 = & 2\cdot \\
\hline
& 3\cdot 8779470 \\
\log 165\cdot 5 = & 2\cdot 2187980 \\
\hline
& 1\cdot 6591490 = \log \text{ of } \underline{45\cdot 62}
\end{array}$$

Calculations for Sheet (J).

Insulation Resistance.

April 9th.

$$\begin{array}{rl}
\log 17{,}100 = & 4\cdot 2329961 \\
\log 173 = & 2\cdot 2380461 \\
\hline
& 2\cdot 9949500 = \log \text{ of } \underline{98\cdot 84} = \text{value of battery} \\
\log 10{,}020 \times 1000 = & 7\cdot 0008677 \\
\log 152 = & 2\cdot 1818436 \\
\hline
& 11\cdot 1776613 = \log \text{ constant} \\
\log 1480 = & 3\cdot 1702617 \\
\hline
& 14\cdot 3479230 \\
\log 6940 = & 3\cdot 8413595 \\
\hline
& 10\cdot 5065635 \\
\log 164\cdot 5 = & 2\cdot 2161659 \\
\hline
& 8\cdot 2903976
\end{array}$$

Res. of Galvanometer and Shunt	=	5·46
„ 1 Cell	=	15
		20·46

$$\frac{G+S}{S} = \frac{5460+1480}{1480} = \frac{6940}{1480}$$

Calculations for Sheet (J)—*continued.*

April 9th.

$$\log 5{\cdot}1194 = {\cdot}7092191$$
$$\log \text{coeff.}^{*}\ 56^\circ = {\cdot}6274066$$
$$\overline{{\cdot}0818125}$$
$$= \log \text{ of } 1{\cdot}2073$$
$$\underline{{\cdot}47527}$$
$$1{\cdot}68257$$

$$\log {\cdot}8732 = \bar{1}{\cdot}9411137$$
$$\log \text{coeff.}^{*}\ {\cdot}67^\circ = {\cdot}2641712$$
$$\overline{\bar{1}{\cdot}6769425}$$
$$= \log \text{ of } {\cdot}47527$$

$$8{\cdot}2903976$$
$$\log 1{\cdot}6826 = {\cdot}2259809$$
$$\overline{8{\cdot}5163785} = \log \text{ of } \underline{328{\cdot}38}$$
$$\log 5{\cdot}9926 = {\cdot}7776153$$
$$\overline{6{\cdot}7387632} = \log \text{ of } \underline{54{\cdot}798}$$

Estimated Insulation Resistance.

$$\log 270{\cdot}3 = 2{\cdot}4318460$$
$$\log 1{\cdot}9952 = {\cdot}2999864$$
$$\overline{2{\cdot}1318596} = \log \text{ of } \underline{135{\cdot}48}$$

Percentage of Increase.

$$54{\cdot}798$$
$$\underline{43{\cdot}735}$$
$$\log 11{\cdot}063 = 1{\cdot}0438729$$
$$\log 100 = 2{\cdot}$$
$$\overline{3{\cdot}0438729}$$
$$\log 54{\cdot}798 = 1{\cdot}7387647$$
$$\overline{1{\cdot}3051082} = \log \text{ of } \underline{20{\cdot}2}$$

---

* Table V.

(G)

THE ______________________ TELEGRAPH COMPANY.

DETAILS OF CONSECUTIVE ORDER OF COILS AND LENGTHS OF ______________ CABLE.

Date.	Time.	Consecutive Order of Coils.	Original Lengths of Coils in Yards.	Lengths cut off in making Joints in Yards.	Corrected Lengths of Coils.		Length of Circuit.					Remarks.
							Total Length of Core in Circuit.		Revolutions of Drum. 274·25 Revolutions = 1 Knot.	Length of Cable Sheathed.	Length of Cable Unsheathed.	
1	2	3	4	5	6	7	8	9	10	11	12	
					yards.	knots.	yards.	knots.		knots.	knots.	
1884. April 7	A.M. 12·15	1	4047	..	..	..	4047	1·9946	286	1·0419	·9527	
„ 8	„	1	4047	7	4040	1·9911						
„ „	„	2	4043	2	4041	2·0064	8111	3·9975	920	3·3546	·6429	
„ 9	„	7	4051	3	4048	1·9952	12159	5·9926	1404	5·1194	·8732	

Signature ______________________

(H)

THE____________________TELEGRAPH COMPANY.

CONDUCTOR RESISTANCE TESTS OF CABLE.

Date.	Consecutive Order of Coils.	Length of Circuit.		Temperature.		Resistance of Leads.	Measured Total Resistance of Conductor and Leads.	Measured Total Resistance of Conductor.	Resistance per Knot reduced to 75° Fahr.	Estimated Resistance of Conductor from Tests of Coils at 75° Fahr.				Remarks.
		Cable (sheathed).	Core (unsheathed).	Cable (sheathed).	Core (unsheathed).					Original Resistances per Knot of Coils.	Estimated Total Resistances of Coils when cut for Jointing.	Total Resistances of Cut Coils.	Mean Resistances per Knot of Coils.	
1	3	11	12	13	14	15	16	17	18	18 (sheet B).	19	20	21	
1884.	No.	knots.	knots.	deg. Fahr.	deg. Fahr.	ohms.	ohms.	ohms.	ohms.	ohms.	ohms.	ohms.	ohms.	
Apr. 7	1	1·0419	·9527	58	63	1·43	21·80	20·37	11·51					
„ 8	1	..	..	..	..	..	..	..	[illegible]	11·52	22·94	..	11·52	
„ „	2	3·3546	·6429	58	62	1·42	44·52	43·10	11·50	11·48	23·03	45·97	11·50	
„ 9	7	5·1194	·8732	56	67	1·42	67·76	66·34	11·49	11·4[illegible]	22·86	68·83	11·49	

Signature____________________

(I)

THE ______________________ TELEGRAPH COMPANY.

INDUCTIVE CAPACITY TESTS OF CABLE.

Date.	Consecutive Order of Coils.	Total Length of Circuit.	No. of Cells.	Resistance of Galvanometer.	Condenser. Capacity ⅓ m.f. Shunt.	Condenser. Capacity ⅓ m.f. Discharge.	Cable. Shunt.	Cable. Immediate Discharge after 10 secs. Electrification. Lead.	Cable. Immediate Discharge after 10 secs. Electrification. Cable and Lead.	Cable. Immediate Discharge after 10 secs. Electrification. Cable.	Cable. Discharge after 10 secs. Electrification and 60 secs. Insulation. Lead.	Cable. Discharge after 10 secs. Electrification and 60 secs. Insulation. Cable and Lead.	Cable. Discharge after 10 secs. Electrification and 60 secs. Insulation. Cable.	Cable. Percentage of Loss.	Cable. Capacity per Knot.	Remarks.
1	3	9	22	23	24	25	26	27	28	29	30	31	32	33	34	
1884. Apr. 7	No. 1	knots. 1·9946	Dnls. 10	5450	$\frac{1}{10}$th	divns. 173	ohms. 330	divns. 4·5	divns. 173·5	divns. 169	divns. 4·0	divns. 85	divns. 81	52·10	m. farads. ·2859	
„ 8	1	..	„	„	„											
„ „	2	3·9975	„	„	„	172·5	160	„	173·5	169	„	90	86	49·10	·2864	
„ „	7	5·9926	„	5460	„	172	100	„	170	165·5	„	94	90	45·62	·2976	

Signature ______________________

(J)

THE ______________________ TELEGRAPH COMPANY.

INSULATION TESTS OF CABLE.

Date.	Consecutive Order of Coils.	Length of Circuit. Cable (sheathed).	Length of Circuit. Core (unsheathed).	Temperature. Cable (sheathed).	Temperature. Core (unsheathed).	Number of Cells.	Resistance of Galvanometer.	Constant. Battery Ratio. Discharge from Condenser. 1 Cell with no Shunt.	Constant. Battery Ratio. Discharge from Condenser. Full Battery with $\frac{1}{100}$ Shunt.	Constant. Battery Ratio. Value of Battery.	Constant. Deflection from 1 Cell through 10,000 ohms, with $\frac{1}{100}$ Shunt.	Constant. Logarithm of Constant.	Current from Cable before Test + −
1	3	11	12	13	14	35	23	36	37	38	39	40	41
1884.	No.	knots.	knots.	deg. F.	deg. F.	Dnls.	ohms.				divisions.		divisions.
April 7	1	1·0419	·9527	58	63	100	5450	172	170	98·84	154	11·1807912	..
,, 8	1	..	..	..	..	..	..	..	..	..	..	..	..
,, ,,	2	3·3546	·6429	58	62	100	5450	173	171	98·84	154	11·1833384	..
,, 9	7	5·1194	·8732	56	67	,,	5460	,,	,,	98·84	152	11·1776613	..

Cable. Shunt.	Cable. Deflection after 1 min. Electrification. Lead.	Cable. Deflection after 1 min. Electrification. Cable and Lead.	Cable. Deflection after 1 min. Electrification. Cable.	Cable. Deflection after 2 mins. Electrification. Lead.	Cable. Deflection after 2 mins. Electrification. Cable and Lead.	Cable. Deflection after 2 mins. Electrification. Cable.	Total Resistance reduced to 75° F.	Resistance per Knot reduced to 75° F.	Estimated Resistance of Cable from Tests of Coils at 75° F. Measured Resistance per Knot of Coils at 75° F. before cut for Jointing.	Estimated Resistance of Cable from Tests of Coils at 75° F. Estimated Total Resistance of Coils at 75° F. when cut for Jointing.	Estimated Resistance of Cable from Tests of Coils at 75° F. Reciprocals	Estimated Resistance of Cable from Tests of Coils at 75° F. Sums of Reciprocals	Estimated Resistance of Cable from Tests of Coils at 75° F. Calculated Total Resistance from Tests of Coils at 75° F.	Total Percentage of Increase in Resistance during Manufacture.	Remarks.
42	43	44	45	46	47	48	49	50	49 (Sheet D.)	51	52	53	54	55	
	divns.	divns.	divns.	divns.	divns.	divns.	meghoms.	megohms.	megohms.	megohms.	× 10 miln.		megohms.		
5200	3·0	176	173	2·5	163·5	161	..	286·14	252·1						
..	..	..	..	..	..	..	..	..	..	126·63	78972				
2300	3·0	171	168	2·5	169·5	157	78·206	312 63	264·5	131·81	75866	154838	64·583	17·4	
1480	3·5	168	164·5	3·0	166	163	54·798	328·38	270·3	135·48	73814	228652	43·735	20·2	

Signature ______________________

(K)

THE ______________ TELEGRAPH COMPANY.

MANUFACTURE OF ______________ SUBMARINE CABLE AT ______________ CABLE WORKS.

RECORD SUMMARY OF TESTS OF CABLE.

Date.	Time.	Consecutive Order of Coils.	Length of Circuit.			Temperature.		Resistance of Conductor.	Inductive Capacity.			Resistance of Dielectric.					Remarks.
			Length of Cable (sheathed).	Length of Cable (unsheathed).	Total Length of Core in Circuit.	Cable.	Core.	Resistance per Knot.	No. of Cells.	Percentage of loss after 10 secs. Electrification and 60 secs. Insulation.	Capacity per Knot.	No. of Cells.	Resistance per Knot Reduced to 75° Fahr.	Total Measured Resistance Reduced to 75° Fahr.	Calculated Total Resistance from Tests of Coils at 75° Fahr.	Total Percentage of Increase in Resistance During Manufacture.	
1	2	3	11	12		13	14	18	22	33	34	35	50	49	54	55	
1884.			knots.	knots.	knots.	deg. F.	deg. F.	ohms.	Dnls.		m.f.	Dnls.	mg.ohm.	mg.ohm.	mg.ohm.		
Apr. 7	12·15	1	1·0419	·9527	1·9946	58	63	11·51	10	52·10	·2859	100	286·14	..	..	..	Electrification steady.
" 8	"	1															
" "	"	2	3·3546	·6429	3·9975	58	62	11·50	"	49·10	·2864	"	312·63	78·206	64·583	17·4	"
" 9	"	7	5·1194	·8732	5·9926	56	67	11·49	"	45·62	·2976	"	328·38	54·798	43·735	20·2	"

Signature ______________

CHAPTER XXVII.

MISCELLANEOUS.

To determine the True Insulation Resistance per Mile of a Telegraph Line.

493. On page 6 it was pointed out that the rule of multiplying the total insulation by the mileage of the wire to get the insulation per mile was not strictly correct. Now, although the leakage in a telegraph line insulated on poles is really a leakage at a series of detached points, and not a uniform leakage, as in a cable, yet practically, and especially in the case of long lines, it may be considered as taking place uniformly, and consequently the solutions of problems dealing with cables also apply with considerable accuracy to land lines. We may therefore consider the case in question by the help of the equations we have obtained in the investigations made in Chapter XXII.

On page 386 we have an equation

$$m^2 = \frac{r}{i},$$

where r is the conductivity resistance per unit length and i the insulation resistance per unit length, of the line. Also on page 390 we have an equation

$$R_e\, R_i = \frac{r^2}{m^2},$$

where R_e is the total resistance of the line when the further end is to earth, and R_i the total resistance when the end is insulated. By combining these two equations we have

$$R_e\, R_i = r^2 \frac{i}{r} = r\, i,$$

or

$$i = R_i \frac{R_e}{r}. \qquad [A]$$

If we take the unit length to be a mile, then r being the true conductor resistance per mile, i will be the insulation resistance per mile.

It will be seen that the mileage of the line does not come into the equation, this quantity being represented by

$$\frac{R_e}{r}.$$

What we do, in fact, in order to obtain the true insulation per mile of a line, is to multiply the total insulation, not by the *absolute* total conductivity divided by the *absolute* conductivity per mile, which is the same thing as the mileage, but by the *observed* total conductivity divided by the true conductivity per mile.

For example.

The resistance of a line, 200 miles long, when the further end was insulated was 4000 ohms. When the end was to earth the resistance was 2400 ohms. The absolute conductor resistance of the wire, at the time the measurements were being made, was known to be 16 ohms per mile. What was the true insulation per mile of the line?

$$i = 4000 \times \frac{2400}{16} = 600{,}000 \text{ ohms.}$$

The value of i given by the ordinary rule would be

$$i = 4000 \times 200 = 800{,}000 \text{ ohms,}$$

a result 200,000 ohms, or 33 per cent., too high.

494. It must be evident that what is ordinarily called the conductor resistance of a line is really the true conductivity resistance diminished by the conducting power of the insulators. Conductivity resistance, therefore, in the case of a land line can only be measured accurately in fine weather, when the insulation is very high. To obtain, then, the value of r from equation [A] it would be necessary to take a conductivity test in fine weather, and to note the temperature at that time; and then when an insulation test is made in wet weather, to observe the temperature, and from this correct the value of r previously obtained in the fine weather.

In the case of a submarine cable, the insulation resistance (when the cable is in good condition) is always so greatly in excess of the conductivity resistance that the true value of the

latter is obtained at once by measuring the resistance of the cable when its end is to earth. Also the insulation per mile is practically equal to the total resistance when the end is insulated, multiplied by the mileage.

Testing Telegraph Lines by Received Currents.

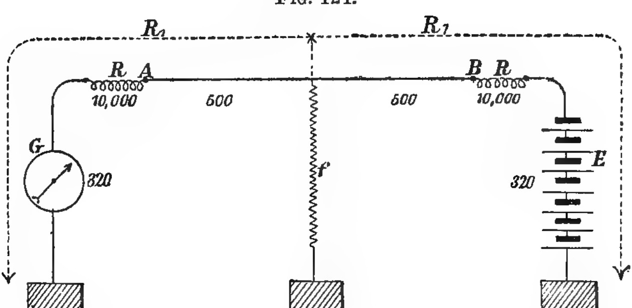
495. The system of daily testing for insulation, described in Chapter I., page 6, and which was in general use on the lines of the Postal Telegraph Department, has now been superseded by a system of testing by received currents, which possesses many advantages over the old method of testing.

Every day at a definite time, currents from batteries, each of an approximately definite electromotive force, are transmitted over the different lines, or sections of lines, and the strengths of the currents received at the further ends are measured. It is evident that the strengths of these currents will vary with the amounts of leakage on the lines, that is with the state of their insulation; if then the battery power employed for transmitting the currents be constant, the strengths of the received currents observed from day to day will give an accurate knowledge of the condition of the lines.

The way in which this general principle is practically carried out is as follows:—

Let A B (Fig. 124) represent the section of line to be tested, then to each end of the latter, resistances, R, R, of 10,000 ohms

Fig. 124.



each are connected, together with a galvanometer G (whose resistance is 320 ohms) and a battery E (whose resistance is also approximately 320 ohms), as shown. Although the sections tested are not all of equal lengths or resistances, yet practically

they are such that they may all be assumed to have a mean conductor resistance of 1000 ohms.

Now it can be demonstrated mathematically that if the resistances R, R, are very great, then a "resultant" fault* f (that is, the total insulation resistance of the line) will produce very nearly the same effect on the current received on the galvanometer G, whether this fault is at the middle, at the end, or at any intermediate point on the line. As a matter of fact the fault has the greatest influence when it is at the middle of the line, and the least influence when it is at either of the ends, but when the resistances R, R, are each about 10 times the conductor resistance of the line, then the difference in the two cases is practically very small. If then we assume, for convenience of calculation, that the resultant fault is at the middle of the line, we have

$$C_r = \frac{E}{R_1 + \dfrac{R_1 f}{R_1 + f}} \times \frac{f}{R_1 + f} = \frac{E f}{R_1^2 + 2 R_1 f},$$

where C_r is the current received on the galvanometer G, E the electromotive force of the battery, and R_1 the total resistance on either side of the fault f.

From this equation we get

$$f = \frac{R_1^2}{\dfrac{E}{C_r} - 2 R_1} = \frac{1}{\dfrac{E}{R_1^2 C_r} - \dfrac{2}{R_1}}.$$

Now the battery from which the current is sent consists of 50 Daniell cells, and if we take the electromotive force of a Daniell cell to be 1·07 volts approximately we have

$$E = 50 \times 1{\cdot}07 = 53{\cdot}5 \text{ volts}.$$

We also have

$$R_1 = 320 + 10{,}000 + 500 = 10{,}820 \text{ ohms};$$

therefore

$$f = \frac{1}{\dfrac{53{\cdot}5}{10{,}820 \times 10{,}820 \times C_r} - \dfrac{2}{10{,}820}}$$

$$= \frac{1}{\dfrac{{\cdot}00000045698}{C_r} - {\cdot}00018484} \text{ ohms},$$

where C_r is measured in ampères.

* See page 230, § 261.

If now we so adjust the galvanometer G by means of the directing magnet, that one milliampère ($\frac{1}{1000}$th ampère) of current gives a deflection of 25°, then if $d°$ be the deflection given by any other current, we must have

$$C_r = \frac{\tan d°}{\tan 25° \times 1000} = \tan d° \times \cdot 0021445 \text{ ampères.}$$

From this last equation, then, we can obtain the strength (C_r) of the received current, in ampères, corresponding to any particular deflection; whilst from the previous equation, by inserting this value of C_r, we can obtain the corresponding value of f, that is the total insulation resistance of the line.

For example.

Suppose $d° = 40°$; then

$$C_r = \cdot 8390996 \times \cdot 0021445 = \cdot 0017994 \text{ ampères,}$$

or 1·80 milliampères, approximately.

Also

$$f = \frac{1}{\frac{\cdot 00000045698}{\cdot 0017994} - \cdot 00018484} = 14{,}468 \text{ ohms,}$$

or 14,500 ohms, approximately.

496. In order to save calculation, a table showing the values of C_r and f corresponding to the various deflections ($d°$), is provided at each of the different test offices; this table is arranged as follows:—

TABLE SHOWING STRENGTHS OF RECEIVED CURRENTS AND EQUIVALENT INSULATION RESISTANCES.

Deflection.	Strength of Received Current.	Equivalent Insulation Resistance.	Deflection.	Strength of Received Current.	Equivalent Insulation Resistance.
	Milliampères.	Ohms.		Milliampères.	Ohms.
49°	2·47	Infinite.	29°	1·19	5,010
48½°	2·42	271,000	28½°	1·16	4,820
48°	2·38	142,000	28°	1·14	4,630
47½°	2·34	96,500	27½°	1·12	4,450
47°	2·30	72,100	27°	1·09	4,290
46½°	2·26	57,500	26½°	1·07	4,120
46°	2·22	47,800	26°	1·05	3,970
45½°	2·18	40,700	25½°	1·02	3,820
45°	2·14	35,400	25°	1·00	3,680

Deflection from standard cell through galvanometer, with both plugs out, to be made 25°.

Battery sending current to give 49½° on galvanometer, with both plugs out and adjusted as above, with 20,000 ohms in circuit.

497. In order that the station transmitting the currents may be able to ascertain whether his 50-cell battery is in proper condition, he can test its electromotive force in the following way:—

The battery being joined up in circuit with the galvanometer and two of the 10,000 ohms resistances, the deflection is noted. Now if the 50 cells are in proper condition, their total electromotive force would be

$$50 \times 1 \cdot 07 = 53 \cdot 5 \text{ volts.}$$

Taking then the resistance of the battery to be 320 ohms approximately, and the resistance of the galvanometer being 1070 ohms,* the current deflecting the needle will be

$$\frac{53 \cdot 5 \times 1000}{320 + 10{,}000 + 10{,}000 + 1070} = 2 \cdot 5012 \text{ milliampères.}$$

But the adjustment of the tangent galvanometer should be such that 1 milliampère of current gives 25°; consequently if the electromotive force of the 50 cells is equal to 53·5 volts, the deflection, d°_1, obtained should be such that

$$\tan d^\circ_1 : \tan 25^\circ :: 2 \cdot 5012 : 1;$$

that is,

$$\tan d^\circ = \tan 25^\circ \times 2 \cdot 5012 = \cdot 4663 \times 2 \cdot 5012 = 1 \cdot 1663$$
$$= \tan 49\tfrac{1}{2}^\circ \text{ approximately.}$$

$49\frac{1}{2}^\circ$ then is the deflection which should be obtained if the battery is in proper condition; if the latter is not the case, however, then the power is brought up to its approximate proper value by adding on an extra cell or two until the deflection is increased to $49\frac{1}{2}^\circ$ as nearly as possible. It is seldom necessary, however, to do this in practice.

498. The measurement of the currents is effected by means of a tangent galvanometer of a pattern nearly similar to that shown on page 18, and the standard cell described on page 118.

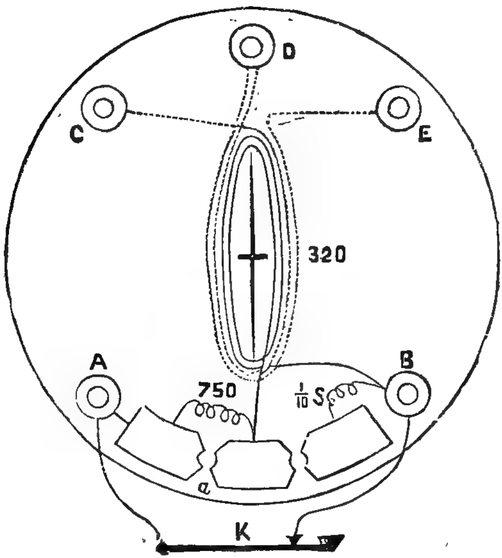
Fig. 125 shows in general plan the arrangement of the galvanometer.

In this instrument there are three coils of wire, the one nearest the needle consisting of No. 35 copper wire wound to a resistance of 320 ohms. The other two coils are of No. 18

* When this test is being made the galvanometer resistance is 320 + 750 = 1070 ohms; the 750 ohms is a resistance, connected to the instrument, whose use will be explained in describing the latter.

gauge—the one between C and D making three turns, and the one between D and E making twelve turns in the opposite direction. The latter coils are for testing batteries. Let c be

FIG. 125.



the current whose strength is to be tested, then if we connect the wires conveying this current on to terminals C and D, we get an effect

$$c \times 3 = 3c.$$

If we connect the wires to terminals C and E the effect will be

$$c \times 12 - c \times 3 = 9c.$$

Again, if we connect the wires to terminals D and E, the effect will be

$$c \times 12 = 12c.$$

Lastly, let terminals C and E be coupled together by a piece of wire, and let the connecting wires conveying the current to be measured be connected to terminals C and D, then the current

c will split, and the amount going through the 12 turns of wire will be

$$c \times \frac{3}{12+3} = c\,\frac{1}{5};$$

and the amount going through the 3 turns of wire will be

$$c \times \frac{12}{12+3} = c\,\frac{4}{5}.$$

The effect produced by the current going through the 12 turns of wire will be

$$c\,\frac{1}{5} \times 12 = c\,\frac{12}{5};$$

and the effect produced by the current going through the 3 turns will be

$$c\,\frac{4}{5} \times 3 = c\,\frac{12}{5}.$$

Therefore since the currents both affect the needle in the same direction, the joint effect will be

$$c\,\frac{12}{5} + c\,\frac{12}{5} = c\,\frac{24}{5} = c\,4 \cdot 8.$$

We can therefore obtain degrees of sensitiveness in the proportions

$$3 : 4 \cdot 8 : 9 : 12$$

or

$$1 : 1 \cdot 6 : 3 : 4.$$

These relative values are, however, only approximate. The resistances of the wires are practically *nil.*

An adjusting magnet, similar to that used on the Thomson galvanometer (page 34), is set on the upper part of the instrument.

499. In testing the strength of a current in milliampères, the standard cell is connected to A and B, and both plugs being removed from the plug-holes *a*, *b*, the key K* is depressed. There is then in circuit a total resistance of 1070 ohms, viz.

* In the more recent instruments this key short circuits the terminals A and B when it is *depressed* and not when it is *raised;* this arrangement is found to be more satisfactory than the old one, since it enables observations to be taken without the necessity of the key being continually held down, whilst the object of the key (the checking of the oscillations of the needle) is effected equally as well by the new arrangement as by the old one.

$750 + 320$. As the electromotive force of the standard cell is 1·07 volts, the resulting deflection of the galvanometer-needle (which is adjusted by means of the adjusting magnet to 25°) will be due to a current of

$$\frac{1 \cdot 07}{1070} = \cdot 001 \text{ ampère, or, 1 milliampère,}$$

and any other deflection obtained with any particular current, compared by direct proportion with the standard deflection (the two deflections being of course reduced to *tangents*), will give the strength of that current in milliampères.

When the standard deflection is obtained, the standard cell is removed and the circuit from which the received current is to be measured is connected to terminal A, terminal B being put to earth.

To determine the Insulation Resistance of a Line when the strengths of the Sent and Received Currents are known.

500. The further end of the line being to earth, and l being the length of the line, we have from equation [2], page 386, by putting $x = l$,

$$\text{Current sent} = \mathrm{C}_s = \frac{m}{r}\left[\mathrm{A}\, e^{ml} - \mathrm{B}\, e^{-ml}\right];$$

and from the same equation by putting $x = 0$,

$$\text{Current received} = \mathrm{C}_r = \frac{m}{r}\left[\mathrm{A} - \mathrm{B}\right];$$

therefore

$$\frac{\mathrm{C}_s}{\mathrm{C}_r} = \frac{\mathrm{A}\, e^{ml} - \mathrm{B}\, e^{-ml}}{\mathrm{A} - \mathrm{B}};$$

but from equation [4], page 387, we have

$$\frac{\mathrm{A}}{\mathrm{B}} = \frac{\sigma \frac{m}{r} + 1}{\sigma \frac{m}{r} - 1}, \quad \text{or,} \quad \mathrm{A}\left(\sigma \frac{m}{r} - 1\right) = \mathrm{B}\left(\sigma \frac{m}{r} + 1\right);$$

therefore

$$\frac{\mathrm{C}_s}{\mathrm{C}_r} = \frac{e^{ml}\left(\sigma \frac{m}{r} + 1\right) - e^{-ml}\left(\sigma \frac{m}{r} - 1\right)}{2};$$

by inserting the values of e^{ml}, e^{-ml}, and $\frac{m}{r}$, given by equations [10] and [12], pages 390 and 391, we get

$$\frac{C_s}{C_r} = \frac{\sqrt{\dfrac{1+\sqrt{\dfrac{R_e}{R_i}}}{1-\sqrt{\dfrac{R_e}{R_i}}}}\left(\dfrac{\sigma}{\sqrt{R_e R_i}}+1\right) - \sqrt{\dfrac{1-\sqrt{\dfrac{R_e}{R_i}}}{1+\sqrt{\dfrac{R_e}{R_i}}}}\left(\dfrac{\sigma}{\sqrt{R_e R_i}}+1\right)}{2}$$

$$= \frac{(\sqrt{R_i}+\sqrt{R_e})\left(\dfrac{\sigma}{\sqrt{R_e R_i}}+1\right) - (\sqrt{R_i}-\sqrt{R_e})\left(\dfrac{\sigma}{\sqrt{R_e R_i}}-1\right)}{2\sqrt{R_i - R_e}}$$

$$= \frac{\sqrt{R_i} + \dfrac{\sigma}{\sqrt{R_i}}}{\sqrt{R_i - R_e}}.$$

The value of R_i although it could be determined from this equation, would be represented by a somewhat complex fraction; if, however, we have $\sigma = 0$, we then get

$$\frac{C_s}{C_r} = \frac{\sqrt{R_i}}{\sqrt{R_i - R_e}}, \quad \text{or,} \quad R_i = R_e \frac{C_s^2}{C_s^2 - C_r^2}. \qquad [B]$$

In which equation, C_s and C_r (being in the form of a proportion) may be measured in ampères or milliampères, or indeed in any multiple or submultiple of an ampère.

For example.

The resistance of a line when to earth at the further end was 1500 ohms (R_e). The strengths of the sent and received currents were 2·8 and 2·6 milliampères respectively. What was the total insulation resistance of the line?

$$R_i = 1500 \frac{2\cdot 8^2}{2\cdot 8^2 - 2\cdot 6^2} = 10{,}908 \text{ ohms}.$$

The measurement of the received current must be made by means of a very low resistance galvanometer in order to avoid the introduction of the quantity σ into the formula.

501. Having obtained R_i, the insulation per mile could be obtained in the manner shown on page 445; a simpler method of doing this is the following:—

If E be the electromotive force of the battery sending the current, then we have

$$C_s = \frac{E}{R_e}, \quad \text{or,} \quad C_s^2 = \frac{E^2}{R_e^2};$$

by substituting this value in equation [B] we get

$$R_i = \frac{E^2}{R_e (C_s^2 - C_r^2)}.$$

Again, for equation [A], page 445, we have

$$i = R_i \frac{R_e}{r}, \quad \text{or,} \quad R_i = \frac{i\,r}{R_e},$$

where i is the true insulation resistance per mile of the line, and r its true conductivity resistance per mile; therefore

$$\frac{i\,r}{R_e} = \frac{E^2}{R_e (C_s^2 - C_r^2)}, \quad \text{or} \quad i = \frac{E^2}{r (C_s^2 - C_r^2)};$$

in which C_s and C_r are in ampères, E in volts, and i and r in ohms. If C_s and C_r are measured in milliampères, then we have

$$i = \frac{(E \times 1000)^2}{r (C_s^2 - C_r^2)} = \frac{E^2 \times 1{,}000{,}000}{r (C_s^2 - C_r^2)} \text{ ohms.}$$

For example.

The strengths of the sent and received currents on a line were 12 and 10 milliampères respectively, the sending battery being a 10-cell Daniell (10 volts approximately); the line had an average estimated conductivity resistance of 14 ohms per mile. What was the insulation per mile of the line?

$$i = \frac{10^2 \times 1{,}000{,}000}{14 (12^2 - 10^2)} = 162{,}000 \text{ ohms.}$$

KIRCHOFF'S LAWS.

502. These laws are two in number, the first is, that

The algebraical sum of the current strengths in all those wires which meet in a point is equal to nothing.

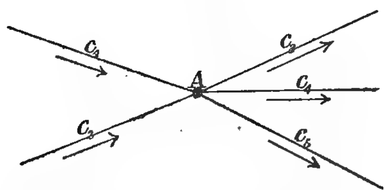
The truth of this law is almost obvious; thus, if we have, say, five wires meeting in a point, as shown by Fig. 126, then as the point A cannot be a reservoir, the sum of the currents c_1, c_2, approaching A must equal the sum of the currents c_3, c_4, c_5, receding from A, that is

$$c_1 + c_2 = c_3 + c_4 + c_5,$$

or

$$c_1 + c_2 - c_3 - c_4 - c_5 = 0.$$

FIG. 126.



It may be as well, perhaps, to point out that although the quantities c_1, c_2, c_3, c_4, c_5, are partly positive and partly negative, yet they together constitute an *algebraical* "sum," for the equation may be written

$$c_1 + c_2 + (-c_3) + (-c_4) + (-c_5) = 0;$$

the quantities c_3, c_4, and c_5, in fact are negative because the currents they represent flow in the opposite direction to the currents c_1, c_2.*

503. The second law of Kirchoff is as follows:—

The algebraical sum of all the products of the current strengths and resistances in all the wires forming an enclosed figure, equals the algebraical sum of all the electromotive forces in the circuit.

The truth of this law follows as a consequence from the laws we investigated on pages 255-258—viz.:

(A) *The difference of the potentials at two points in a resistance (in which no electromotive force exists) is equal to the product of the current and the resistance between the two points.*

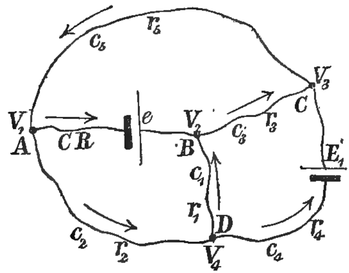
(B) *The difference of the potentials at two points in a resistance in which an electromotive force exists, is equal to the product of the current and the resistance between the two points, added to the electro-*

* It is important that *algebraical* sum should not be confounded with *arithmetical* sum; the latter signifies a number of quantities connected by *plus* signs, whilst in the former the signs may be partly negative and partly positive, or indeed all negative. As a rule when the word "sum" is used in stating a law, it is the *algebraical* sum which is meant.

motive force in the resistance, this electromotive force being negative if it acts with the current and positive if it opposes it.

If we refer to Fig. 127 and we consider any closed circuit in it, then we can see that the sum of the differences of the

FIG. 127.



potentials between the points in that circuit must be equal to 0; thus if we take the closed current formed by the sections A B, B C, C D, D A, for example, then it is evident that

$$(V_1 - V_2) + (V_2 - V_3) + (V_3 - V_4) + (V_4 - V_1)$$

is the same as

$$V_1 - V_1 + V_2 - V_2 + V_3 - V_3 + V_4 - V_4,$$

which equals 0.

Now from laws (A) and (B) we have

$$V_1 - V_2 = C\,R - e$$
$$V_2 - V_3 = c_3\,r_3$$
$$V_3 - V_4 = -\,(c_4\,r_4 - E_1)\,*$$
$$V_4 - V_1 = -\,c_2\,r_2\,;\,*$$

therefore, by addition, we get

$$C\,R - e + c_3\,r_3 - c_4\,r_4 + E - c_2\,r_2 = 0;$$

or

$$C\,R + c_3\,r_3 - c_4\,r_4 - c_2\,r_2 = e - E,$$

which proves the law.

As in the case of Kirchoff's first law, we have, in the last equation, *algebraical* sums, for this equation may be written:

$$C\,R + c_3\,r_3 + (-\,c_4\,r_4) + (-\,c_2\,r_2) = e + (-\,E);$$

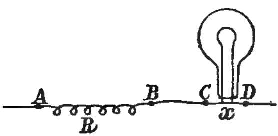
* These quantities are *negative* because the currents c_4 and c_2 flow in the reverse direction to the currents c_1 and c_3.

c_4, c_2, and E in fact, are negative, because the currents in the sections (C D and D A) in which these quantities occur are in the reverse direction to the currents in the other sections (A B and B C).

A Method of measuring the Resistance of, and the Current flowing through, Electric Lamps when burning.

504. This method is an adaptation of the methods given on page 275, § 319, and page 336, § 396, and is as follows:—

FIG. 128.



A resistance, R (Fig. 128), is inserted in the circuit of the lamp whose resistance is to be measured, and then the potential, V, between the points, A and B, is measured. A similar measurement is then taken of the potential, V_1, between the terminals, C and D, of the lamp. We then have—

$$x = R\frac{V_1}{V}.$$

For example.

Suppose the resistance, R, were 1 ohm, and suppose that the discharge deflection obtained by the condenser from the points, A and B, was 250 divisions, there being no shunt to the galvanometer; also suppose that the discharge deflection obtained from between the points, C and D, was 60 divisions, the galvanometer, whose resistance was 6100 ohms, being shunted with a shunt of 200 ohms; then we have

$$V = 250$$

$$V_1 = 260 \times \frac{6100 + 200}{200} = 8190;$$

therefore,

$$x = 1 \times \frac{8190}{250} = 32 \cdot 8 \text{ ohms.}$$

If the discharge given by a standard cell (page 118) were 140, then we should have

$$\text{Electromotive force between A and B} = 1 \cdot 079 \times \frac{250}{140} = 1 \cdot 92.$$

The current flowing, therefore, equals

$$\frac{1 \cdot 91}{1} = 1 \cdot 92 \text{ ampères.}$$

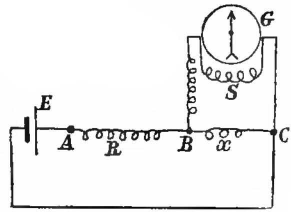
In cases where the current is powerful, and where it is not advisable to introduce so high a resistance as 1 ohm into the circuit, R could be made, say, $\frac{1}{10}$th of an ohm.

A Method of measuring Low Resistances.

505. This method, like the foregoing, is merely an adaptation of the method given on page 336, § 396, and is shown in principle by Fig. 129.

E is a single Daniell cell, R a resistance of 1 ohm, and B C the resistance, x, to be measured. Between B and C a Thomson galvanometer (page 31) in circuit with a resistance is connected.

Fig. 129.



Now, taking the resistance of the cell E to be, say, 4 ohms, then if x be $\frac{1}{100}$th of an ohm, the potential between B and C will be approximately $\frac{1}{500}$th of a volt, and the potential between A and B, $\frac{1}{4}$th of a volt, consequently if we can measure these two potentials accurately we can determine the value of a resistance of $\frac{1}{100}$th of an ohm to an equal degree of accuracy. Now a Thomson galvanometer, wound to about 5000 ohms resistance, will give a deflection of 100 divisions with one Daniell cell, there being in circuit a total resistance of 10,000,000 ohms. If there be no resistance in the circuit beyond that of the galvanometer itself (5000 ohms) the deflection would be

$$100 \times \frac{10{,}000{,}000}{5000} = 200{,}000 \text{ divisions,}$$

representing an electromotive force, or potential, of 1 volt approximately; hence 200 divisions would represent a potential of $\frac{1}{1000}$th of a volt. We can easily, therefore, measure a potential of $\frac{1}{500}$th of a volt.

In order to make a measurement we should proceed as follows:—

The battery, resistances, etc., being connected up as shown in Fig. 129, and the shunt being removed from the terminals of the galvanometer, the resistance in circuit with the latter must be varied until a good deflection (about 300 divisions) is obtained. Let d_1 be this deflection, and let G and R_1 be the respective resistances of the galvanometer and the resistance in circuit with the latter, then if v_1 be the difference of potential between

B and C, the current c_1 flowing through the galvanometer will be

$$c_1 = \frac{v_1}{R_1 + G}.$$

The galvanometer and the resistance in its circuit are now disconnected from B and C, and are connected to A and B, the $\frac{1}{1000}$th shunt being joined up to the terminals of the instrument. The resistance in its circuit is then varied until a deflection d_2, approximately the same as d_1, is obtained; then if R_2 be this resistance, and if v_2 be the potential between A and B, and further if c_2 be the current producing the deflection d_2, we have

$$c_2 = \frac{v_2}{R_2 + g} \times \frac{1}{1000},$$

where g is the combined resistance of the galvanometer and shunt.

We have therefore

$$\frac{c_1}{c_2} = \frac{(R_2 + g)\,1000}{(R_1 + G)} \times \frac{v_1}{v_2};$$

but

$$v_1 : v_2 :: x : R$$

or

$$\frac{v_1}{v_2} = \frac{x}{R},$$

and as

$$c_1 : c_2 :: d_1 : d_2,$$

or

$$\frac{c_1}{c_2} = \frac{d_1}{d_2},$$

we get

$$\frac{d_1}{d_2} = \frac{(R_2 + g)\,1000}{(R_1 + G)} \times \frac{x}{R},$$

or

$$x = R\,\frac{(R_1 + G)}{(R_2 + g)\,1000} \cdot \frac{d_1}{d_2}.$$

For example.

The deflection obtained between the points B and C was equal to 320 divisions (d_1), there being a resistance of 8000 ohms (R_1) inserted in the circuit of the galvanometer. When the latter was

connected between A and B, the $\frac{1}{1000}$th shunt was inserted, together with a resistance of 1200 ohms (R_2); the deflection obtained was then equal to 310 divisions (d_2). The resistance of the galvanometer was 5000 ohms (G), and the resistance, R, 1 ohm. What was the value of x?

$$x = 1 \frac{(8000 + 5000)}{(1200 + 5)\,1000} \cdot \frac{320}{310} = \cdot 0111 \text{ ohms}.$$

We are not, of course, necessarily bound to use the $\frac{1}{1000}$th shunt, but in practice it would almost always have to be employed.

506. The degree of accuracy with which the test could be made would depend entirely upon the values of the deflections d_1 and d_2, and as we should endeavour to make them both as high as possible, that is to say, both as nearly equal as possible, the "Percentage of accuracy" would practically be $\frac{\delta\, 200}{d_1}$, where δ is the fraction of a division to which each of the deflections could be read.

The Silvertown Compound Key for Cable Testing.

507. This key, which is in general use in the testing rooms of the Silvertown Telegraph Works, is an excellent arrangement, and greatly facilitates the execution of the "Inductive capacity" and "Insulation" tests of insulated wires or of cables; it is particularly useful when a large number of wires have to be tested. The apparatus (Fig. 130) consists of two keys, of the form shown by Figs. 82 and 83, pages 245 and 246, mounted on one base.

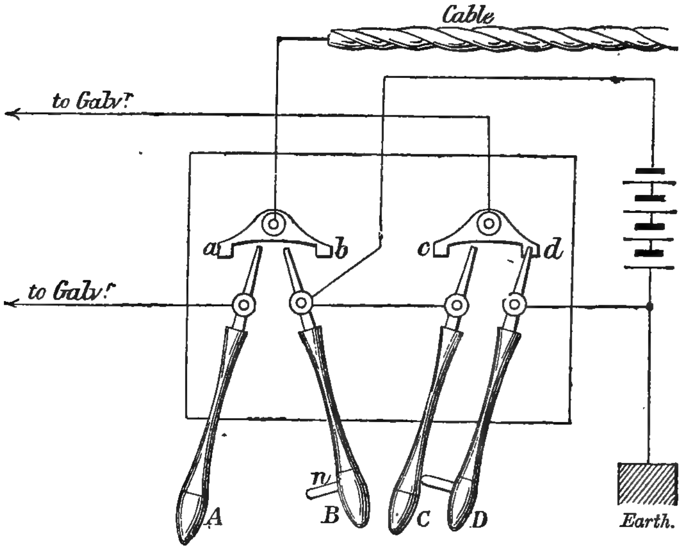
Supposing the connections to be made as shown by the figure, then in order to measure the "discharge" from the cable, levers C and D are set in the positions shown. Lever B is now pressed to the right so that its projecting piece n comes in contact with lever A; the brass tongue of lever B is then in contact with b, so that the battery, whose zinc pole is joined to lever B, is connected to the cable. If now lever A is pressed over to the left, then lever B is also moved and the tongue of the latter consequently leaves b whilst the tongue of A comes in contact with a, and thus puts the cable in connection with the galvanometer. As the second terminal of the galvanometer is connected to the piece $c\,d$, the circuit is completed to earth through d and the tongue of lever D.

To measure the discharge from a condenser, one terminal of the former would be connected to the piece $c\,d$ and the other

terminal to earth; the manipulation of the levers would of course be the same as in the case of the cable.

To take the "Insulation" test (p. 331) of the cable, levers A and B would be set over to the right so that the tongue of lever

FIG. 130.



A is in contact with a whilst the tongue of B is disconnected from b. The short-circuit key of the galvanometer being closed, lever C is now pressed over to the right, so that the tongue of lever C comes in contact with c, whilst the tongue of lever D becomes disconnected from d; the zinc pole of the battery thus becomes connected through c with one terminal of the galvanometer, and as the other terminal is connected (through lever A and a) with the cable, the circuit is complete. The short-circuit key of the galvanometer is now depressed, and the deflection noted in the usual manner (p. 332). As soon as the observations are completed the short-circuit key of the galvanometer is raised, and lever D being pressed over to the left the battery becomes disconnected from the galvanometer terminal and the latter is connected to earth, so that the cable discharges itself.

Particular care must be taken that the short-circuit key of

the galvanometer is raised before lever D is pressed over to the left, otherwise the whole discharge from the cable will pass through the galvanometer coils, and the needles may either be demagnetised, or at least the "constant" of the instrument be altered.

508. The battery power with which the "Insulation" test is taken is much greater than that required for the "Inductive Capacity" test, consequently after the latter test has been made (with about 10 Daniell cells usually), the battery power has to be changed to the required larger amount.

Method of testing Batteries in the Postal Telegraph Department.

509. The apparatus employed in the Postal Telegraph Department for battery testing is shown by Figs. 131 and 132. It consists of two sets of resistance coils R_1, R_2, the former being in the direct circuit of a tangent galvanometer* G, and the latter being a shunt between the terminals of the battery x when the shunt plug S is inserted. The values of the resistance coils A, B,

Fig. 131.

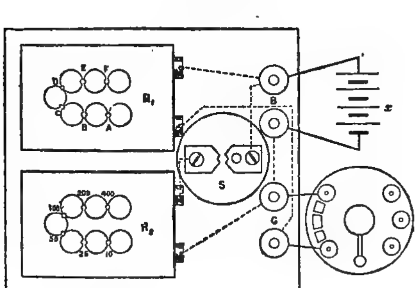
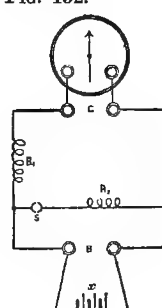


Fig. 132.



C, D, E and F, in R_1, are 1070, 3210, 4280, 8560, 17,120, and 34,240, ohms, respectively, that is A, B, C, D, E, and F are in the proportion of 1 : 3 : 4 : 8 : 16 : 32.

Test for Electromotive Force.

510. The principle of the method of testing for electromotive force is as follows:

If the standard cell (page 118) is joined up in circuit with the

* This galvanometer is the same as that employed for making the daily morning tests by received currents (page 451, Fig. 125).

tangent galvanometer, both plugs being out, then the deflection obtained is that due to an electromotive force of 1·070 volts (the approximate electromotive force of the standard cell) acting through a resistance of 1070 ohms. If, say, five Daniell cells, were in circuit, and also a total resistance of 5 × 1070 ohms, then the deflection obtained should be the same as that given by the standard cell, provided the total electromotive force of the five cells was five times that of the standard cell, or, in other words, if the average electromotive force per cell were 1·070 volts; and it is evident that if with a still larger number of cells there was placed in circuit a total resistance as many times greater than 1070 ohms as there are cells to be tested, then if the average electromotive force per cell of the battery were equal to the electromotive force of the standard cell, the deflection obtained would be the same as that given by the latter. If the deflection were less, it would show that the average electromotive force per cell of the battery must be proportionately less.

For example.

Suppose the standard cell gave a deflection of 25°, then, if, say, a 30-cell battery with a total resistance in circuit of 30 × 1070, or 32,100, ohms, gave a deflection of 22°, the average electromotive force per cell of the battery would be ·928 volts, thus,

$$1{\cdot}070 \times \frac{\tan 22^\circ}{\tan 25^\circ} = 1{\cdot}070 \times \frac{\cdot 404}{\cdot 466} = \cdot 928 \text{ volts.}$$

Now, if instead of the resistance in circuit being increased in exact proportion to the number of cells tested, it had been increased in a *less* proportion, then the deflection representing an electromotive force of 1·070 volts would be correspondingly higher.

For example.

If, when the 30 cells were tested there were in the circuit, not 30 × 1070 ohms, but 12 × 1070 ohms, then the deflection which would indicate that the average electromotive force per cell of the battery is 1·070 volts would be $49\frac{1}{2}^\circ$, thus,

$$\tan 25^\circ \times \frac{30}{12} = \cdot 466 \times \frac{5}{2} = 1{\cdot}165 = \tan 49\tfrac{1}{2}^\circ.$$

If, therefore, the *total* resistance in the circuit of the battery tested is made equal to

$$1070 \times \text{number of cells tested} \times \tfrac{2}{5}, \qquad [A].$$

and if 25° is the deflection given by the standard cell through a total resistance of 1070 ohms, then $49\frac{1}{2}°$ will be the deflection given by a battery whose average electromotive force per cell is 1·070 volts, and any deflection other than $49\frac{1}{2}°$ will (by proportion of the tangents of the deflections) represent the actual electromotive force per cell of the battery.

For example.

If the deflection obtained were 40°, then the electromotive force per cell of the battery would be ·767 volts, thus,

$$1 \cdot 070 \times \frac{\tan 40°}{\tan 49\frac{1}{2}°} = 1 \cdot 070 \times \frac{\cdot 839}{1 \cdot 171} = \cdot 767.$$

If the total resistance in the circuit of the battery tested is made equal to

$$1070 \times \text{number of cells tested} \times \tfrac{4}{5}, \qquad [B]$$

then the deflections obtained will represent average electromotive forces per cell which are double those which they represent when the resistance in circuit is that indicated by formula [A]. So that if formula [A] is applied when Daniell cells are tested, and formula [B] when Bichromates are tested, the range of deflections required in the two cases will be the same, since the electromotive force of a Bichromate battery is double that of a Daniell.

511. In order to facilitate calculation, tables constructed on the foregoing principles are employed; portions of these tables are as shown:—

TABLE I.

Number of Cells to be Tested.			Coils to be placed in Circuit in R_1.
Daniells.	Bichromates.	Leclanchés.	
5	..	3	A
10	5	6	B
..	..	8	C
15	..	10	A + C
20	10	12	B + C
25	..	16	A + D
30	15	18 and 20	B + D
35	..	..	A + C + D

Table II.

Observed Deflection (D°).	Equivalent Electromotive Force per Cell.					Percentage of Fall from Normal Electromotive Force.					Observed Deflection (D°).
	Number and Description of Cells Tested.					Number and Description of Cells Tested.					
	Daniells.	Bi-chromates.	Leclanchés.			Daniells.	Bi-chromates.	Leclanchés.			
	5 to 160.	5 to 80.	3, 6, 12, 18, 24, 36.	8, 16, 32, 40, 48.	10, 20, 30, 50, 60.	5 to 160.	5 to 80.	3, 6, 12, 18, 24, 36.	8, 16, 32, 40, 48.	10, 20, 30, 50, 60.	
49½°	1·070	2·140	..	..	1·612	0·00	0·00	..	..	..	49½°
..	..	..	..	..	1·600	..	..	..	..	0·00	..
49°	1·051	2·102	..	..	1·584	1·80	1·80	..	..	1·00	49°
48½°	1·033	2·066	..	1·621	1·556	3·46	3·46	..	..	2·75	48½°
..	..	..	..	1·600	..	..	..	..	0·00	..	..
48°	1·015	2·030	..	1·593	1·529	5·14	5·14	..	·44	4·44	48°
47½°	·996	1·992	..	1·565	1·502	6·92	6·92	..	2·19	6·12	47½°
47	·980	1·960	..	1·538	1·476	8·41	8·41	..	3·88	7·75	47°
46½°	·963	1·926	1·612	1·511	1·451	10·00	10·00	..	5·56	9·31	46½°
..	..	..	1·600	..	..	..	..	0·00	..	..	..
46°	·946	1·892	1·584	1·485	1·426	11·99	11·99	1·00	7·19	10·87	46°
45½°	·930	1·860	1·557	1·459	1.401	13·09	13·09	2·69	8·81	12·44	45½°
45°	·914	1·828	1·530	1·434	1·377	14·58	14·58	4·38	10·38	13·94	45°

The underlined figures show the Normal Forces of the Cells.

"Constant" with Standard Cell through Galvanometer with both plugs out to be 25°.

The way in which these tables would be used would be as follows:—

The 25° constant deflection having been obtained correctly, the standard cell is removed from terminals B, and the battery to be tested joined in its place, resistances having been previously inserted in resistance coils R_1, according to Table I. For example, if 35 Daniells are to be tested, the resistances to be inserted would be A, C, and D. The two plugs in the galvanometer must still remain out so that the resistance of the latter (1070 ohms) is included in the circuit.

The deflection obtained being now noted, the electromotive force per cell of the battery is given by Table II.; thus if the deflection is $45\frac{1}{2}°$, the electromotive force per cell is ·930, and the percentage of fall from the normal electromotive force is 13·09.

512. It will be observed that in the case of Leclanché batteries, the resistances to be placed in circuit and the deflections corresponding to the various electromotive forces, have to be taken in a somewhat different proportion from that adopted in the case of Daniell or Bichromate batteries, as the cells are made up in sets of 6, 8, and 10, and not in sets of 5, and moreover the normal electromotive force of a Leclanché is intermediate in value between a Daniell and a Bichromate battery; the general principle, however, upon which the resistances and deflections are arranged is similar to that adopted in the case of the latter batteries.

513. The accuracy of the method of testing electromotive force depends upon the resistance of the batteries being small in proportion to the external resistance, and this is attained by making the latter very large, so as to reduce the error beyond sensible limits.

Test for Internal Resistance.

514. This test is made by the "Diminished deflection shunt method" described in Chapter VI., page 114. The resistance R_1 being very high, the resistance of the battery is given by formula [G], page 116, in the test referred to, that is to say we have

$$x = R_2\left(\frac{\tan D°}{\tan d°} - 1\right).$$

For example.

If by the insertion of a shunt R_2 of 25 ohms, the deflection D° of $45\frac{1}{2}^\circ$ were reduced to 23° (d°), then resistance (x) of battery would be 35·0 ohms, thus,

$$x = 25\left(\frac{\tan 45\frac{1}{2}^\circ}{\tan 23^\circ} - 1\right) = 25\left(\frac{1\cdot018}{\cdot424} - 1\right) = 35\cdot0 \text{ ohms}.$$

515. To facilitate calculation, a table giving values of $\left(\frac{\tan D^\circ}{\tan d^\circ} - 1\right)$ for various values of D° and d°, is employed; hence it is only necessary to multiply the corresponding quantity by R_2, and the result is the total resistance of the battery.

516. In exceptional cases where an odd number of cells have to be tested for electromotive force, i. e. a number which is not included in Table I., the resistances inserted in R_1 are those corresponding to the number in the table next above the odd number; thus if 13 Bichromates are to be tested, the resistances corresponding to 15 cells, viz. B and D, are inserted in R_1. The deflection obtained having been noted, the result corresponding to that deflection in Table II. is multiplied by the even number of cells and divided by the odd number, the result being the electromotive force per cell of the battery.

517. It may be remarked that the *range* of the apparatus is considerable, it being possible to test from 5 to 160 Daniell cells, or 5 to 80 Bichromate cells, with an equal degree of accuracy, and with equal facility.

Combined Resistances.

518. Problem—Required the joint resistance of the resistances, a, b, c, d, and g, between the points A and B (Fig. 133).

If we call R the resistance of the combined resistances between the points A and B, then what we have to do is to obtain an equation of the form

$$c_5 = \frac{E}{r + R}.$$

Now it is obvious that the value of R can be in no way dependent upon the value of r, hence in order to simplify the problem we may assume r to be equal to 0.

By Kirchoff's laws (page 455) we have the following six equations, showing the connection between the resistances

a, b, c, d, and g, the current strengths c_1, c_2, c_3, c_4, c_5, and c_6, and the electromotive force E:—

$$c_5 - c_1 - c_2 = 0 \qquad [1]$$

$$c_4 - c_6 - c_1 = 0 \qquad [2]$$

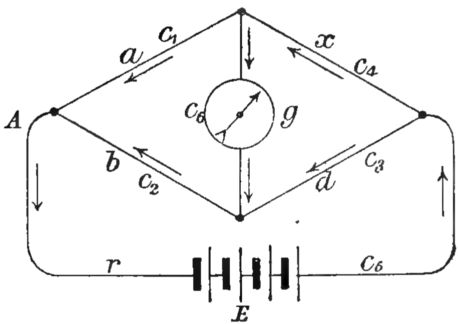
$$c_3 + c_6 - c_2 = 0 \qquad [3]$$

$$c_3 d + c_2 b - \mathrm{E} = 0 \qquad [4]$$

$$c_1 a - c_2 b - c_6 g = 0 \qquad [5]$$

$$c_3 d - c_4 x - c_6 g = 0. \qquad [6]$$

FIG 133.



In order to determine the value of c_5 from these six equations we must first find the value of c_1 from, say, equation [1], and substitute this value in the other equations, thereby getting rid of c_1; again in like manner, if we find the value of c_2 from, say, equation [3], and substitute throughout, we get rid of c_2, and so on. As it will be unnecessary to show all these substitutions, we shall confine ourselves to one or two only; thus from equation [1] we have

$$c_5 - c_1 - c_2 = 0, \quad \text{or,} \quad c_1 = c_5 - c_2;$$

therefore we get

$$c_4 - c_6 - c_5 + c_2 = 0 \qquad [2]$$

$$c_3 + c_6 - c_2 = 0 \qquad [3]$$

$$c_3 d + c_2 b - \mathrm{E} = 0 \qquad [4]$$

$$c_5 a - c_2 a - c_2 b - c_6 g = 0 \qquad [5]$$

$$c_3 d - c_4 x - c_6 g = 0. \qquad [6]$$

By continuing this process, we at length get

$$c_5 a - c_6 a - c_6 b - c_6 g - (a + b)\frac{E - c_6 b}{b + d} = 0$$

and

$$c_5 x + c_6 g - (d + x)\frac{E - c_6 b}{b + d} = 0;$$

therefore

$$c_6 (a d + b d + b g + d g) = c_5 (a b + a d) - E (a + b)$$

and

$$c_6 (b g + d g + b x + b d) = - c_5 (b x + d x) + E (d + x).$$

By dividing one equation by the other, c_6 is eliminated, that is we get

$$\frac{a d + b d + b g + d g}{b g + d g + b x + b d} = \frac{c_5 (a b + a d) - E (a + b)}{- c_5 (b x + d x) + E (d + x)},$$

or

$$c_5 = \frac{E}{\dfrac{(a b + a d)(b g + d g + b x + b d) + (b x + d x)(a d + b d + b g + d g)}{(d + x)(a d + b d + b g + d g) + (a + b)(b g + d g + b x + b d)}}.$$

By dividing the numerator and denominator of the fraction below the thick line by $a + x$, we finally get

$$c_5 = \frac{E}{\dfrac{g[(a + x)(b + d)] + a b (d + x) + d x (a + b)}{g[(a + x) + (b + d)] + (a + b)(d + x)}};$$

that is to say,

$$\left.\begin{array}{c}\text{The combined resistance of the resistances, } a, b, c, d, x, \\ \text{and } g\text{, between A and B}\end{array}\right\} =$$

$$\frac{g[(a + x)(b + d)] + a b (d + x) + d x (a + b)}{g[(a + x) + (b + d)] + (a + b)(d + x)}.$$

It will be observed that if $g = \infty$, that is to say if we remove g, then we get

$$\left.\begin{array}{l}\text{Combined}\\ \text{resistance}\end{array}\right\} = \frac{g[(a + x)(b + d)]}{g[(a + x) + (b + d)]} = \frac{(a + x)(b + d)}{(a + x) + (b + d)}$$

which is the joint resistance of $(a + x)$ and $(b + d)$.

If we have $g = 0$, that is to say if we join together the two points connected by g, then we get

$$\left.\begin{array}{l}\text{Combined}\\\text{resistance}\end{array}\right\} = \frac{a\,b\,(d+x) + d\,x\,(a+b)}{(a+b)\,(d+x)} = \frac{a\,b}{a+b} + \frac{d\,x}{d+x},$$

which is the joint resistance of a and b, added to the joint resistance of d and x.

The truth of these simplifications is obvious.

Combined Condensers.

519. Problem—Required the joint electrostatic capacity of two or more condensers joined up in "cascade."

Let a, b, and c, f, Fig. 134, be the plates of the two condensers, then if we suppose these plates to be of equal size, and d_1, and d_2 to be the distances separating them, the respective capacities C_1 and C_2 will be in the proportion

$$C_1 : C_2 :: d_2 : d_1,$$

or

$$\frac{d_2}{C_1\,d_1} = \frac{1}{C_2}.$$

Now the plates b and c, being joined together, may be considered to be one plate as shown by the dotted line $b\,c$, Fig. 135; moreover as the latter plate is in no way connected with either

Fig. 134.

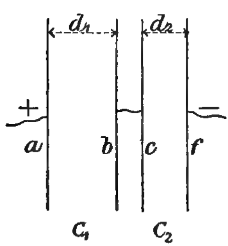
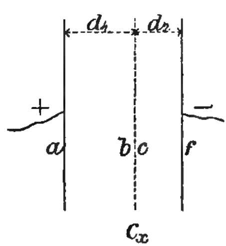


Fig. 135.



of the charging wires + and −, it practically does not affect the joint capacity of the arrangement; hence we can represent this joint capacity as being due to a condenser formed of the plates a and f, separated by a distance $d_1 + d_2$. The capacity

C_x of the combination must therefore be given by the proportion

$$C_x : C_1 : : d_1 : d_1 + d_2,$$

or

$$C_x = \frac{C_1\,d_1}{d_1 + d_2} = \frac{1}{\frac{1}{C_1} + \frac{d_2}{C_1\,d_1}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}.$$

If we had a third condenser of a capacity C_3, in the circuit of C_1 and C_2, then the joint capacity $C'x$, of this condenser in combination with C_x must be

$$C'_x = \frac{1}{\frac{1}{C_x} + \frac{1}{C_3}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

and so on with any number of condensers. Hence we have the law :—

The joint electrostatic capacity of any number of condensers joined together in "cascade" is equal to the reciprocal of the sum of the reciprocals of their respective capacities.

TABLES.

TABLE I.—NATURAL TANGENTS.

Degrees.	Tangents.	Degrees.	Tangents.	Degrees.	Tangents.	Degrees.	Tangents.	Degrees.	Tangents.	Degrees.	Tangents.
1·00	·0175	16·00	·2867	31·00	·6009	46·00	1·0355	61·00	1·8040	76·00	4·0108
1·25	·0218	16·25	·2915	31·25	·6068	46·25	1·0446	61·25	1·8228	76·25	4·0867
1·50	·0262	16·50	·2962	31·50	·6128	46·50	1·0538	61·50	1·8418	76·50	4·1653
1·75	·0306	16·75	·3010	31·75	·6188	46·75	1·0630	61·75	1·8611	76·75	4·2468
2·00	·0349	17·00	·3057	32·00	·6249	47·00	1·0724	62·00	1·8807	77·00	4·3315
2·25	·0393	17·25	·3105	32·25	·6310	47·25	1·0818	62·25	1·9007	77·25	4·4194
2·50	·0437	17·50	·3153	32·50	·6371	47·50	1·0913	62·50	1·9210	77·50	4·5107
2·75	·0480	17·75	·3201	32·75	·6432	47·75	1·1009	62·75	1·9416	77·75	4·6057
3·00	·0524	18·00	·3249	33·00	·6494	48·00	1·1106	63·00	1·9626	78·00	4·7046
3·25	·0568	18·25	·3298	33·25	·6556	48·25	1·1204	63·25	1·9840	78·25	4·8077
3·50	·0612	18·50	·3346	33·50	·6619	48·50	1·1303	63·50	2·0057	78·50	4·9152
3·75	·0655	18·75	·3395	33·75	·6682	48·75	1·1403	63·75	2·0278	78·75	5·0273
4·00	·0699	19·00	·3443	34·00	·6745	49·00	1·1504	64·00	2·0503	79·00	5·1446
4·25	·0743	19·25	·3492	34·25	·6809	49·25	1·1606	64·25	2·0732	79·25	5·2672
4·50	·0787	19·50	·3541	34·50	·6873	49·50	1·1708	64·50	2·0965	79·50	5·3955
4·75	·0831	19·75	·3590	34·75	·6937	49·75	1·1812	64·75	2·1203	79·75	5·5301
5·00	·0875	20·00	·3640	35·00	·7002	50·00	1·1918	65·00	2·1445	80·00	5·6713
5·25	·0919	20·25	·3689	35·25	·7067	50·25	1·2024	65·25	2·1692	80·25	5·8197
5·50	·0963	20·50	·3739	35·50	·7133	50·50	1·2131	65·50	2·1943	80·50	5·9758
5·75	·1007	20·75	·3789	35·75	·7199	50·75	1·2239	65·75	2·2199	80·75	6·1402
6·00	·1051	21·00	·3839	36·00	·7265	51·00	1·2349	66·00	2·2460	81·00	6·3138
6·25	·1095	21·25	·3889	36·25	·7332	51·25	1·2460	66·25	2·2727	81·25	6·4971
6·50	·1139	21·50	·3939	36·50	·7400	51·50	1·2571	66·50	2·2998	81·50	6·6912
6·75	·1184	21·75	·3990	36·75	·7467	51·75	1·2685	66·75	2·3276	81·75	6·8969
7·00	·1228	22·00	·4040	37·00	·7536	52·00	1·2799	67·00	2·3559	82·00	7·1154
7·25	·1272	22·25	·4091	37·25	·7604	52·25	1·2915	67·25	2·3850	82·25	7·3479
7·50	·1317	22·50	·4142	37·50	·7673	52·50	1·3032	67·50	2·4142	82·50	7·5958
7·75	·1361	22·75	·4193	37·75	·7743	52·75	1·3151	67·75	2·4443	82·75	7·8606

8·00	·1405	23·00	·4245	38·00	·7813	53·00	1·3270	68·00	2·4751	83·00	8·1443
8·25	·1450	23·25	·4296	38·25	·7883	53·25	1·3392	68·25	2·5065	83·25	8·4490
8·50	·1495	23·50	·4348	38·50	·7954	53·50	1·3514	68·50	2·5386	83 50	8·7769
8·75	·1539	23·75	·4400	38·75	·8026	53·75	1·3638	68·75	2·5715	83·75	9·1309
9·00	·1584	24·00	·4452	39·00	·8098	54·00	1·3764	69·00	2·6051	84·00	9·5144
9·25	·1629	24·25	·4505	39·25	·8170	54·25	1·3891	69·25	2·6395	84·25	9·9310
9·50	·1673	24·50	·4557	39·50	·8243	54·50	1·4019	69·50	2·6746	84·50	10·3854
9·75	·1718	24·75	·4610	39·75	·8317	54·75	1·4150	69·75	2·7100	84·75	10·8829
10·00	·1763	25·00	4663	40·00	·8391	55·00	1·4281	70·00	2·7475	85·00	11·4301
10·25	·1808	25·25	·4716	40·25	·8466	55·25	1·4415	70·25	2·7852	85·25	12 0346
10·50	·1853	25·50	·4770	40·50	·8541	55·50	1·4551	70·50	2·8239	85·50	12·7062
10·75	·1899	25·75	·4823	40·75	·8617	55·75	1·4687	70·75	2·8636	85·75	13·4566
11·00	·1944	26·00	·4877	41·00	·8693	56·00	1·4826	71·00	2·9042	86·00	14·3007
11·25	·1989	26·25	·4931	41·25	·8770	56·25	1·4966	71·25	2·9460	86·25	15·2571
11·50	·2035	26·50	·4986	41·50	·8847	56·50	1·5108	71·50	2·9887	86·50	16·3499
11·75	·2080	26·75	·5040	41·75	·8925	56·75	1·5253	71·75	3·0326	86·75	17·6106
12·00	·2126	27·00	·5095	42·00	·9004	57·00	1·5399	72·00	3·0777	87·00	19·0811
12·25	·2171	27·25	·5150	42·25	·9083	57·25	1·5547	72·25	3·1240	87·25	20·8188
12·50	·2217	27·50	·5206	42·50	·9163	57·50	1·5697	72·50	3·1716	87·50	22·9038
12·75	·2263	27·75	·5261	42·75	·9243	57·75	1·5849	72·75	3·2205	87·75	25·4517
13·00	·2309	28·00	·5317	43·00	·9325	58·00	1·6003	73 00	3·2709	88·00	28·6363
13·25	·2355	28·25	·5373	43·25	·9407	58·25	1·6160	73·25	3·3226	88·25	32·7303
13·50	·2401	28·50	·5430	43·50	·9490	58·50	1·6319	73·50	3·3759	88·50	38·1885
13·75	·2447	28·75	·5486	43·75	·9573	58·75	1·6479	73·75	3 4308	88·75	45·8294
14·00	·2493	29·00	·5543	44·00	·9657	59·00	1·6643	74·00	3·4874	89·00	57·2900
14·25	·2540	29·25	·5600	44·25	·9742	59·25	1·6808	74·25	3·5457	89·25	76·3900
14·50	·2586	29·50	·5658	44·50	·9827	59·50	1·6977	74·50	3·6059	89·50	114·5887
14·75	·2633	29·75	·5715	44·75	·9913	59·75	1·7147	74·75	3·6680	89·75	229·1817
15·00	·2679	30·00	·5774	45·00	1·0000	60·00	1·7321	75·00	3·7321	90·00	∞
15·25	·2726	30·25	·5832	45·25	1·0088	60·25	1·7450	75·25	3·7983		
15·50	·2773	30·50	·5890	45·50	1·0176	60·50	1·7675	75·50	3·8667		
15·75	·2820	30·75	·5949	45·75	1·0265	60·75	1·7856	75·75	3·9375		

TABLE II.*—RESISTANCE OF A KNOT-POUND of COPPER WIRE of various CONDUCTIVITIES, at 75° FAHR.

Percentage of Conductivity.	Resistance.	Percentage of Conductivity.	Resistance.	Percentage of Conductivity.	Resistance.	Percentage of Conductivity.	Resistance.
100·0	1196·7	97·5	1227·4	95·0	1259·7	92·5	1293·4
99·9	1197·9	97·4	1228·6	94·9	1261·0	92·4	1294·8
99·8	1199·1	97·3	1229·9	94·8	1262·4	92·3	1296·1
99·7	1200·3	97·2	1231·2	94·7	1263·7	92·2	1297·4
99·6	1201·5	97·1	1232·5	94·6	1264·0	92·1	1298·8
99·5	1202·7	97·0	1233·7	94·5	1266·4	92·0	1300·1
99·4	1203·9	96·9	1235·0	94·4	1267·7	91·9	1301·6
99·3	1205·1	96·8	1236·2	94·3	1269·1	91·8	1303·1
99·2	1206·4	96·7	1237·5	94·2	1270·4	91·7	1304·6
99·1	1207·6	96·6	1238·8	94·1	1271·8	91·6	1306·1
99·0	1208·8	96·5	1240·1	94·0	1273·1	91·5	1307·6
98·9	1210·0	96·4	1241·4	93·9	1274·5	91·4	1309·1
98·8	1211·2	96·3	1242·7	93·8	1275·8	91·3	1310·6
98·7	1212·5	96·2	1244·0	93·7	1277·2	91·2	1312·1
98·6	1213·7	96·1	1245·3	93·6	1278·6	91·1	1314·6
98·5	1214·9	96·0	1246·6	93·5	1280·0	91·0	1315·1
98·4	1216·2	95·9	1247·9	93·4	1281·3	90·9	1316·5
98·3	1217·3	95·8	1249·2	93·3	1282·7	90·8	1318·0
98·2	1218·6	95·7	1250·5	93·2	1284·0	90·7	1319·4
98·1	1219·9	95·6	1251·8	93·1	1285·4	90·6	1320·9
98·0	1221·1	95·5	1253·1	93·0	1286·8	90·5	1322·4
97·9	1222·4	95·4	1254·4	92·9	1288·1	90·4	1323·8
97·8	1223·6	95·3	1255·7	92·8	1289·4	90·3	1325·3
97·7	1224·9	95·2	1257·0	92·7	1290·8	90·2	1326·8
97·6	1226·1	95·1	1258·4	92·6	1292·1	90·1	1328·2

Resistance of "statute-mile-pound" equals resistance of "knot-pound" multiplied by ·752422. log ·752422 = $\bar{1}$·8764614.

* See p. 368 (§ 443).

TABLE III.*—SHOWING THE RELATIVE DIMENSIONS, LENGTHS, RESISTANCES (AT 60° FAHR.), AND WEIGHTS, OF PURE COPPER WIRE.

B.W.G.	Diameter.		Area.		Length and Weight.						Length and Resistance.						Resistance and Weight.		B.W.G.
No.	Mils.	Millimetres.	Square Inches.	Square Millimetres.	Pounds per Foot.	Pounds per Yard.	Pounds per Mile.	Feet per Pound.	Yards per Pound.	Miles per Pound.	Feet per Ohm.	Yards per Ohm.	Miles per Ohm.	Ohms per Foot.	Ohms per Yard.	Ohms per Mile.	Ohms per Pound.	Pounds per Ohm.	No.
0000	454	11·53	·1619	10·44	·6239	1·872	3294	1·603	·5343	·0003036	1997	6656	3·782	·00005008	·0001503	·2644	·00008017	12460	0000
000	425	10·80	·1419	9·152	·5468	1·640	2887	1·829	·6097	·0003464	1750	5832	3·314	·00005715	·0001715	·3018	·0001046	9567	000
00	380	9·652	·1134	7·317	·4371	1·311	2308	2·288	·7626	·0004333	1399	4663	2·649	·00007149	·0002145	·3775	·0001636	6114	00
0	340	8·636	·09079	5·857	·3499	1·050	1848	2·858	·9526	·0005412	1120	3733	2·121	·00008930	·0002679	·4715	·0002552	3919	0
1	300	7·620	·07069	4·560	·2724	·8173	1438	3·671	1·224	·0006952	8718	2906	1·651	·0001147	·0003441	·6056	·0004210	2375	1
2	284	7·213	·06335	4·087	·2442	·7325	1289	4·096	1·365	·0007757	7814	2605	1·480	·0001280	·0003840	·6758	·0005242	1908	2
3	259	6·578	·05269	3·399	·2031	·6092	1072	4·925	1·642	·0009327	6498	2166	1·231	·0001539	·0004617	·8125	·0007579	1320	3
4	238	6·045	·04449	2·870	·1715	·5144	905·3	5·832	1·944	·001105	5487	1829	1·039	·0001822	·0005467	·9623	·001063	940·8	4
5	220	5·588	·03801	2·452	·1465	·4395	773·6	6·826	2·275	·001293	4689	1563	·8880	·0002133	·0006399	1·126	·001456	686·9	5
6	203	5·156	·03237	2·088	·1247	·3742	658·6	8·017	2·672	·001518	3992	1331	·7560	·0002506	·0007515	1·323	·002008	498·0	6
7	180	4·572	·02545	1·642	·09808	·2942	517·8	10·20	3·399	·001931	3139	1046	·5944	·0003185	·0009558	1·682	·003149	317·8	7
8	165	4·191	·02138	1·379	·08241	·2472	435·1	12·13	4·045	·002298	2637	879·1	·4995	·0003792	·001138	2·002	·004601	217·3	8
9	148	3·759	·01720	1·110	·06631	·1989	350·1	15·08	5·027	·002856	2122	707·3	·4019	·0004715	·001414	2·488	·007108	140·7	9
10	134	3·404	·01410	·9098	·05436	·1631	287·0	18·40	6·133	·003485	1739	579·8	·3294	·0005749	·001725	3·036	·01058	94·54	10
11	120	3·048	·01131	·7296	·04359	·1308	230·2	22·94	7·647	·004345	1394	465·0	·2642	·0007169	·002151	3·785	·01645	60·80	11
12	109	2·770	·009331	·6020	·03596	·1079	189·9	27·81	9·268	·005266	1151	383·6	·2180	·0008689	·002607	4·588	·02416	41·39	12
13	95·0	2·413	·007088	·4571	·02732	·08196	144·2	36·60	12·20	·006933	874·3	291·4	·1656	·001144	·003431	6·039	·04187	23·88	13
14	83·0	2·108	·005411	·3491	·02085	·06256	110·1	47·95	15·98	·009082	667·3	222·4	·1264	·001498	·004495	7·912	·07186	13·92	14
15	72·0	1·829	·004072	·2486	·01569	·04708	82·86	63·73	21·24	·01207	502·2	167·4	·09511	·001991	·005974	10·51	·1268	7·887	15
16	65·0	1·651	·003318	·2141	·01279	·03837	67·53	78·19	26·06	·01481	409·3	136·4	·07751	·002443	·007330	12·90	·1910	5·234	16
17	58·0	1·473	·002642	·1705	·01018	·03055	53·77	98·20	32·73	·01859	325·9	108·6	·06171	·003069	·009200	16·20	·3014	3·318	17
18	49·0	1·245	·001886	·1217	·007268	·02180	38·37	137·6	45·86	·02606	232·6	77·53	·04405	·004300	·01290	22·70	·5916	1·690	18
19	42·0	1·067	·001385	·08940	·005340	·01602	28·19	187·3	62·43	·03547	170·9	56·96	·03236	·005852	·01756	30·90	1·096	·9124	19
20	35·0	·8890	·0009621	·06207	·003708	·01112	19·58	269·7	89·89	·05108	149·4	49·80	·02248	·008427	·02528	44·49	2·273	·4400	20
21	32·0	·8128	·0008043	·05188	·003100	·009299	16·37	322·6	107·6	·06110	99·20	33·07	·01879	·01008	·03024	53·23	3·252	·3075	21
22	28·0	·7112	·0006158	·03972	·002373	·007119	12·53	421·4	140·5	·07981	75·95	25·32	·01438	·01317	·03950	69·52	5·548	·1802	22
23	25·0	·6350	·0004909	·03167	·001892	·005676	9·989	528·6	176·2	·1001	60·54	20·18	·01147	·01652	·04955	87·21	8·730	·1145	23
24	22·0	·5588	·0003801	·02452	·001465	·004395	7·736	682·6	227·5	·1293	46·89	15·63	·008880	·02133	·06399	112·6	14·56	·06869	24
25	20·0	·5080	·0003142	·02027	·001211	·003632	6·393	825·9	275·3	·1564	38·75	12·92	·007339	·02581	·07742	136·3	21·31	·04692	25
26	18·0	·4571	·0002545	·01642	·0009808	·002942	5·178	1020	339·9	·1931	31·39	10·46	·005944	·03186	·09558	168·3	32·49	·03078	26
27	16·0	·4064	·0002011	·01297	·0007749	·002325	4·092	1290	430·1	·2444	24·80	8·266	·004697	·04032	·1210	212·9	52·04	·01922	27
28	14·0	·3556	·0001539	·00993	·0005933	·001780	3·133	1685	561·8	·3192	18·99	6·329	·003596	·05267	·1580	278·1	88·77	·01127	28
29	13·0	·3302	·0001327	·00856	·0005116	·001535	2·701	1955	651·6	·3702	16·37	5·457	·003101	·06108	·1833	322·5	119·4	·008375	29
30	12·0	·3048	·0001131	·007296	·0004359	·001308	2·302	2294	764·7	·4345	13·95	4·650	·002642	·07169	·2151	378·5	164·5	·006080	30

PURE COPPER weighs 555 lb per cubic foot. The resistance of one mil-foot at 60° Fahr. is, according to Dr. Matthiessen, 10·323 ohms. Upon these data the above Table has been calculated.

The *resistance* of Copper varies with the temperature at about 0·38 per cent. per degree Centigrade, or 0·21 per cent. per degree Fahrenheit.

STRANDED WIRES.—With a conductor of *stranded* wires of a definite length the total *weight* is *greater*, and the *resistance* is *less*, than with a similar number and equal length of wires *not* stranded.

To convert:
- Inches to millimetres, multiply by 25·3994.
- Feet to metres, " ·3048.
- Yards to metres, " ·9144.
- Miles to kilometres, " ·6214.
- Pounds to kilogrammes, " ·45359.

* This Table is abbreviated from one compiled by Messrs. W. T. Glover, wire makers, of Manchester, and is inserted by permission.

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TABLE IV.*—COEFFICIENTS for correcting the OBSERVED RESISTANCE of PURE COPPER WIRE at any TEMPERATURE to 75° FAHR.

Temperature in Degrees Fahr.	Coefficient.	Temperature in Degrees Fahr.	Coefficient.	Temperature in Degrees Fahr.	Coefficient.	Temperature in Degrees Fahr.	Coefficient.
100	·9484	83	·9832	66	1·0193	49	1·0567
99·5	·9494	82·5	·9842	65·5	1·0204	48·5	1·0578
99	·9504	82	·9853	65	1·0214	48	1·0589
98·5	·9514	81·5	·9863	64·5	1·0225	47·5	1·0601
98	·9524	81	·9874	64	1·0236	47	1·0612
97·5	·9534	80·5	·9884	63·5	1·0247	46·5	1·0623
97	·9544	80	·9895	63	1·0258	46	1·0634
96·5	·9554	79·5	·9905	62·5	1·0269	45·5	1·0646
96	·9564	79	·9916	62	1·0280	45	1·0657
95·5	·9575	78·5	·9926	61·5	1·0290	44·5	1·0668
95	·9585	78	·9937	61	1·0301	44	1·0679
94·5	·9595	77·5	·9947	60·5	1·0312	43·5	1·0690
94	·9605	77	·9958	60	1·0323	43	1·0702
93·5	·9615	76·5	·9968	59·5	1·0334	42·5	1·0714
93	·9626	76	·9979	59	1·0345	42	1·0725
92·5	·9636	75·5	·9990	58·5	1·0356	41·5	1·0736
92	·9646	75	1·0000	58	1·0367	41	1·0748
91·5	·9656	74·5	1·0011	57·5	1·0378	40·5	1·0759
91	·9666	74	1·0021	57	1·0389	40	1·0771
90·5	·9677	73·5	1·0032	56·5	1·0400	39·5	1 0782
90	·9687	73	1·0042	56	1·0411	39	1·0793
89·5	·9697	72·5	1·0053	55·5	1·0422	38·5	1·0804
89	·9708	72	1·0064	55	1·0433	38	1·0816
88·5	·9718	71·5	1·0074	54·5	1·0444	37·5	1·0828
88	·9728	71	1·0085	54	1·0455	37	1·0839
87·5	·9738	70·5	1 0096	53·5	1·0466	36·5	1·0851
87	·9749	70	1·0106	53	1·0478	36	1·0862
86·5	·9759	69·5	1·0117	52·5	1·0489	35·5	1·0873
86	·9769	69	1·0128	52	1·0500	35	1·0885
85·5	·9780	68·5	1·0139	51·5	1·0511	34·5	1·0896
85	·9790	68	1·0149	51	1·0522	34	1·0908
84·5	·9801	67·5	1·0160	50·5	1·0533	33·5	1·0920
84	·9811	67	1·0171	50	1·0544	33	1·0932
83·5	·9821	66·5	1·0182	49·5	1·0556	32·5	1·0943

See p. 373.

TABLE IV.*—COEFFICIENTS for correcting the OBSERVED RESISTANCE of PURE COPPER WIRE at any TEMPERATURE to 75° FAHR.

Temperature in Degrees Fahr.	Coefficient.	Temperature in Degrees Fahr.	Coefficient.	Temperature in Degrees Fahr.	Coefficient.	Temperature in Degrees Fahr.	Coefficient.
100	·9484	83	·9832	66	1·0193	49	1·0567
99·5	·9494	82·5	·9842	65·5	1·0204	48·5	1·0578
99	·9504	82	·9853	65	1·0214	48	1·0589
98·5	·9514	81·5	·9863	64·5	1·0225	47·5	1·0601
98	·9524	81	·9874	64	1·0236	47	1·0612
97·5	·9534	80·5	·9884	63·5	1·0247	46·5	1·0623
97	·9544	80	·9895	63	1·0258	46	1·0634
96·5	·9554	79·5	·9905	62·5	1·0269	45·5	1·0646
96	·9564	79	·9916	62	1·0280	45	1·0657
95·5	·9575	78·5	·9926	61·5	1·0290	44·5	1·0668
95	·9585	78	·9937	61	1·0301	44	1·0679
94·5	·9595	77·5	·9947	60·5	1·0312	43·5	1·0690
94	·9605	77	·9958	60	1·0323	43	1·0702
93·5	·9615	76·5	·9968	59·5	1·0334	42·5	1·0714
93	·9626	76	·9979	59	1·0345	42	1·0725
92·5	·9636	75·5	·9990	58·5	1·0356	41·5	1·0736
92	·9646	75	1·0000	58	1·0367	41	1·0748
91·5	·9656	74·5	1·0011	57·5	1·0378	40·5	1·0759
91	·9666	74	1·0021	57	1·0389	40	1·0771
90·5	·9677	73·5	1·0032	56·5	1·0400	39·5	1 0782
90	·9687	73	1·0042	56	1·0411	39	1·0793
89·5	·9697	72·5	1·0053	55·5	1·0422	38·5	1·0804
89	·9708	72	1·0064	55	1·0433	38	1·0816
88·5	·9718	71·5	1·0074	54·5	1·0444	37·5	1·0828
88	·9728	71	1·0085	54	1·0455	37	1·0839
87·5	·9738	70·5	1 0096	53·5	1·0466	36·5	1·0851
87	·9749	70	1·0106	53	1·0478	36	1·0862
86·5	·9759	69·5	1·0117	52·5	1·0489	35·5	1·0873
86	·9769	69	1·0128	52	1·0500	35	1·0885
85·5	·9780	68·5	1·0139	51·5	1·0511	34·5	1·0896
85	·9790	68	1·0149	51	1·0522	34	1·0908
84·5	·9801	67·5	1·0160	50·5	1·0533	33·5	1·0920
84	·9811	67	1·0171	50	1·0544	33	1·0932
83·5	·9821	66·5	1·0182	49·5	1·0556	32·5	1·0943

* See p. 373.

TABLE V.*—LOGARITHM COEFFICIENTS for correcting the OBSERVED RESISTANCE of "SILVERTOWN" GUTTA-PERCHA at any TEMPERATURE to 75° FAHR.

Temp.	Logarithm.	Temp.	Logarithm.	Temp.	Logarithm.	Temp.	Logarithm.
100	$\bar{1}$·1744650	82·5	$\bar{1}$·7523395	65	0·3302140	47·5	0·9080885
99·5	·1909757	82	·7688502	64·5	·3467247	47	·9245992
99	·2074864	81·5	·7853609	64	·3632354	46·5	·9411099
98·5	·2239971	81	·8018716	63·5	·3797461	46	·9576206
98	·2405078	80·5	·8183823	63	·3962568	45·5	·9741313
97·5	·2570185	80	·8348930	62·5	·4127675	45	·9906420
97	·2735292	79·5	·8514037	62	·4292782	44·5	1·0071527
96·5	·2900399	79	·8679144	61·5	·4457889	44	·0236634
96	·3065506	78·5	·8844251	61	·4622996	43·5	·0401741
95·5	·3230613	78	·9009358	60·5	·4788103	43	·0566848
95	·3395720	77·5	·9174465	60	·4953210	42·5	·0731955
94·5	·3560827	77	·9339572	59·5	·5118317	42	·0897062
94	·3725934	76·5	·9504679	59	·5283424	41·5	·1062169
93·5	·3891041	76	·9669786	58·5	·5448531	41	·1227276
93	·4056148	75·5	·9834893	58	·5613638	40·5	·1392383
92·5	·4221255	75	0·0000000	57·5	·5778745	40	·1557490
92	·4386362	74·5	·0165107	57	·5943852	39·5	·1722597
91·5	·4551469	74	·0330214	56·5	·6108959	39	·1887704
91	·4716576	73·5	·0495321	56	·6274066	38·5	·2052811
90·5	·4881683	73	·0660428	55·5	·6439173	38	·2217918
90	·5046790	72·5	·0825535	55	·6604280	37·5	·2383025
89·5	·5211897	72	·0990642	54·5	·6769387	37	·2548132
89	·5277004	71·5	·1155749	54	·6934494	36 5	·2713239
88·5	·5542111	71	·1320856	53·5	·7099601	36	·2878346
88	·5707218	70·5	·1485963	53	·7264708	35·5	·3043453
87·5	·5872325	70	·1651070	52·5	·7429815	35	·3208560
87	·6037432	69·5	·1816177	52	·7594922	34·5	·3373667
86·5	·6202539	69	·1981284	51·5	·7760029	34	·3538774
86	·6367646	68·5	·2146391	51	·7925136	33·5	·3703881
85·5	·6532753	68	·2311498	50·5	·8090243	33	·3868938
85	·6697860	67·5	·2476605	50	·8255350	32·5	·4034095
84·5	·6862967	67	·2641712	49·5	·8420457	32	·4109202
84	·7028074	66·5	·2806819	49	·8585564		
83·5	·7193181	66	·2971926	48·5	·8750671		
83	·7358288	65·5	·3137033	48	·8915778		

* See p. 377.

TABLE VI.* — LOGARITHM COEFFICIENTS for correcting the OBSERVED RESISTANCE OF "WILLOUGHBY SMITH'S" GUTTA-PERCHA at any TEMPERATURE to 75° FAHR.

Temp.	Logarithm.	Temp.	Logarithm.	Temp.	Logarithm.	Temp.	Logarithm.
100	$\bar{1}$·2992893	82·5	$\bar{1}$·7897772	65	0·3362596	47·5	0·9246514
99·5	·3133241	82	·8037984	64·5	·3530159	47	·9414617
99	·3273589	81·5	·8178021	64	·3697723	46·5	·9582628
98·5	·3413094	81	·8318058	63·5	·3866180	46	·9750640
98	·3552599	80·5	·8458321	63	·4034637	45·5	·9918959
97·5	·3692683	80	·8589585	62·5	·4202899	45	1·0087279
97	·3832767	79·5	·8738785	62	·4371161	44·5	·0255335
96·5	·3973640	79	·8878985	61·5	·4539102	44	·0423392
96	·4114513	78·5	·9019128	61	·4707044	43·5	·0591617
95·5	·4259710	78	·9159272	60·5	·4875196	43	·0759842
95	·4394906	77·5	·9299283	60	·5043349	42·5	·0927813
94·5	·4534753	77	·9439395	59·5	·5211399	42	·1095785
94	·4674601	76·5	·9579554	59	·5379450	41·5	·1263903
93·5	·4814522	76	·9719713	58·5	·5292687	41	·1432022
93	·4955443	75·5	·9859856	58	·5715924	40·5	·1600083
92 5	·5095153	75	0·0000000	57·5	·5883948	40	·1768145
92	·5234863	74·5	·0167119	57	·6051973	39·5	·1936334
91·5	·5375291	74	·0334238	56·5	·6220428	39	·2104523
91	·5515720	73·5	·0502473	56	·6388884	38·5	·2272657
90 5	·5655635	73	·0670709	55·5	·6556773	38	·2440791
90	·5795550	72·5	·0838930	55	·6724673	37·5	·2608900
89·5	·5935586	72	·1007151	54·5	·6892937	37	·2777009
89	·6075622	71·5	·1126055	54	·7061201	36·5	·2945154
88·5	·6215732	71	·1344959	53·5	·7229253	36	·3113300
88	·6355843	70·5	·1513498	53	·7397305	35·5	·3281361
87·5	·6495950	70	·1682027	52·5	·7565390	35	·3449422
87	·6636067	69·5	·1849364	52	·7733475	34·5	·3617609
86·5	·6776534	69	·2016702	51·5	·7901705	34	·3785796
86	·6917002	68·5	·2184731	51	·8069935	33·5	·3953858
85·5	·7057228	68	·2352759	50·5	·8238007	33	·4121917
85	·7197455	67·5	·2521608	50	·8406079	32·5	·4289991
84·5	·7337340	67	·2690457	49·5	·8573998	32	·4458065
84	·7477225	66·5	·2857965	49	·8741918		
83·5	·7617392	66	·3025474	48·5	·8910164		
83	·7757560	65·5	·3194037	48	·9078411		

* See p. 377.

TABLE VII.*—Of the MULTIPLYING POWER of SHUNTS EMPLOYED with a GALVANOMETER of 6000 OHMS RESISTANCE.

Resistance of Shunt.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.	Resistance of Shunt.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.	Resistance of Shunt.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.
ohms.		ohms.	ohms.		ohms.	ohms.		ohms.
1	3·7782236	1·0	70	1·9380892	69·2	850	·9062704	744·5
2	3·4772660	2·0	75	1·9084850	74·1	900	·8846085	782·6
3	3·3012471	3·0	80	1·8808136	79·0	950	·8642618	818·3
4	3·1763807	4·0	85	1·8548402	83·8	1000	·8450980	857·2
5	3·0795430	5.0	90	1·8303769	88·7	1100	·8098626	929·6
6	3·0004341	6·0	95	1·8072508	93·5	1200	·7781513	1000·0
7	2·9335581	7·0	100	1·7853298	98·4	1300	·7493807	1068·5
8	2·8756399	8·0	110	1·7446450	108·0	1400	·7231107	1135·7
9	2·8245619	9·0	120	1·7075702	117·7	1500	·6989700	1200·0
10	2·7788745	10·0	130	1·6735185	127·2	1600	·6766936	1263·2
11	2·7375504	11·0	140	1·6420488	136·8	1700	·6560407	1324·7
12	2·6998377	12·0	150	1·6127839	146·3	1800	·6368188	1384·6
13	2·6651493	13·0	160	1·5854607	155·9	1900	·6188636	1443·1
14	2·6320441	14·0	170	1·5598348	165·3	2000	·6020600	1500·0
15	2·6031444	15·0	180	1·5357118	174·8	2200	·5713943	1609·7
16	2·5751878	16·0	190	1·5129244	184·2	2400	·5440680	1714·3
17	2·5489296	17·0	200	1·4913617	193·6	2600	·5195201	1814·0
18	2·5241753	17·9	220	1·4513719	212·2	2800	·4973306	1909·1
19	2·5007578	18·9	240	1·4149733	230·8	3000	·4771213	2000·0
20	2·4785665	19·9	260	1·3816024	249·2	3300	·4499718	2129·0
22	2·4373224	21·9	280	1·3508099	267·5	3600	·4259742	2250·0
24	2·3996737	23·9	300	1·3222193	285·7	4000	·3979400	2400·0
26	2·3650572	25·9	330	1·2828939	312·8	4300	·3793780	2504·8
28	2·3330239	27·9	360	1·2461628	340·4	4600	·3625579	2603·7
30	2·3031961	29·9	400	1·2041200	375·0	5000	·3424227	2727·3
33	2·2620237	32·8	430	1·1747574	401·2	5500	·3182929	2883·1
36	2·2244554	35·8	460	1·1461280	428·6	6000	·3010300	3000·0
40	2·1789769	39·7	500	1·1139434	461·5	6500	·2840019	3120·0
43	2·1477999	42·7	550	1·0722867	508·0	7000	·2688353	3230·8
46	2·1187276	45·6	600	1·0413927	545·5	7500	·2552725	3333·3
50	2·0827854	49·8	650	1·0131744	582·1	8000	·2430380	3428·6
55	2·0378646	54·5	700	·9809755	617·6	8500	·2319536	3517·2
60	2·0043214	59·5	750	·9542425	666·7	9000	·2218574	3600·0
65	1·9699189	64·3	800	·9294189	705·9	9500	·2126137	3677·4
						10000	·2041200	3750·0

* See p. 335 (§ 394).

TABLE VIII.*—Of the MULTIPLYING POWER of SHUNTS EMPLOYED with a GALVANOMETER of 10,000 OHMS RESISTANCE.

Resistance of Shunt.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.	Resistance of Shunt.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.	Resistance of Shunt.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.
ohms		ohms.	ohms.		ohms.	ohms.		ohms.
1	3·0000434	1·0	70	2·1579315	69·5	850	1·1056104	784·1
2	3·6990569	2·0	75	2·1281838	74·4	900	1·0831840	825·7
3	3·5230090	3·0	80	2·1003705	79·4	650	1·0636691	863·6
4	3·3981137	4·0	85	2·0742570	84·3	1000	1·0453230	900·9
5	3·3012471	5·0	90	2·0496487	89 2	1100	1·0078253	982·1
6	3·2221092	6·0	95	2·0264827	94·1	1200	·9738972	1061·7
7	3·1552059	7·0	100	2·0043214	99·0	1300	·9429615	1140·4
8	3·0972573	8·0	110	1·9633585	108·8	1400	·9107769	1228·1
9	3·0461482	9·0	120	1·9259993	118·6	1500	·8846065	1304·4
10	3·0004341	10·0	130	1·8916660	128·3	1600	·8603380	1379·3
11	2·9590848	11·0	140	1·8599100	138·1	1700	·8377370	1453·0
12	2·9213396	12·0	150	1·8303747	147·8	1800	·8166095	1525·4
13	2·8866208	13·0	160	1·8027737	157·5	1900	·7967934	1596·8
14	2·8544796	14·0	170	1·7768721	167·2	2000	·7781512	1666·7
15	2·8245597	15·0	180	1·7524753	176·8	2200	·7439371	1803·3
16	2·7965743	16·0	190	1·7294206	186·5	2400	·7132105	1935·5
17	2·7702888	17·0	200	1·7075702	196·1	2600	·6853972	2063·5
18	2·7455085	18·0	220	1·6670282	215·3	2800	·6600520	2187·5
19	2·7220708	19·0	240	1·6300888	234·4	3000	·6368221	2307·7
20	2 6998377	20·0	260	1·5961741	253·4	3300	·6053377	2481·2
22	2·6585137	22·0	280	1·5648351	272·4	3600	·5772364	2647·1
24	2·6208299	23·9	300	1·5357159	291·3	4000	·5440680	2857·1
26	2·5861544	25·9	330	1·4955864	319·5	4300	·5218675	3007·0
28	2·5540563	27·9	360	1·4590573	347·5	4600	·5015951	3150·7
30	2·5241796	29·9	400	1·4149733	384·6	5000	·4771213	3333·3
33	2·4829169	32·9	430	1·3848158	412·2	5500	·4499690	3548·4
36	2·4452582	35·9	460	1·3567739	439·8	6000	·4259687	3750·0
40	2·3996737	39·8	500	1·3222193	476·2	6500	·4045705	3939·4
43	2·3683950	42·8	550	1·2828898	521·3	7000	·3853509	4117·6
46	2·3392354	45·8	600	1·2471546	556·0	7500	·3679767	4285·7
50	2·3031961	49·8	650	1·2144362	610·3	8000	·3521825	4444·4
55	2·2620194	54·7	700	1·1842858	654·2	8500	·3377528	4594·6
60	2·2244467	59 6	750	1·1563472	697·7	9000	·3245111	4736·8
65	2·1899004	64·6	800	1·1303338	740·7	9500	·3123110	4871·8
						10000	·3010300	5000·0

* See p. 335 (§ 394).

TABLE IX.—NEW STANDARD WIRE GAUGE.†

No.	Diameters.			No.	Diameters.		
	Mils.*	Differences.	Millimetres.		Mils.*	Differences.	Millimetres.
0,000,000	500		12·70	23	24	4	·610
000,000	464	36	11·78	24	22	2	·559
00,000	432	32	10·97	25	20	2	·508
0,000	400	32	10·16	26	18	2	·457
000	372	28	9·45	27	16·4	1·6	·417
00	348	24	8·84	28	14·8	1·6	·376
0	324	24	8·23	29	13·6	1·2	·345
1	300	24	7·62	30	12·4	1·2	·315
2	276	24	7·01	31	11·6	·8	·295
3	252	24	6·40	32	10·8	·8	·274
4	232	20	5·89	33	10·0	·8	·254
5	212	20	5·38	34	9·2	·8	·234
6	192	20	4·88	35	8·4	·8	·213
7	176	16	4·47	36	7·6	·8	·193
8	160	16	4·06	37	6·8	·8	·173
9	144	16	3·66	38	6·0	·8	·152
10	128	16	3·25	39	5·2	·8	·132
11	116	12	2·95	40	4·8	·4	·122
12	104	12	2·64	41	4·4	·4	·112
13	92	12	2·34	42	4·0	·4	·102
14	80	12	2·03	43	3·6	·4	·0914
15	72	8	1·83	44	3·2	·4	·0813
16	64	8	1·63	45	2·8	·4	·0711
17	56	8	1·42	46	2·4	·4	·0610
18	48	8	1·22	47	2·0	·4	·0508
19	40	8	1·016	48	1·6	·4	·0406
20	36	4	·914	49	1·2	·4	·0305
21	32	4	·813	50	1·0	·2	·0254
22	28	4	·711				

* 1 Mil. = $\frac{1}{1000}$th of an inch.

† This gauge is now (March 1st, 1884) the only legal standard wire gauge for the United Kingdom.

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LONDON: PRINTED BY WILLIAM CLOWES AND SONS, LIMITED, STAMFORD STREET AND CHARING CROSS.